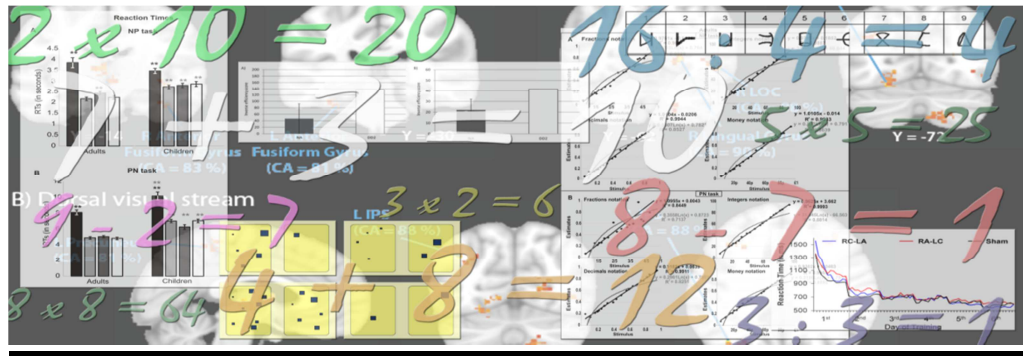


Good and bad at numbers: typical and atypical development of number processing and arithmetic



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Statement of Originality

I, Teresa Iuculano, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the thesis. Please note that the work presented in this thesis on symbol learning with brain stimulation (Chapter VIII) was a collaborative project led by Dr. Roi Cohen Kadosh.

Abstract

This thesis elucidates the heterogeneous nature of mathematical skills by examining numerical and arithmetical abilities in typical, atypical and exceptional populations. Moreover, it looks at the benefits of intervention for remediating and improving mathematical skills. First, we establish the nature of the ‘number sense’ and assess its contribution to typical and atypical arithmetical development. We confirmed that representing and manipulating numerosities approximately is fundamentally different from the ability to manipulate them exactly. Yet only the exact manipulation of numbers seems to be crucial for the development of arithmetic. These results lead to a better characterization of mathematical disabilities such as Developmental Dyscalculia and Low Numeracy. In the latter population we also investigated more general cognitive functions demonstrating how inhibition processes of working memory and stimulus-material interacted with arithmetical attainment. Furthermore, we examined areas of mathematics that are often difficult to grasp: the representation and processing of rational numbers. Using explicit mapping tasks we demonstrated that well-educated adults, but also typically developing 10 year olds and children with low numeracy have a comprehensive understanding of these types of numbers. We also investigated exceptional maths abilities in a population of children with Autism Spectrum Disorder (ASD) demonstrating that this condition is characterized by outstanding arithmetical skills and sophisticated calculation strategies, which are reflected in a fundamentally different pattern of brain activation. Ultimately we looked at remediation and learning. Targeted behavioural intervention was beneficial for children with low numeracy but not in Developmental Dyscalculia. Finally, we demonstrated that adults’ numerical performance can be enhanced by neural stimulation (tDCS) to dedicated areas of the brain. This work sheds light on the entire spectrum of mathematical skills from atypical to exceptional development and it is extremely relevant for the advancing of the field of mathematical cognition and the prospects of diagnosis, education and intervention.

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Publications

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Iuculano, T., Moro, R., Butterworth, B. (2011). Updating Working Memory and arithmetical attainment in school. *Learning and Individual Differences*

Cohen Kadosh, R., Soskic, S., Iuculano, T., Kanai, R., Walsh, V. (2010). Modulating Neuronal Activity Produces Specific and Long-Lasting Changes in Numerical Competence. *Current Biology*

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Invited Talks

Iuculano, T., Karolis, V., & Butterworth, B. 'Constructing the mental number line using rational numbers'. *28th Workshop on Cognitive Neuropsychology*. Bressanone, Italy. 2010

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Conference Abstracts

Iuculano, T., Rosenberg-Lee, M., Khouzam, A., Philips, J.M., Hardan, A., Uddin, L.Q., & Menon, V. (2011). Neural correlates of calculation abilities in children with autism. *Frontiers in Human Neuroscience*. Conference Abstract: *XI International Conference on Cognitive Neuroscience (ICON XI)*. doi: 10.3389/conf.fnhum.2011.207.00101

Iuculano, T., Moro, R., Butterworth, B. (2008). Arithmetical impairments related to domain-specific working memory updating. *Frontiers in Human Neuroscience*. Conference Abstract: *10th International Conference on Cognitive Neuroscience*. doi:10.3389/conf.neuro.09.2009.01.282

Poster Presentations

Iuculano, T., Dowrick, N., & Butterworth, B. Numeracy intervention in Developmental Dyscalculia and Low Numeracy: Preliminary results from a pilot study. *30th Workshop on Cognitive Neuropsychology*. Bressanone, Italy. 2012

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Individual Contribution to Research Chapters

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Chapter VI. ‘If’ and ‘how’ of exceptional maths abilities in Autism Spectrum Disorder

Please note that this study was performed during a 6-month Bogue Research Fellowship at Stanford University. The contribution therefore reflects the time constraints of the visit.

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Analysed data: T. Iuculano

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Supervised project: V. Menon

Chapter VII. Numeracy intervention in Developmental Dyscalculia and Low Numeracy

Please note that this study was carried out in the framework already developed by the UK government and Edge Hill University, the Department of Education and the Every Child a Chance Trust. Both the assessment methodology and the intervention were in place before the study began.

Contributing researchers: Dr. Nick Dowrick, Prof. Brian Butterworth

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Chapter VIII. Neural stimulation for enhancing numerical performance

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To grandma

“Whenever you can, count”
-- Sir Francis Galton

I

Introduction

*"Numbers are the highest degree of knowledge.
It is knowledge itself." -- Plato*

Numbers are one of the most pervasive stimulus categories in our environment and an integral foundation of modern society. The ability to understand and represent numbers (or their numerosities) has been thought to be innate and phylogenetically specified through neural mechanisms grounded in the intraparietal sulcus and the prefrontal cortex. A well anchored processing system for numbers (i.e. ‘the number sense’) is what is meant to derive the foundations of arithmetic. Over the past years, the field of numerical cognition has had a long lasting debate on how numerosities are cognitively represented: as approximate ‘activations’ on an analogue mental number line, or as exact discrete elements? The question that arises is how these abilities are linked to one another and ultimately to arithmetic. Moreover, what are the implications of these theories on the typical and atypical development of maths? Mathematical proficiency requires mastering of numerous skills from counting and enumeration all the way to higher level reasoning. Indeed, the development of maths abilities is very heterogeneous both within and between individuals. This leads to the issue of better defining the behavioural phenotype of maths abilities along the entire spectrum (i.e. from low to average up to exceptional attainers). Two models have so far been proposed to account for the variance in the population and for the specific impairment in maths. Yet, none of them seems to be able to account for the occurrence of both scenarios in the empirical data. In this Chapter, we will briefly introduce the different types of numerical properties and formats that can be manipulated by human (and sometimes non-human) cognition. We will subsequently lay down the different theories on numbers representation - from a behavioural as well as neuroimaging perspective - and the implications these have for inferring how we learn arithmetic in typical and atypical development. Finally, we will describe the diagnostic criteria for our experimental populations, and the methodological and theoretical framework which will be the basis of our investigations in subsequent chapters.

1.1. Introduction

‘*Mathematics*’ is a term that is derived from the Greek word (*máthēma*), which literally means *to learn*. The etymology of the word is interesting as it encompasses the idea that mathematics is a broad discipline which comprises many different aspects *to be learnt*. Indeed, the maths curriculum over the school years covers various areas of mathematics such as geometry, algebra, probability and statistics, analysis, and so on. Yet, when we refer to mathematics in the beginning of the school career we often refer to lessons aimed at mastering numbers and arithmetic, or otherwise termed *Numeracy*.

There seems to be some sort of hierarchy within the discipline of mathematics since unlike for other disciplines (i.e. reading), the next ability to be learnt usually builds upon the previous one. The best example of this hierarchical component of mathematics is indeed numbers and arithmetic. In order to be able to learn and solve arithmetical operations it is essential to be able to grasp the property of numbers (*see next paragraph*), or in other words, to be able to understand what numbers are and mean.

To understand a large and complex domain, scientists try to break it down into its component parts. Thus in this thesis I am only going to focus on the last described meaning of mathematics (i.e. numbers and arithmetic) and explore its heterogeneity in typical, atypical and exceptional development. Particularly, I will use a series of behavioural and cognitive approaches in order to investigate the nature of the representation of numbers in their various forms, and its possible ‘disruptions’. I will then explore arithmetical abilities in populations who show a superior ‘maths phenotype’ and finally I will look at possible remediation approaches in order to help learners who suffer from different maths impairments.

1.2. The heterogeneity of numbers

In our society number terms are used in many different ways. For example, it is very important to distinguish between their use to denote magnitudes, and specifically, the cardinality of sets, their use to indicate order in a sequence and their use as labels. In this thesis the focus will be on numbers as abstract properties of sets, for example the ability to characterize the number of candies in a bag, the number of books read for an exam, or the number of chapters in a thesis. We will call these cardinal magnitudes, *numerosities*. Cardinality is an abstract property of sets (i) the set denoted by the greater number includes the set denoted by the smaller one (e.g. a set of four will include a set of three); and (ii) two sets have the same *numerosity* (i.e. are exactly equal) when the members of one can be put in one-to-one correspondence with the members of the other (i.e. a set of mugs has the same numerosity as a set of teabags just in case there is one teabag for each cup). This means that adding a number to a set or subtracting a member from a set will affect the *numerosity* of that set (i.e. there is not going to be enough teabags for each mug). This is the type or meaning of ‘number’ that is relevant to arithmetic. However, there is some disagreement about how magnitudes are cognitively represented and how we come to have representations of exactly four, which in turn has consequences for the idea on how arithmetic develops (see **paragraph 1.3**, and also **Chapter II**).

Numbers are also used to order things – such as the pages of a thesis. Page 45 of this thesis does not have a larger magnitude than page 44, and does not include it. These types of numbers are called *ordinal* numbers, or rank (Nieder, 2005), and they are not ordered by magnitude though they correspond to the cardinals that are (i.e. the next number on the counting list is always greater than the previous one). It is important to note that counting a set of objects involves putting the objects into one-to-one correspondence with the ordered counting words (e.g. ‘one, two, three, four’) and such counting procedure can result both in a numerosity outcome – ‘cardinality principle’ (i.e. the number of objects in the set is four) or an ordinal outcome – ‘ordinality principle’ (i.e. ‘this is the fourth object’). Interestingly, children often confuse ordinal and cardinal, as it is evident in the following example reported by Gelman and Gallistel (1978).

“Experimenter (E): So how many are there?

Adam (A): [A grabber, counting three objects ...] One, two, five!

E: [Pointing towards the three items] So there’s five here?

A: No, that’s five [pointing to the item he’d tagged ‘five’] ...

E: What if you counted this way, one, two, five? [Experimenter counts the objects in a different order than Adam has been doing]

A: No, this is five [pointing to the one he has consistently tagged five]”

Numbers are also frequently used as *labels* for example bus numbers, telephone numbers, brands (e.g. Levis 501) and so on. In these use, neither the magnitude of the number nor its order is relevant.

In this thesis we will focus on cardinality, or numerosity, since this property of sets is the logical and ontogenetic foundation of arithmetic. Thus, the sum of an addition, for example, can be thought of as the numerosity of the union of two, or more, disjoint sets; similarly, subtraction, multiplication and division can be thought of in terms of the results of operations on sets (Giaquinto, 1995).

Even if we narrow down our investigation to the one property of numbers that is thought of as foundational for arithmetic, the heterogeneity of numbers does not entirely vanish. We can in fact have different types of cardinal numbers such as integers, decimals, fractions. Moreover, cardinality can be expressed in many different ways: *symbolically*, using Arabic digits and number words, or *non-symbolically*, for example as sets of objects like the dots on a dice.

One of the problems for the learner is to distinguish these types and uses of numbers. For example, it is important for the learner to understand the correct mapping between numerals (i.e. Arabic digits such as 2) and numerosities (i.e. sets of two elements). Moreover, it is important to be able to differentiate the meaning of a quantity expressed as fractions to the meaning of one expressed as integers. In this sense, it is crucial to understand that when dealing with fractions, the larger the denominator of the fraction, the smaller the magnitude that the fraction represents, while this logic does not apply to integers, where the larger the number the larger the magnitude represented. Critically, the field of numerical cognition has mostly focused on the investigation of the representation and processing of integers, while other forms of numerical representations, such as rational numbers have been neglected.

Even more importantly, how are all these abilities linked to one another, and ultimately to arithmetic?

It has been suggested that arithmetical abilities are built on an inherited system for representing numerosities (i.e. the ‘number sense’). Yet, there are different views on the nature of this system. The next paragraph will: (i) provide evidence for the innate nature of the ‘number sense’; (ii) describe the different views on the nature of this system; and (iii) it will discuss the theoretical implications of these distinct views on the development of arithmetic.

1.3. The ‘number sense’ and the development of arithmetic

According to a growing consensus in the field of numerical cognition, humans and also other species are endowed with a core capacity to understand numerosities (Butterworth, 1999; Carey, 2004; Dehaene, 1997). There is now extensive evidence that this core capacity is innate. Infants can discriminate displays with different small numerosities - e.g. they respond when the display changes from 2 objects to 3, or from 3 objects to 2 (*first demonstrated by* Starkey & Cooper, 1980; *but see also* Starkey, Spelke, & Gelman, 1990; van Loosbroek & Smitsman, 1990). Furthermore, infants are aware of changes in the numerosity of a set, even when these changes take place behind a screen, suggesting that as well as a basic representation of small numerosities, infants have an arithmetic expectancy regarding numerical transformations of the number of objects in a set (Wynn, 1992, 1998).

Beyond the ontogenetic evidence, the hypothesis of an endowed capacity for numbers should be grounded in evidence that such a capacity has an evolutionary value. Indeed, a growing body of literature on animal studies supports this idea. Numerical discrimination abilities have been found in rats (Church & Meck, 1984; Mechner, 1958; Meck & Church, 1983), orangutans (Shumaker, Palkowich, Beck, Guagnano, & Morowitz, 2001), monkeys (Brannon & Terrace, 1998), birds (Emmerton, Lohmann, & Niemann, 1997; Koehler, 1951), fishes (Agrillo, Dadda, & Bisazza, 2007; Agrillo, Dadda, Serena, & Bisazza, 2008; Piffer, Agrillo, Hyde, 2011) and also bees (Dacke & Srinivasan, 2008). Moreover, field studies have shown numerical processing abilities in animals independent of training (McComb, Packer, & Pusey, 1994; Wilson, Hauser, & Wrangham, 2001).

Furthermore, it has been elegantly demonstrated that the visual perception of numerosity is susceptible to adaptation (Burr, & Ross, 2008). Thus, it can be considered as a primary visual property of a scene to the same extent as colour is.

These results suggest the existence of a phylogenetically specified mechanism for the representation and processing of numerical magnitude (i.e. ‘the number sense’). Critically, the nature of this core capacity is in dispute with obvious implications for understanding the development of the next level of knowledge. What kind of system is arithmetic built upon? On the one hand, it has been argued that the core capacity uses an internal ‘numerosity code’ that represents numerosities exactly – exactly fiveness, exactly sixness, and so on – along with concepts of sets and their numerosities (Butterworth, 1999, 2005, 2010; Butterworth & Reigosa-Crespo, 2007). Thus some operations on sets (such as adding a member) will change the numerosity of the set, while other operations (such as rearranging the elements) will not. Numerosities are properties of sets not individual objects, and are hence by nature abstract. This idea is similar to the idea of innate non-verbal enumeration tokens, ‘numerons’, proposed by Gelman & Gallistel (1978). Sets, according to this account, can comprise any type of entity that can be individuated, including sounds, events, or even abstract entities such as thoughts or ideas (Butterworth, 1999). On the other hand, it has been claimed that we are born with two ‘core systems of knowledge’ with ‘numerical content’ that provide the basis for arithmetic (Dehaene, Molko, & Cohen, 2004; Feigenson, Dehaene, & Spelke, 2004). The first is the ‘object-file system’, an attentional mechanism which supports exact enumeration up to about three objects (Carey, 2004; Le Corre & Carey, 2007; Hyde, 2011) and thereby solves numerical problems on small quantities. The second is the ‘approximate system’ (or the ‘analogue magnitude system’), which only deals with ‘approximate numerosities’ and comes into play when there are too many items to be tracked by the ‘object-file system’ (Feigenson, Spelke, & Carey, 2002). Under this account, we would not be endowed with the capacity to represent exactly fiveness, but only with the capacity to represent approximately fiveness. Moreover, according to this theory, we would use an ‘analog-magnitude mechanism’ to represent large sets and their approximate numerosity (Feigenson et al., 2002). This notion is based on the metaphor of the mental number line in its logarithmic nature: analogue magnitudes are distributions of activation on a continuous mental number line in a way

that the larger the magnitude, the more approximate the activation (i.e. the representation for fiveness will overlap with the representations for fourness and sixness) (Dehaene, 2004; Dehaene & Changeux, 1993; Feigenson et al., 2004; see also **paragraph 1.5.1**). Interestingly, in some accounts, numerosities are described as the same as other continuous quantities such as area (Feigenson et al., 2002) or time (Gallistel & Gelman, 1992) and therefore similarly represented.

Indeed, the nature of the ‘number sense’ has been challenged by infant studies showing that infants respond not to numerosity per se, but rather to changes in non-numerical dimensions, such as surface area and contour length, which can be defined as continuous quantities (i.e. analogue magnitudes) (Clearfield & Mix, 1999; Feigenson et al., 2002). Even children as old as three have been shown to rely on non-numerical visual cues in order to discriminate between sets of objects (Rousselle, Palmer, & Noël, 2004). Yet, empirical evidence on this matter has been rather inconsistent as several studies have shown that infants still respond to changes in numerosities even when non-numerical continuous variables (i.e. area, density) are strictly controlled (Brannon, 2002; Lipton & Spelke, 2003; Wynn, Bloom, & Chiang, 2002; Xu, 2003; Xu & Spelke, 2000).

As mentioned above the capacity to represent and mentally manipulate numerosities is the key to learning arithmetic as the usual arithmetical operations of addition, subtraction, multiplication and division can be defined in terms of operations on sets and their numerosities. It follows that learners that struggle with the grasping of the basic concept of numerosity and its properties will consequently have difficulties in learning how to perform arithmetical operations on them. This brings us back to the intrinsic hierarchy of maths and its constructs. Indeed, in formal curricula, multiplication, division and fractions typically follow addition and subtraction, and are explained in terms of them. For example, the mathematics curriculum in the UK begins in Reception (4–5 years) and Year 1 (5–6 years) with counting, adding and subtracting, and then in Year 2 introduces “the operation of multiplication as repeated addition or as describing (e.g., counting) an array”(DfEE, 1999, Key Objectives, p. 3).

Thus, the critical question is which of these systems forms the basis of arithmetic. Some authors have claimed that approximate arithmetic forms the basis of

exact arithmetic (Barth, La Mont, Lipton, & Spelke, 2005; Barth, La Mont, Lipton, Dehaene, Kanwisher, & Spelke, 2006; Gilmore, McCarthy, & Spelke, 2007). Moreover, it has been suggested that the approximate system also forms the basis of mathematical knowledge (Halberda, Mazocco, & Feigenson, 2008). In this very influential longitudinal study it has been shown that the level of accuracy on a task of approximate enumeration (i.e. judging whether there were more blue or yellow dots in a visual display) was retroactively predictive of maths performance on a standardized maths task taken years later when participants were 14 years old.

Yet, it does seem reasonable that in order to add or subtract ‘exactly’ one would need to represent numbers exactly. Indeed, it has been argued that representations of exact sets are foundational for arithmetic (Butterworth, 2010). On the contrary, in order to solve arithmetic problems approximately, one might rely on a more approximate representation. This view presumes the idea that the two systems could be endowed, one for the exact representation of numerosity, and one for the approximate representation of it: the former would serve to perform arithmetic exactly, while the latter would aid the manipulation on ensembles that are approximate. In this view it is important to distinguish between arithmetical operations on approximations and approximate arithmetic on exact numbers, such as roughly computing the product of 91 and 39, which typically depends on knowledge of exact arithmetic (Case & Sowder, 1990; Dowker, 2003; LeFevre, Greenham, & Waheed, 1993).

Another possibility is that the ability to represent large quantities depends on the acquisition of culture-specific numeric symbols. Thus, the acquisition and mastering of symbolic meanings for numerical expression (i.e. Arabic digits) may play a crucial role in the acquisition of exact numbers (Ansari, 2008). Moreover, it has been proposed that proficiency in maths is strongly associated with better mapping abilities between the symbolic and non-symbolic system for numbers (Castronovo & Göbel, *personal communication*). It follows that different sets of abilities seem to interact in the development of arithmetical knowledge. Hence, the child needs to start off with a well anchored representation for numerosities, upon which he or she can develop a good understanding of operations on sets and their numerosities. Moreover, from the representation of numerosities he or she needs to understand the relationship between

number symbols and quantities in order to be able to perform arithmetical operations with the symbols.

1.4. Neural correlates of maths abilities

The capacity to understand and manipulate numerosities and numbers has been consensually linked to a specialized neural network, which seems to be orchestrated by the parietal lobes, and particularly the intraparietal sulcus (IPS) (Castelli, Glaser, & Butterworth, 2006; Dehaene, Piazza, Pinel, & Cohen, 2003; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004; Piazza, Mechelli, Butterworth, & Price, 2002; Piazza, Mechelli, Price, & Butterworth, 2006). Healthy adults have consistently demonstrated activations in the horizontal segment of the intraparietal sulcus (hIPS) when dealing with the manipulation of numerical quantity (Dehaene, et al., 2003) of both symbolic and non-symbolic stimuli (Cohen-Kadosh, Cohen-Kadosh, Kaas, Henik, & Goebel, 2007; Piazza, Pinel, Le Bihane, & Dehaene, 2007). Moreover, the parietal lobes are systematically activated in tasks involving arithmetical problem solving (van Eimeren, Grabner, Koschutnig, Reishofer, Ebner, & Ansari, 2010; Zago, Pesenti, Mellet, Crivello, Mazoyer, & Tzourio-Mazoyer, 2001; *see also* Ansari, 2008 and Zamarian, Ischebeck & Delazer, 2009 *for a review*) and they have also been reported to play a role in exceptional calculators such as calendrical savants (Cowan & Frith, 2009). Additionally, the processing of mathematical information seems to recruit areas such as the dorsal visual stream encompassing the superior parietal lobule (SPL), the angular gyrus (AG) and the supramarginal gyrus regions of the posterior parietal cortex (PPC) (Delazer, Domahs, Barth, Brenneis, Lochy, Trieb, & Benke, 2003; Grabner, Ansari, Koschutnig, Reishofer, Ebner, & Neuper, 2009; Rickard, Romero, Basso, Wharton, Flitman, & Grafman, 2000; Wu, Chang, Majid, Caspers, Eickhoff, & Menon, 2009). In addition to the parietal lobes, the prefrontal cortex also seems to be important in mathematical cognition. Indeed, an emerging body of single cell recording research has shown that when monkeys were made to judge between two consecutive displays containing a different number of items, neurons in the lateral prefrontal cortex selectively responded to numerical information (Nieder, Freedman, & Miller, 2002). In humans, the prefrontal cortex has been associated with more abstract mathematical thinking (Shallice & Evans, 1978).

As we have seen above, mathematical proficiency requires mastering of numerous skills from counting and enumeration, to the proper understanding of maths symbols, all the way to higher level reasoning. So even if the parietal lobes (and to some extent the prefrontal lobes) seem to be the key areas which orchestrate these sets of abilities, it is quite unlikely that they do so independently from the rest of the brain. Indeed, most recently, there seems to be converging evidence for the involvement of a more distributed network of regions responsible for maths abilities. These include the frontal and temporal lobes and the cerebellum for calculation and problem solving skills, and areas in the anterior ventral visual stream for the processing of symbols (Rykhlevskaia, Uddin, Kondos, & Menon, 2009). Particularly, the right fusiform gyrus (rFG) has been associated with the visual processing of mathematical symbols (Rykhlevskaia, et al., 2009; *but see* Ansari, 2007; Cohen-Kadosh, Cohen Kadosh et al., 2007; Piazza, et al., 2007 *for a different account* on processing information of numerical stimuli and their neural correlates).

1.5. Typical development

Studies on infants have reported a developmental trajectory where the basic understanding of numerosities evolves over time. By the age of 5-months, babies are sensitive to changes of small numerosities of visual displays of objects (Starkey & Cooper, 1980). Yet, some researchers have shown this sensitivity to be present already by the first week of life (Antell & Keating, 1983). By the age of 6-months, babies can discriminate even between larger numerosities of sets (up to 16 elements), yet their discrimination acuity is minimal (i.e. they can discriminate between a set of 8 objects and a set of 16 objects but not between a set of 8 objects and a set of 12 objects) (Xu & Spelke, 2000). Infants will habituate to familiar displays as measured by reduced looking time, and will dishabituate to novel displays as indicated by increased looking time. This behavior has enabled researchers to assess whether infants are sensitive to numerosity as a property of visual display, and to evaluate developmental changes in sensitivity, as indicated by the size of the distance effect in comparing non-symbolic stimuli such as arrays of dots, and symbolic stimuli (digits). Interestingly, it has been shown that numerosity discrimination improves with development even at older ages (*see for example* Piazza, Facoetti, Trussardi, Berteletti, Conte et al., 2010).

In order to assess the representation of numbers and number processing in older children, teenagers and adults, two well established behavioural effects have been standardly used. One is the *distance effect* which postulates that when comparing two numbers (or numerosities) the larger the difference in magnitude, the faster the response time (Moyer & Landauer, 1967). Usually depending on the age of the subjects (but not always), the distance effect can be measured either when participants are dealing with non-symbolic numerosities (e.g. sets of dots) or with symbolic numerosities (e.g. Arabic digits) and it has been used in order to investigate natural number representations. A non-symbolic distance effect has been reported already by the age of 6 (Ansari & Dhital, 2006) and it has been shown that it gets even more pronounced with age (Landerl & Kölle, 2009; Piazza et al., 2010). Moreover, a symbolic distance effect has been found from kindergarten (Sekuler & Mierkiewicz, 1977) and it has also been reported to increase with age (Holloway & Ansari, 2009; Landerl & Kölle, 2009). Finally, both types of distance effects have been showed to be correlated with arithmetical performance (Halberda, Mazocco, & Feigenson, 2008; Holloway & Ansari, 2009).

The other effect that has been used to measure numerical proficiency is the *congruity effect* that almost always gets elicited by *Numerical Stroop* paradigms. In these types of paradigms subjects are presented with two stimuli expressed as numerical digits¹ and are required to compare the stimuli according to their physical size (having to ignore the numerical meaning of the symbols presented). The stimuli can be incongruent (the physically larger digit is numerically smaller, e.g. 2 4), neutral (the stimuli differ only in the relevant dimension, e.g. 2 2), or congruent (the physically larger digit is also numerically larger, e.g. 2 4). The *congruity effect* capitalizes on the fact that reaction times on incongruent trials are slower compared to congruent trials. This effect has been interpreted as an indicator of automatic processing of numbers (Schwarz, & Ischebeck, 2003; Tzelgov, Meyer, & Henik, 1992) and more generally as an index of proficiency with numbers (Rubinsten & Henik, 2006). Indeed, the congruity effect has been shown to be modulated by development as typically developing children in their first year of school do not show it (Girelli, Lucangeli, & Butterworth, 2000).

¹ Though variants of this paradigm have also been used. Particularly, some researchers have used it with non-symbolic stimuli (Fabbri, Tang, Zorzi, & Butterworth, 2009).

In summary, from the behavioural account on the development of numerical abilities there seems to be a refinement of the symbolic and non-symbolic representation of numbers (Piazza et al., 2010) and their automaticity (Girelli et al., 2000) with age.

1.5.1. The ‘mental number’ line in typical development

Another signature of numerical proficiency seems to be an accurate mapping of numbers to space. The proposal that number representations are spatially organized dates back to Sir Francis Galton (1880), who reported that the conscious visualization of numbers on a mental number line were oriented from left to right. However, it wasn’t until 1992 that the construct of a ‘mental number line’ was formally included in a model of mathematical cognition. This model, named the Triple Code Model, described three systems for the mental representation of numbers: the visual system encoding numbers as strings of Arabic numerals, the verbal system encoding numbers as syntactically organized sequences of words, and the analogical quantity system which represents numbers as activations oriented upon a ‘mental number line’ (Dehaene, 1992). This third coding system is thought to obey the *Weber-Fechner law*, where the precision of numerical representations are logarithmically determined upon the number line (Dehaene, 2003, 2004; Dehaene & Changeux, 1993; Feigenson et al., 2004; Siegler & Opfer, 2003). Moreover, it has been proposed that a representation in the form of a mental number line is obligatorily activated whenever numbers are processed (Dehaene, 1992, 1997). Evidence for mental number line representations have relied on two very well established behavioural findings in the field of numerical cognition: - the *distance effect* (see **paragraph 1.5**) and – the *size effect* (i.e. responses are slower and less accurate when the numbers are larger) (Moyer & Landauer, 1967).

In Western cultures, the spatial organisation of the mental number line has been thought to be oriented from left to right (Dehaene, Bossini, & Giraux, 1993). Evidence for this notion, now known as the SNARC effect (*Spatial Numerical Association of Response Codes*), was initially based on evidence that subjects respond faster with the left hand to small numbers and faster with the right hand to relatively large numbers (Dehaene et al., 1993). The SNARC effect has since been replicated numerous times and has been shown to be unrelated to handedness, as well as to hemispheric dominance (Dehaene et al., 1993, *but see* Ito & Hatta, 2004). SNARC effects in numerical tasks

have provided additional support for the idea of numerical representations upon a mental number line (Fias, Brysbaert, Geypens, & d'Ydewalle, 1996; Fias, 2001). Recent cognitive experiments have complemented this view by showing that relatively large numbers are associated with the right side of a mental representational space (de Hevia, Girelli, & Vallar, 2006; *see de Hevia & Spelke, 2009 for a developmental account*). Finally, when patients with Neglect syndrome are tested on numerical bisection tasks (i.e. identify the middle number between 1 and 9) they systematically report a rightward bias (Zorzi, Priftis, & Umiltà, 2002). This suggests that patients with hemi-field perceptual deficits perform the numerical bisection tasks only in one hemi-field representational space as if they were bisecting physical lines in perceptual space (Zorzi et al., 2002; *see also* Rossetti, Jacquin-Courtois, Rode, Ota, Michel, & Boisson, 2004; Vuilleumier, Ortigue, & Brugger, 2004). Moreover, it has been reported that healthy enumerate adults show a systematic spatial bias towards the larger number (Zorzi et al., 2002), while children show a bias towards the small number (de Hevia & Spelke, 2009).

The concept of a number line is introduced to individuals at a very young age: children are taught to represent numbers upon a line from constant exposure to rulers, tape measures, thermometers, and other measuring devices. Yet, it has been shown that preliterate children display an intuition for the left to right organization of numerical magnitude (Opfer & Thompson, 2006) supporting the notion of a number to space mapping prior to education. Furthermore, it has been claimed that the subjective magnitude scale is represented logarithmically in younger children and uneducated adults, but linearly in older educated individuals (Dehaene, Izard, Spelke, & Pica, 2008; Siegler & Opfer, 2003; *but see* Karolis, Iuculano, & Butterworth, 2011 *about the kind of evidence needed to establish the nature of the subjective scale*). This finding has led to the consensus that a good mastering of numerical information is characterized by a linear mapping of numbers to space. Finally, it has been shown that individual differences in number line estimation tasks correlate strongly with maths achievement test scores (Booth & Siegler, 2006, 2008) hinting at the idea of a possible link between the ability to linearly (and accurately – or in other words exactly) represent numbers and the ability to perform arithmetic.

1.5.2. The typical acquisition of arithmetic

Different proposals - mostly based on correlational studies - have been made on the acquisition of symbolic numerical knowledge and arithmetic (*see* Piazza, 2010 *for a review*). Some authors have highlighted the development of counting principles as a critical foundation for later maths achievement, in particular for early arithmetic (Butterworth, 2005; Fuson, 1988; Gelman & Gallistel, 1978). Indeed, mastery of the *how-to-count* principles has notably been found to predict children's later abilities in maths and particularly in arithmetic (Passolunghi, Vercelloni, & Schadee, 2007). On the other hand, as we previously mentioned (*see* **paragraph 1.3**) some authors have highlighted how the approximate number system should be essential for the acquisition of numerical knowledge and ultimately arithmetic (Dehaene & Changeux, 1993; Verguts & Fias, 2004); while others have focused on the 'object-tracking system' (Carey, 2004), or both (Feigenson et al., 2004; Spelke & Kinzle, 2007; *see* Hyde, 2011 *for an attempt to combine the two views*).

In recent years, many factors have been shown to correlate with maths achievement. Notably, both domain-general and domain-specific factors have been associated with individual differences in mathematical abilities. In a recent longitudinal study, Geary has highlighted the importance of the executive as well as the visual components of Working Memory in maths proficiency (Geary, 2011a). Yet, in the same study, a factor named 'early quantitative competencies' was also uniquely predictive of maths achievement (Geary, 2011a), and indeed numerous studies have showed domain-specific factors to be correlated with maths achievement during development: estimation abilities (Dowker, 2005; Ebersbach, Luwel, Frick, Onghena, & Verschaffel, 2008; LeFevre, Smith-Chant, Hiscock, Daley, & Morris, 2003; Schneider, Grabner, & Paetsch, 2009; Siegler & Booth, 2004), non-symbolic arithmetic ability (Gilmore, McCarthy, & Spelke, 2010), symbolic and non-symbolic comparison accuracy (Mundy & Gilmore, 2009) and the symbolic numerical distance effect (i.e., the greater mathematics achievement, the smaller the symbolic distance effect) (De Smedt, Verschaffel, Ghesquière, 2009; Holloway & Ansari, 2009). Together, this evidence suggests that even if the variance on tests of maths achievement can be successfully explained by both domain-general and domain-specific factors, there still seems to be some residual domain-specificity to maths achievement. One of the most influential

studies on this topic has reported a strong (and highly predictive) link between higher order maths skills and numerical abilities with the level of acuity of the approximate number system (Halberda et al., 2008). Yet, these authors have failed to specify the mechanisms by which the approximate system could form the basis of arithmetic. On the other hand, some studies have failed to find a significant association between the ability to deal with approximate numerosities and arithmetic (Mazzocco, Feigenson, & Halberda, 2011; Piazza et al., 2010). Instead, in these two studies arithmetic performance was significantly correlated with the ability to deal with exact numerosity and mapping abilities between symbolic (i.e. number words) and non-symbolic material (*see also* Barth, Starr, & Sullivan, 2009; Mundy & Gilmore, 2009, De Smedt, et al., 2009 *on the link between maths abilities and mapping abilities*).

As we can see, previous studies have been sparse and do not allow making strong conclusions on what exactly is foundational in typical development for the acquisition of arithmetic.

1.5.3. Neural correlates of typical development

In previous paragraphs we have discussed the evidence to suggest that the parietal lobes are a key area of the brain to modulate the representation and processing of numerical magnitude in the adult brain. However, the most interesting question is how these areas come to specialize over the course of development. In recent years, application of neuroimaging techniques to the study of development has finally made it possible to answer such questions.

In a cross-sectional study of individuals aged between 8 and 19, Rivera and colleagues demonstrated age-related increases in the recruitment of the left inferior parietal cortex (specifically the left supramarginal gyrus - SMG) during a calculation task (Rivera, Reiss, Eckert, & Menon, 2005). Since the SMG, together with the angular gyrus had previously been implicated in mental arithmetic tasks in adults (Dehaene, Tzourio, Frak, Raynaud, Cohen, et al., 1996; Gruber, Indefrey, Steinmetz, Kleinschmidt, 2001; Rickard, et al., 2000; Rueckert, Lange, Partiot, Appollonio, Litvan, et al., 1996), this suggested that there is a process of developmental neural specialization related to the acquisition of arithmetic. Moreover, the increase in activity in posterior regions was coupled with reductions of activity in bilateral regions of the

pre-frontal cortex. So, even more specifically, it is the fronto-parietal shift that could reflect the development of arithmetic at the brain level. Moreover, Menon and colleagues (2000) have extended these findings by reporting a fronto-parietal neuronal shift with increased age but also arithmetic proficiency (Menon, Rivera, White, Glover, & Reiss, 2000). The increased activity in the angular gyrus or more generally in occipito-temporal regions suggests that there is an age-related increase in the specialization of these regions that is related to the refinement of the processing of symbolic visual stimuli with age (Ansari, 2008). Moreover, the decrease in frontal activity has been interpreted as a reduced reliance on cognitive processes such as attention and working memory with age for comparable tasks (Ansari, 2008).

Additionally, evidence for a fronto-parietal shift has been documented for more basic tasks such as magnitude comparison tasks with symbolic stimuli (Ansari, Garcia, Lucas, Hamon, & Dhital, 2005; Kaufmann, Koppelstaetter, Siedentopf, Haala, Haberlandt, et al., 2006); while no such shift has been documented for non-symbolic stimuli (Ansari & Dhital, 2006). Interestingly, the reported '*neural distance effect*' in the bilateral IPS for adults has not been reported for children of 10 years of age, who instead showed a '*neural distance effect*' (i.e. decreasing activation with increasing distance) in a network of right prefrontal areas including the right IFG (Ansari et al., 2005). Similar to monkeys, children seem to rely more on prefrontal areas when dealing with numerical stimuli (Diester & Nieder, 2007) which suggest a less specialized neural organization that supports numerosity processes in these populations.

Recent studies have also reported increased activations in the medial temporal lobe (MTL) coupled with decreased activation in frontal areas that was modulated by strategy refinement (i.e. the more children relied on retrieval rather than counting strategies, the more the MTL would activate) (Cho, Ryali, Geary, Menon, 2011). Furthermore, by testing a population of children in Grade 2 and a population of children in Grade 3, Rosenberg-Lee and colleagues have demonstrated greater connectivity between fronto-parietal areas as a consequence of schooling (Rosenberg-Lee, Barth, & Menon, 2011). Finally, learning studies in adults have shown a more selective involvement and an increased specialization of the parietal lobes along with a reduced activation in frontal regions associated with the refinement of strategies for problem solving (Ischeback, Zamarian, Eger, Schocke, & Delazer, 2007; Grabner et al., 2009).

On a different account, it has been proposed that the refinement of number understanding is coupled with a progressive shift from predominance of the right intraparietal sulcus to the involvement of the left intraparietal sulcus (Ansari, Dhital, & Siong, 2006; Cantlon, Brannon, Carter, & Pelphrey, 2006; Izard, Dehaene-Lambertz, & Dehaene, 2008; Piazza et al., 2007). These changes have been interpreted as reflecting a progressive refinement of the approximate system for numbers into a second number system dealing with symbolic numbers for exact number processing, which might be lateralized in the left hemisphere (Ansari, 2007; Dehaene, 2009; Piazza et al., 2007; Piazza & Izard, 2009; *but see* Cohen-Kadosh et al., 2007). This idea is supported by the fact that Arabic numerals are coded with greater precision than sets of objects, particularly in the left hemisphere (Piazza et al., 2007; *but see* Cohen-Kadosh et al., 2007). Therefore, the idea that emerges is that following the acquisition of arbitrary cultural symbols for numbers, the approximate number system would be refined in the left parietal lobe into a formal, symbolic system, allowing the automatic access from symbolic numbers to their corresponding magnitude; while a dormant approximate (and logarithmic) number system would ‘remain’ in the right intraparietal sulcus (Dehaene, 2009). This notion has gained further support by behavioural data on numerical representations and their linearization over the course of development (Siegler & Opfer, 2003, *but see* Karolis et al., 2011) and could represent the neurobiological mechanism that interacts with the development of arithmetic (Ansari, 2008). Indeed an increase activation in and around the left intraparietal sulcus for arithmetic processing has been reported with increasing age (Rivera et al., 2005).

Altogether these results demonstrate the involvement of a more distributed network for numerical and arithmetic processing. Moreover, they show how the recruitment of such a network is modulated by age, performance, and education. Additionally, these findings indicate that dynamic increases and decreases in activation occur in a large network of regions when individuals deal with numerosities, numerical symbols, and arithmetic, highlighting the importance of considering networks of activation rather than focusing on a selected set of brain regions.

1.6. Atypical development

Even if studies on typical development have been very influential and informative on the understanding of the development of the human capacity for numbers and the subsequent development of arithmetic, they do not provide a clear picture on what are the crucial skills (and processes) for such development. In this sense, investigations of atypical development are the most revealing in terms of what capacities are foundational for learning arithmetic.

Critically, research into the characterization of atypical development has been hindered by highly variable selection criteria and terminology across studies, reducing the coherent impact of multiple sets of results. Here we will list the different definitions given for atypical maths development and the theoretical implications of the broad terminology used in the literature. Moreover, we will describe the different diagnostic criteria for atypical development in its different forms. We will then focus on the two main conditions that represent the central investigation of the present thesis (i.e. Developmental Dyscalculia and Low Numeracy) and describe them in details in terms of behavioural and neuronal ‘abnormalities’.

1.6.1. Terminology and diagnosis of atypical development

1.6.1.1. Terminology

There is considerable confusion in the terminology and the criteria used to describe atypical maths development. Thus far, experimental investigations on atypical math development have talked about: ‘*Mathematical Learning Disabilities*’ (Geary, 2010) or ‘*Mathematics Learning Disability*’ (Mazzocco & Myers, 2003; Rousselle & Noël, 2007), ‘*Low Numeracy*’ (Räsänen, Salminen, Wilson, Aunio, & Dehaene, 2009), ‘*Low Achieving*’ (Geary, 2008), ‘*Low math achievement*’ (Mazzocco & Devlin, 2008), ‘*Mathematics Disorder*’ (American Psychiatric Association, 1994), ‘*Arithmetic Learning Disabilities*’ (Koontz & Berch, 1996), ‘*Mathematics Disabilities*’ (Geary, 1993; Geary, Hamson, & Hoard, 2000), ‘*Learning Disabilities in Arithmetic and Mathematics*’ (Geary & Hoard, 2005), ‘*Specific Arithmetic Difficulties*’ (Hitch & McCauley, 1991; Lewis, Hitch, & Walker, 1994), ‘*Specific disorder of arithmetical*

skills’ (International Classification of Diseases – 10), ‘*Calculation Difficulties*’ (Isaacs, Edmonds, Lucas, & Gadian, 2001), ‘*Arithmetic Deficit*’ or ‘*Mathematics Difficulties*’ (Jordan, Kaplan, & Hanich, 2002), and *Developmental Dyscalculia*’ (Butterworth, 2005, 2010; Butterworth, Varma, & Laurillard, 2011; Gross-Tsur, Manor, & Shalev, 1996; Kosci, 1974; Landerl, Bevan & Butterworth, 2004; Piazza et al., 2010; Shalev & Gross-Tsur, 1993, 2001; Shalev, 2007). Moreover, some have used multiple/combined definitions such as ‘*Mathematical Learning Disability (Dyscalculia)*’ (Mazzocco, Feigenson, & Halberda, 2011).

While the terms mathematics, arithmetic, math, arithmetical are essentially used interchangeably across the literature, and tend to denote the same area of study, there is an essential difference between the terms disability and difficulty (Mazzocco, 2007). While the terms ‘disability’ and ‘disorder’ reflect an inherent inability to acquire the necessary skills within a given learning domain, and suggests a biologically based disorder, the term ‘difficulty’ has been explicitly referred to poor achievement deriving from various causes, with no presumed biological basis (Hanich, Jordan, Kaplan, & Dick, 2001). It follows that ‘mathematical difficulties’ is a term that encompasses not only a broader range of causes, but also a much wider range of performance compared to the terms ‘disability’ and ‘disorder’. Moreover, this has critical implications for the inclusion (i.e. cut-off) criteria based on standardized tests. Indeed, ‘mathematical difficulties’ are often operationalized as those scores which fall below the 35th percentile on standardized scores, that is, the lowest 35% of performers, a cut-off which sits high above the estimated prevalence rate for more biologically based disorders such as Developmental Dyscalculia (Mazzocco, 2007).

1.6.1.2. Diagnosis

Diagnosis of atypical development can be given either (i) in terms of a selection criteria based on a certain cut-off below the control mean on standardized arithmetic tests in the absence of other types of cognitive or behavioural disorders; or (ii) on the basis of a discrepancy criteria whereby a child’s score on a standardized test of arithmetic is compared to non-arithmetical intelligence measures. The *International Classification of Diseases – 10* (Section 8.21) defines a ‘Specific disorder of arithmetical skills’ as involving a specific impairment in arithmetical skills that is not

solely explicable on the basis of general mental retardation or of inadequate schooling. The deficit concerns mastery of basic computational skills of addition, subtraction, multiplication, and division rather than of the more abstract mathematical skills involved in algebra, trigonometry, geometry, or calculus' (World Health Organization, 1994).

On the other hand, the DSM-IV definition (American Psychiatry Association, 1994, Section 315.1) implies a 'discrepancy criteria': 'Mathematical ability, as measured by individually administered standardized tests, is substantially below that expected given the person's chronological age, measured intelligence, and age-appropriate education'. However, both definitions depend explicitly or implicitly on poor performance on standardized tests of arithmetic. Even more critically, what counts as poor performance varies widely from study to study: the lowest quartile (Koontz & Berch, 1996; Siegel & Ryan, 1989); a standard score of less than 90 (Mazzocco & McCloskey, 2005); and below the 11th or 12th percentile (Geary, Hoard, Nugent, Byrd-Craven, 2008; Mazzocco & Devlin, 2008).

In this work we take a different approach for the characterization of atypical development of maths abilities. Specifically, we base our diagnosis on a grounded theory that defines Developmental Dyscalculia as a core deficit in the basic understanding of numbers and numerosities which in turn affects the ability to acquire arithmetic skills (Butterworth, 2003, 2005, 2010; *see also* Department for Education and Skills – DfES, 2001). Moreover, we use a very stringent cut-off criteria based on the prevalence estimate of the disorder (Butterworth, Reigosa-Crespo, 2007; Shalev, 2007), that is we only take children at the bottom 7% of the population. Our diagnosis of Low Numeracy is also based on a grounded theory that defines it as a specific impairment in arithmetic, but not in the basic understanding of numerosity. Moreover, our cut-off was equally stringent (7% of the population).

This approach on diagnosis and selection cut-off criteria was taken in order to better differentiate between the core impairment in the representation and processing of numerosity (Developmental Dyscalculia) and the selective impairment in arithmetic without core impairments (Low Numeracy). Moreover, drawing this differentiation in

our clinical populations allowed us to directly test the predictive factors of arithmetic development.

1.6.2. Developmental Dyscalculia and Low Numeracy

Poor numerical skills are a more severe handicap than most people realize. A recent cohort study of the effects of low functional numeracy shows that ‘it is more of a handicap in the workplace than poor literacy’ (Bynner & Parsons, 1997). ‘People with poor numerical skills (26% of the population), are more than two and half times as likely to be unemployed, and more than three and half time as likely to be depressed’ (Parsons, & Bynner, 2005). ‘Compared with their numerically competent peers, fewer than half are in employment by 30 years old, fewer than half are home owners, and twice as many are in poor physical health’ (Parsons & Bynner, 2005).

A major cause of poor numerical skills is Developmental Dyscalculia (DD), which, according to the current best estimates, affects about 3 to 7% of the population (Butterworth & Reigosa-Crespo, 2007; Reigosa-Crespo, Valdés-Sosa, Butterworth, Estévez, Rodríguez, et al., 2012; Shalev, 2007). The usual presenting symptoms of DD are poor performance in school maths tests, failing to understand numerical concepts, losing track in maths lessons, often in the presence of good marks in other school subjects, inability to deal with numbers in everyday life situations such as shopping, telling the time, and remembering phone numbers. However, these can be symptoms with other causes, including poor or inappropriate maths teaching, missing lessons, anxiety about numbers and mathematics, behavioural problems, poor working memory, attentional problems, and some language-related impairments including dyslexia (Butterworth, 2003; *see also* Reigosa-Crespo et al., 2012). Mathematics seems particularly vulnerable to any kind of stress on learning, perhaps because of the cumulative structure of its content: failing to understand one concept can mean that the learner will fail to understand concepts that depend on it, as we have previously noted. DD also persists into adulthood. In a six-year prospective follow-up study, Shalev and colleagues showed that of the learners diagnosed as DD at age 11, over 40% were still in the DD category (i.e. two years behind the control population according to their criteria) at age 17, and 95% were still in the lowest quartile of their age group (Shalev, Manor, & Gross-Tsur, 2005).

There is now evidence that Developmental Dyscalculia is a domain-specific condition (Butterworth, 2005; Butterworth, 2010). Thus, one of the striking things about DD is that it can be highly selective. Sufferers can be average or even excellent at all school subjects apart from mathematics. Normal or superior IQ does not protect one against it (Landerl, et al., 2004), and the common DD symptom of poor memory for arithmetical facts does not need to be part of a wider impairment in either long-term memory or working memory. Moreover, it appears that DD children are not only poor on school arithmetic and on standardized tests of arithmetic, they are slower and less efficient at very basic numerical tasks, such as recognizing the numerosities of displays of objects (typically dots), and at comparing numerosities in a variety of number comparison tasks with non-symbolic and symbolic material (Butterworth, 2005; Landerl et al., 2004; Landerl & Kölle, 2009; *but see also* Butterworth, 2010 *for a review*). This suggests a deficit in the core capacity for representing and manipulating numerosities (Butterworth, 1999; Dehaene, 1997). Indeed, it has been argued that Developmental Dyscalculia is a congenital deficit in this core capacity which, in many cases, will be inherited (i.e. the '*defective number module hypothesis*') (Butterworth, 2005, 2010). Indeed genetic evidence for the inheritance of basic numerical abilities has been reported. For example, if one twin is DD, then 58% of monozygotic co-twins and 39% of dizygotic co-twins are also DD (Alarcon, Defries, Gillis-Light, & Pennington, 1997); and nearly half of siblings of dyscalculics are also dyscalculic (5 to 10 times greater risk than controls) (Shalev, Manor, Kerem, Ayali, Badichi, et al., 2001). Although a substantial proportion of the variance in a twin study of mathematics can be attributed to some general factor (e.g. *g* or 'generalist genes'), nevertheless approximately a third of the variance seems to be specific to mathematical ability, though not exclusively numerical ability (Kovas, Harlaar, Petrill, & Plomin, 2006). Moreover, various X-chromosome disorders seem to affect numeracy more than other cognitive functions (Rovet, Szekely, & Hockenberry, 1994; Mazzocco & McCloskey, 2005 *for a review*). Interestingly, Turner Syndrome subjects have been consistently reported to be slower on dot estimation tasks (Bruandet, Molko, Cohen, & Dehaene, 2004; Butterworth et al., 1999). Bruandet and colleagues (2004) noted that females with Turner syndrome were slower than control subjects even with two dots. Furthermore poor number skills are

also found in Fragile X (Mazzocco & McCloskey, 2005), Klinefelters and other extra X conditions (Semenza, *personal communication*).

Butterworth (2005, 2010) proposes that this core deficit on the inherited capacity to deal with numerosities will in turn affect the learning of mathematics, and more specifically, the learning of arithmetic.

Another possibility that has been discussed in the literature is that Developmental Dyscalculia is a deficit in mapping number symbols onto intact representations of numerical magnitude. This proposal was initially put forward by Rousselle and Noël (2007) who compared a group of typically developing children in Year 2 with a group of age and education matched DD children. The DD group was slower (yet equally accurate) than the control group in symbolic comparison tasks but not in non-symbolic comparison tasks. This ‘mapping hypothesis’ is consistent with the findings that maths achievement in elementary school is predicted by the size of the symbolic distance effect (Holloway & Ansari, 2009; De Smedt et al., 2009). However, other studies have argued against this hypothesis as their results show a significant difference between children with DD and their control group on both symbolic and non-symbolic tasks (Landerl et al., 2004; Landerl & Kölle, 2009).

Finally, it has been proposed that mathematical learning disabilities are the result of difficulties in understanding basic numerosities concepts and retrieving arithmetical facts from long term memory, which ultimately are related to Working Memory impairments in these children (Geary, 2011b).

Low numeracy (LN) which is not the result of DD may have different causes, yet it might apparently result in the same behavioural outcome as reflected by poor performance in arithmetic tasks. This is particularly true if poor performance is only measured in terms of arithmetical skills rather than on the basis of tests of simple numerosity processing and representation. Thus, there have many proposals as to what the causes of low numeracy might be, including poor working memory (Geary, 1993; Geary & Hoard, 2005; Geary, Hoard, & Hamson, 1999; Hitch & McCauley, 1991; McLean & Hitch, 1999), poor long-term memory (Geary, 1993; Geary & Hoard, 2005), poor language skills (Donlan, Bishop, & Hitch, 1998; Donlan, Cowan, Newton &

Lloyd, 2007; *but see* Gelman & Butterworth, 2005), and poor cognitive ability (Kovas et al., 2006; O'Connor, Cowan, & Samella, 2000).

1.6.3. The ‘mental number’ line in atypical development

Even though proponents of the ‘*defective number module hypothesis*’ have not directly addressed the question of number representations, we could infer that such representations might be disrupted, or somehow defective in Developmental Dyscalculia. Moreover, the question of magnitude representation is an area of theoretical interest as ‘the mechanisms that support number line representations are often thought as based on a potentially inherent number-magnitude system that is supported by specific areas in the parietal cortices’ (Geary, 2011b). Moreover, examining the mapping of numbers to space in critical populations can have important implications on the nature of the representational system itself (i.e. exact and linear *versus* approximate and logarithmically represented as analogue activations on a mental number line- see **paragraphs 1.5** and **1.5.1**).

The question that arises is whether mapping numbers on a physical line would result in a pattern that conforms to the natural logarithm of the number (Feigenson et al., 2004) or to the exact value of the number (Gallistel & Gelman, 1992, 2000; Zorzi & Butterworth, 1999; *but see also* Siegler & Opfer, 2003 *for an attempt to combine the two views*). Moreover, if children with poor numerical skills have a deficit in the number-magnitude representational system, then their performance on a number line task might show less precision compared to typically developing children and therefore might not conform to a linear representation (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Geary, Hoard, Nugent, & Byrd-Craven, 2008).

One study has addressed this issue by showing that 6 year old children with low numerical skills and mathematical learning disabilities (who were not DD) were less accurate in their placement of numbers on a physical line compared to their typically developing peers (Geary et al., 2007). Thus, these authors have proposed that the delayed learning of these children may be the consequence of a problem in basic numerical representations, including the mental number line (Geary et al., 2007, *see* Geary 2011b *for a review*).

As previously mentioned (see **paragraph 1.5.1**), individual differences in children's refinement of the mapping of numbers onto a physical line (i.e. from logarithmic to linear) are correlated with maths achievement (Booth & Siegler, 2006) and indeed it has been recently shown that children with mathematical learning disabilities tend to be less accurate in their number line representations (Geary et al., 2007; *see also* Geary et al., 2008).

1.6.4. The 'atypical' acquisition of arithmetic

As previously mentioned, research on typical development has not been conclusive on what are the foundational skills that support arithmetical development. Moreover, as we have seen in **paragraph 1.3**, the nature of the 'number sense' is disputed: is the core system for numbers (i) an approximate system that can only deal with approximate numerosities; or (ii) an exact system that can deal with numerosities exactly and encompasses the properties of cardinality; or finally (iii) a system that includes two core systems of knowledge, with both exact and approximate features? As previously discussed, these views have critical implications on what is essential in order to build a good understanding of arithmetic.

Although proponents of the two core systems approach (Carey, 2004; Dehaene et al., 2004; Feigenson et al., 2004; Le Corre & Carey, 2007) have not addressed the issue of Developmental Dyscalculia (DD) or Low Numeracy (LN), it is reasonable to infer that DD, a congenital condition, will have a deficit in one the core systems of knowledge (see **paragraph 1.3**). Presumably it could not be a deficit to the '*object-file system*' since this would have far-reaching effects on every-day attention, and there is no evidence that attentional deficits are sufficient to cause DD, nor that DD sufferers have attentional deficits (Monuteaux, Faraone, Herzig, Navsaria, & Biederman, 2005). Therefore, since there are only two core systems, the only other possibility is that DD is caused by a deficit in the approximate system, which could lie in the ability to extract approximate numerosities from the stimulus, or in the ability to represent it as an analogue magnitude. This may show itself in one of two ways: either in the ability to estimate numerosities or in the ability to estimate analogue quantities. Indeed, recent studies have argued in favor of the foundational role of the approximate number system for the development of higher level numerical abilities, by reporting the existence of a

link between DD and an impaired approximate number sense (Piazza et al., 2010). Supporting these results, Mazzocco and colleagues (2011) conducted a study on ninth grade students with either dyscalculia or with low, typical or high mathematics achievement and also found that dyscalculia is linked with an impaired performance on approximate comparison tasks (Mazzocco, Halberda, & Feigenson, 2011). However, both these studies found that performance on an approximate comparison task was predictive of performance on symbolic comparison tasks but it did not correlate with children's maths achievement (Mazzocco et al., 2011; Piazza et al., 2010).

On the other hand, according to the '*defective number module hypothesis*' (Butterworth, 2005, 2010), the cognitive deficit in DD is a disability in dealing with exact numerosities, which implies that sufferers may be entirely normal on tests of approximate numerosity and on tests of analogue magnitude.

Finally, according to the '*defective mapping hypothesis*', DD would arise from a disconnection between concepts of numerosity, which are intact (and presumably exact), and the symbols that denote them (Rousselle & Noël, 2007). Indeed Mazzocco and colleagues have showed that DD is associated with impaired mapping abilities between the ability to represent approximate numerosities and number words (Mazzocco et al., 2011). Yet, none of these studies have demonstrated a link between mapping abilities and arithmetical performance.

1.6.5. Neural correlates of atypical development

The first neuroimaging studies which investigated atypical numerical processing looked at a population with numerical and visuo-spatial impairments occurring in the context of genetic developmental syndromes, such as Turner syndrome (TS) and Fragile X syndrome (fraX). Using both functional and structural neuroimaging methods, Molko and colleagues compared TS patients to typically developing controls during an exact calculation task (Molko, Cachia, Rivière, Mangin, Bruandet, et al., 2003). While healthy participants showed increased activation in the bilateral intraparietal sulcus (IPS) as the difficulty of the calculation problems increased, the TS subjects failed to show the same modulation. Moreover, the lack of modulation was coupled with a less accurate performance by the TS group. Subsequent structural analyses revealed decreased grey

matter density of the right IPS in the TS subjects (Molko et al., 2003; *see also* Molko, Cachia, Riviere, Mangin, Bruandet, et al., 2004). Furthermore, a study on fraX conducted by Rivera and colleagues also showed that the activation of the right IPS was not modulated by problems' difficulty in fraX subjects (Rivera, Menon, White, Glaser, & Reiss, 2002).

These findings were the first to demonstrate that atypical development of maths abilities showed an abnormal recruitment of parietal regions. Moreover, these studies suggest that an atypical development of parietal regions, particularly the right IPS, may be the neurobiological marker of arithmetical disabilities. Hence, in a seminal study by Isaacs and colleagues (2001), it was found that adolescents of very low birth weight, who showed deficits in maths as determined by standardized tasks, had reduced grey matter volume in the left IPS (Isaacs, et al., 2001). In a later investigation, Rotzer and colleagues demonstrated reduced grey matter density in the right IPS in younger subjects (Rotzer, Kucian, Martin, von Aster, Klaver, & Loenneker, 2008). These results reflect the developmental trajectory mentioned earlier (see **paragraph 1.5.3**) which indicates that non-symbolic number processing migrate from a predominantly right hemisphere locus to a more left lateralized or at least a bilateral one.

Critically, few studies so far have investigated the neural correlates of numerical processing in children with atypical maths development (including Developmental Dyscalculia) rather than wider genetic syndromes. Kucian and colleagues (2006) conducted an fMRI experiment with developmental dyscalculics of 11 years of age defined by discrepancy scores on a battery of mathematical and reading tests and general IQ. The in-scanner tasks included approximate and exact calculation, and a non-symbolic magnitude comparison task. The results showed similar levels of brain activations between DD and controls, even though the pattern of activation was generally weaker and more diffuse for DD in all conditions (Kucian, Loenneker, Dietrich, Dosch, Martin, & von Aster, 2006). So, even if the regions recruited when solving numerical tasks seem to be the same for children with Developmental Dyscalculia and their typically developing peers – and these include a network of frontal and parietal areas – , the degree to which this network of regions is activated discriminates the groups. This view is supported by the finding of a lack of neuronal modulation (as measured by the neuronal distance effect on a non-symbolic comparison

task) in the right IPS of a group of 12-year old children with Developmental Dyscalculia (Price, Holloway, Räsänen, Vesterinen, & Ansari, 2007).

These findings are further supported by an ERP study that investigated the neural correlates of the distance effect in DD children using a symbolic number comparison task (Soltész, Szucs, Dékány, Márkus, & Csépe, 2007). This study observed no significant differences in the distance effect at an early time window, but at a later time window (between 400 and 440 milliseconds), the control group showed a significant distance effect over right parietal areas, while the DD group did not.

All the aforementioned studies seem to consensually demonstrate right (or left) parietal lobes abnormalities associated with atypical maths development.

However, recent findings of structural brain imaging data have started to suggest that the brain abnormalities in Developmental Dyscalculia would instead be associated with the disruption of a more distributed brain network that involves the parietal lobes, but also the fusiform gyrus, the parahippocampal gyrus, and the right anterior temporal cortex, as DD children of 7-to-9 year old showed reduced grey matter density in these regions (Rykhlevskaia et al., 2009). Interestingly, reductions in white matter volumes have also been reported and these abnormalities include regions in the right temporo-parietal cortex, which also happened to show reduced fractional anisotropy (FA) in the DD population as measured by DTI analyses (Rykhlevskaia et al., 2009). Moreover, FA in this region was positively correlated with performance on a standardized maths task, suggesting that the neuroanatomical correlates of Developmental Dyscalculia might be reflected in grey matter abnormalities in a network of right temporo-parietal areas, but also in the white matter pathways associated with it.

1.6.6. Remediation studies of atypical development

The first studies that have informed the remediation of atypical development are learning studies on healthy adult populations. In these studies adults usually practice solving arithmetic problems that are not normally memorized in school and are subsequently tested with trained *versus* untrained lists of problems. These studies usually involve a neuroimaging component, that is, the test phase usually is done in the scanner so that the hemodynamic signal changes happening in the brain while participants perform the task can be measured. In a very influential study, Delazer and

colleagues (2003) had healthy adults practice a set of 18 complex multiplication problems and subsequently tested their latency and accuracy contrasting the problems that have been practiced with a set of unpracticed problems. The results of their study showed that participants were faster and less error prone on the 18 complex problems that were previously practiced. Moreover, the effects of practice (or training) were also evident in the neuroimaging data as trained problems activated different brain regions compared to untrained problems. Particularly, trained problems activated the left Angular Gyrus (AG), a region that had been previously associated with the retrieval of arithmetic problems (Dehaene et al., 2003); while untrained problems activated a more distributed network of regions that included the bilateral intraparietal sulcus and the bilateral inferior frontal gyrus (Delazer et al., 2003; *see also* Delazer, Ischebeck, Domahs, Zamarian, Koppelstaetter, et al., 2005). Using a similar paradigm, the same group of researchers demonstrated that the effects of practice were modulated by the type of arithmetical operation performed. That is, subtraction problems, even though improved with training, still activated a more distributed neural network including fronto-parietal regions, compared to multiplication problems (Ischebeck, Zamarian, Siedentopf, Koppelstätter, Benke, et al., 2006).

Behavioural remediation research of Developmental Dyscalculia has focused on the use of concrete material and informational feedback to the learner. Specifically, the use of Cuisenaire rods, number tracks and number cards has been incentivized in order to help the learner discover from direct manipulations of concrete objects (Butterworth & Yeo, 2004).

Two studies have looked at the effects of software games for the remediation of atypical maths development. One of the studies tested the effects of two computerized intervention programs in a group of kindergarten children. One of the software was specifically designed for the training of ‘number sense’ (*The Number Race*), while the other one was developed for the training of mapping numerosities to numerical symbols and number words (*Graphogame-Math*). Interesting, both programs improved children’s numerical understanding as measured by a number comparison task, yet no interaction was found in terms of type of software used (Räsänen et al., 2009). The other study tested the effect of one of the aforementioned software (*The Number Race*) on a group of 7-9 year old children with mathematical difficulties and showed a significant

improvement in tasks tapping the ‘number sense’ after intervention. Moreover, a transfer on subtraction, but not addition, was found (Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006).

Finally, the only study that has looked at the effects of intervention from a behavioural as well as neuroimaging perspective comes from Kucian and colleagues (2011). These authors conducted an fMRI study on 9 year olds who were trained using a specially designed computer game that stressed number to space associations. The effects of intervention were effective for both dyscalculic and typical learners both behavioural and neurologically. However, the dyscalculic group did not improve to the level of the typically developing children in terms of their behavioural nor neuronal profile (Kucian, Grond, Rotzer, Henzi, Schönmann, et al., 2011). Particularly, the dyscalculic group displayed more frontal, rather than parietal activation compared to their typically developing peers, suggesting that they fail to recruit efficient, specialized and mature brain processes to the same extent (Rivera, et al., 2005; *see also* Butterworth & Laurillard, 2010 *for a review*).

Altogether the remediation studies are still in their infancy. Yet, they seem to be producing promising results, on both behavioural as well as neuroimaging measures.

1.7. Individual differences in maths

Besides the classification of typical or atypical development, the heterogeneity of performance that is reflected in individual differences in maths abilities is large, and it seems to be equally common in typical and atypical development. Why are some people better at maths than others? The question of individual differences can be critical for mathematical education and for understanding the development of arithmetic from a more heterogeneous perspective. An interesting example of massive individual difference in the population is given by the distance effect. In **paragraph 1.5** we have described the importance of the distance effect in understanding proficiency with numbers and their representation. Indeed, a large proportion of individuals display a clear distance effect. Yet, not all individuals show a distance effect and even more surprisingly, a few people show an inverse distance effect (i.e. they are faster for near-distance versus far-distance comparisons). Far more important is the question on whether individual differences in understanding numbers and numerosities are reflected

in individual difference in maths achievement. In a study of 6-7 and 8-year olds, it has been reported that the size of the symbolic distance effect predicted maths proficiency in a task of arithmetic problem solving in all age groups. By contrast, the size of the non-symbolic distance effect was not predictive of arithmetical performance (Holloway & Ansari, 2009). This finding seems to suggest that symbolic, but not non-symbolic, number processing is important for arithmetical achievement. This has been replicated in a longitudinal study by De Smedt and colleagues (2009). On the other hand, a longitudinal study of 14-year olds has found the opposite: individual differences in the size of the non-symbolic distance effect were predictive of individual differences in maths achievement (Halberda et al., 2008). Yet, it is important to note that this contradictory finding might have depended on the type of measurement: reaction times in the case of Holloway & Ansari (*see also* De Smedt, Verschaffel, & Ghesquière, 2009), and accuracy in the case of Halberda and colleagues (2008).

Individual differences are also evident in neuroimaging studies. Cho and colleagues have looked at strategy use in a population of 7 to 8 year olds and found that the activation of Medial Temporal Lobe (MTL) areas was modulated by the type of strategy used by the child (i.e. either retrieval or counting) (Cho et al., 2011). Moreover, individual differences in arithmetic abilities of healthy adults with no history of learning disabilities were coupled with more efficient strategy use supported by memory retrieval (Grabner, Ansari, Reishofer, Stern, Ebner, & Neuper, 2007).

These examples clearly demonstrate the wide range of individual differences in the general population and more importantly how influential they can be in our understanding of maths processes and proficiency.

1.8. Exceptional development

So far we have described typical, atypical development and individual differences. At the other end of the spectrum there are mathematical talents, or individuals who show a great proficiency in maths. For example, Euler, Newton, Gauss, Turing, and Dalton are only a few cases of people who have advanced the quality of human life and knowledge through their exceptional numerical abilities. Interestingly, these ‘expert’ people seemed to have achieved their mastering of superior abilities in a way that suggests exceptional cognitive abilities (Mitchell, 1907). Yet, is this truly the

case? Moreover, what is the (standardized) criterion for defining someone as ‘expert’ in one domain? According to Ericsson & Charness (1994), the ‘expert’ candidate should demonstrate ‘reproducible superior performance’. It goes without saying that in experimental settings this usually refers to a reproducible superior performance compared to a group of people of the same, age, socio-economic background, education, and more often than not, comparable in terms of other types of abilities. In the case of maths experts we should refer to people who present ‘superior’ maths abilities in the context of otherwise normal intelligence, and compared to a group of people matched on age, socio-economic status and education.

In previous paragraphs we have presented evidence that humans are endowed with a specific capacity for representing and processing numerosities, which seems to have its distinct neural correlate in the brain. Moreover, we have seen how this capacity can be disrupted (as in cases of Developmental Dyscalculia), or not as well developed (as demonstrated by individual differences in the population). Even more than the basic capacity for numbers, arithmetical abilities can be low, average, or superior. So, even if the ‘number sense’ can predict the development of low and average ability, this core ability may have little or nothing to do with superior maths performance. Interestingly, one of the factors for excellence has been proposed to be motivation (*first proposed by* Smith, 1983). According to this idea, calculating prodigies, expert mathematicians, or even individuals who show exceptional performance in maths (or just arithmetic) derive their abilities from a great interest in numbers (*concept adopted from* Smith, 1983). This is for example the case of autistic savants (Horwitz, Deming, & Winter, 1969; *see also* Cowan & Frith, 2009; Wallace, Happè, & Giedd, 2009) and indeed many calculating prodigies often report that their abilities come from their great interest in numbers (Butterworth, 2006). Another view is that these people present exceptional skills in other cognitive domains which they then apply to the domain of mathematics. So exceptional memory (the calculating prodigy Bidder, *quoted by* Smith, 1983, p.53; *see also* Ericsson & Kintsch, 1995) or visual abilities (the calculating prodigy Bidder, *quoted by* Smith, 1983, p.212) would boost maths performance in people that can successfully apply them to the domain of mathematics.

Another possibility is that maths expertise can exist as a domain-specific achievement (*see* Gardner, 1983 on maths as part of logico-mathematical intelligence;

see also Mitchell, 1907), as it is has been proposed to be the case for core maths impairments (see **paragraph 1.6.2**).

Finally, it is possible that an interaction of motivational skills – triggered by a great interest for numbers and maths - and exceptional abilities in certain cognitive domains could together explain the ‘phenomenon’ of maths expertise. Indeed, in his book ‘Hereditary Genius’, Galton defines exceptional maths skills as the result of “zeal and the ability to do a very great deal of hard work”, coupled with a general capacity that seems to anticipate Spearman’s *g* (Galton, 1979, *originally published in 1869*)².

Critically, there has been very limited neuroscientific investigation on expert calculation abilities. A seminal study by Pesenti and colleagues involved the behavioural and cognitive examination of calculating prodigy Rüdiger Gamm (Pesenti, Zago, Crivello, Mellet, Samson, et al., 2001). Using Positron Emission Tomography (PET), Pesenti and colleagues found that Gamm utilized a different brain network compared to controls when solving calculation problems. This network encompassed the right prefrontal and medial temporal areas but also posterior visual areas bilaterally. According to the authors, Gamm was able to use well developed cognitive functions such as long-term memory (*see* Ericsson & Kintsch, 1995), and short term memory to achieve excellent performance (*see* Pesenti, Seron, Samson, & Duroux, 1999 *for a behavioural account on Gamm’s digit span*). Moreover, when asked to solve unmemorized number facts, Gamm (and to some extent control participants as well) activated an extensive visual processing system bilaterally which suggests that numbers were manipulated in some sort of visual representational way. The latter idea contrasts with the more consensual idea that mental arithmetic requires sub-vocal rehearsal (Logie, Gilhooly, & Wynn, 1994). Yet, their finding could be explained by the fact that Gamm was relying on good visualization abilities and applying them to solve the arithmetic problem presented. Indeed, many calculators report that their early experiences involve manipulables that are usually visual (Smith, 1983, p.212; *see also* Butterworth, 2006).

As we have seen, studies on maths expertise are only few and sparse. Moreover, they mainly rely on descriptive evidence of laboratory studies that were not well controlled, and most of them are single case studies. However, from the evidence we

² Although Galton thought each of these factors was highly inherited.

have gathered, it seems that an interaction of motivational factors and, depending on the individual, other highly developed cognitive abilities put to use in the maths domain is what is need to ‘make a prodigy’. Indeed, there is now extensive evidence for activity dependent brain plasticity (Pascual-Leone & Torres, 1993; Schlaug, Jancke, Huang, Staiger, & Steinmetz, 1995; Schlaug, Jancke, Huang, & Steinmetz, 1995) that is modulated by level of expertise (Gauthier, Tarr, Anderson, Skudlarski, & Gore, 1999; Gauthier, Skudlarski, Gore, & Anderson, 2000).

1.8.1. Autism Spectrum Disorder: a model for exceptional mathematical abilities

In the previous paragraph we described the different hypotheses that have been proposed in order to explain cases of exceptional maths skills. In this paragraph we take the neurodevelopmental condition of Autism Spectrum Disorder and describe it as a potential model for studying exceptional maths abilities.

Autism Spectrum Disorder (ASD) is a neurodevelopmental disorder of an estimated prevalence of 1:100 (Baron-Cohen, Scott, Allison, Williams, Bolton, et al., 2009; Fernell & Gillberg, 2010; Rice, 2009). ASD is characterized by a complex phenotype that includes social and emotional processing deficits -which manifest in impairments in social interaction and verbal communication-, and a distinct behaviour mostly characterized by restricted and repetitive actions (Volkmar, Lord, Bailey, Shultz, & Klin, 2004). Yet, it has been noted that the ‘altered developmental trajectory that defines ASD can also lead to remarkable cognitive strengths’ (Baron-Cohen & Belmonte, 2005) and that children with ASD might present ‘islets of ability’ in various domains (Happè & Frith, 2010), suggesting that we should consider autism as a different type of information-processing system (Jolliffe & Baron-Cohen, 1997).

One of the domains in which individuals with ASD seem to present exceptional abilities is mathematics. Hence, one of the most cited examples is the one of calendrical savants (Cowan & Frith, 2009; Frith, 1989; Happè & Frith, 2010; Hermelin & O’Connor, 1986; Wallace, et al., 2009). However, support for the claim of a link between ASD and exceptional maths abilities has a much long history and goes back to the original studies of Kanner (1943) who observed that children with ASD performed well on some parts of tests of intelligence. He particularly reported that parents of

children with ASD often described them as ‘serious minded, perfectionist individuals, with an intense interest in abstract and mathematical ideas’. Moreover, in one of his first papers, the pediatrician Hans Asperger wrote that ‘for success in science or art a dash of autism is essential’ recognizing this tight connection between the syndrome and mathematical talent and its great implications in the everyday life of individuals with ASD. Particularly, he noted that the professions of a large proportion of people with ASD were determined by their mathematical abilities (Asperger, 1944, *translated* by Frith, 1991).

More recently, Baron-Cohen and colleagues (2001, 2003) have tried to quantify this relation (i.e. between Autism and maths talent) in a descriptive manner by devising a self-administered questionnaire for measuring the degree to which an adult with normal intelligence has the traits associated with the autistic spectrum. Interestingly scientists scored higher than non-scientists for ‘autism associated traits’. Even more interestingly, within the sciences, mathematicians, computer scientists, and engineers presented more ‘autistic traits’ than the individuals in the life sciences such as medicine and biology (Baron-Cohen, Wheelwright, Skinner, Martin, & Clubley, 2011; Baron-Cohen, Richler, Bisarya, Gurunathan, & Wheelwright, 2003). Their research was taken a step further in study that looked at a large sample of undergraduate students from Cambridge University (378 mathematics and 414 control disciplines) which revealed ‘a three to sevenfold increase for autism spectrum condition amongst the mathematicians’ (Baron-Cohen, Wheelwright, Burtenshaw, & Hobson, 2007). Moreover, the same study suggests that mathematical abilities may be genetically influenced, as it has been reported that a significant proportion of close relatives of individuals with ASD achieve considerable success in mathematics, science and engineering (Baron-Cohen et al., 2007).

In the previous paragraph we have highlighted the criteria for defining expertise in a given domain. One of the criteria mentioned was that expertise is defined by superior skills in one domain with otherwise general normal abilities. Indeed, Jones and colleagues, reported better than expected, given IQ, mathematical abilities in ASD. Specifically, these authors quantified the discrepancy (i.e. discrepancy scores) between IQ and achievement skills (i.e. reading and mathematics) and found that the largest

proportion of individuals with ASD with a high discrepancy score showed it in the mathematical domain (Jones, Happè, Golden, Marsden, Tregay, et al., 2009).

One of the hypotheses for expert calculators is to possess exceptional skills in other cognitive domains such as visualization abilities that can be successfully applied when solving maths problems. Interestingly, higher visual processes skills and visuo-spatial peaks have been consistently reported in Autism Spectrum Disorder (Happè, 1994). Moreover, individuals with autism often perform better than controls on the Embedded Figures Test, where a simple shape needs to be identified within a more complex figure (Shah & Frith, 1983). This pattern of results has been explained in terms of the ‘weak central coherence’ theory which suggests that individuals with autism tend to favor local over global processing (*first proposed by* Frith, 1989; *see* Happè & Frith, 2006 *for a review*), or in other words, they tend to process information at an analytical rather than global level. Interestingly in a behavioural study on counting skills of dot patterns presented in a canonical way, Jarrold and Russell showed that children with autism differed to controls and to children with moderate learning disabilities in their counting speeds confirming their tendency towards an analytic level of processing (Jarrold & Russell, 1997).

Furthermore, a recent behavioural study has demonstrated that teenagers with autism were more accurate in the formation and comparison of mental images than controls. Moreover, this effect was modulated by visuo-spatial abilities, suggesting that the ability to form, access and manipulate visual mental representations may be more developed in this population (Soulières, Zeffiro, Girard, & Mottron, 2011). Interestingly, it has been reported that calendrical calculators tend to break down calendars into ‘fragments’ of dates (Heavey, Pring, & Hermelin, 1999). Indeed, a recent proposal has highlighted the fact that calendrical calculation skills might emerge from combining the natural tendency of these individuals to segment large chunks of information with their idiosyncratic interest in dates (Wallace et al., 2009).

Finally, in his very influential theoretical account of Autism Spectrum Disorder, Baron-Cohen and Belmonte (2005) have proposed that systematic, logical and analogical thinking is enhanced in individuals with ASD. Specifically, ‘systemizing’ has been demonstrated to be a strength in Autism Spectrum Disorder and could be related to savant skills development (Baron-Cohen, Ashwin, Ashwin, Tavassoli, & Chakrabarti,

2009). Interestingly, ‘systemizing’ includes implicit or explicit rule extraction as applied to predictable and lawful ‘systems’, which is typical of mathematics.

1.8.2. Neurobiology of Autism Spectrum Disorder

The neural correlates of Autism Spectrum Disorder (ASD) have been widely studied from an anatomical as well as functional perspective (i.e. functional Magnetic Resonance Imaging). One of the reasons for this is that many theories of the disorder have grounded their explanations of the behavioural manifestations of ASD on functional disruptions or aberrancies of brain mechanisms that are often coupled with anatomical differences in the cortical thickness of brain structures. I here provide a brief review of the literature of the neurobiology of the disorder mostly to provide evidence for the idea that ASD manifest itself as a different processing system compared to the typical brain. Hence, both grey and white matter abnormalities, together with task-based functional aberrancies have been identified throughout the brain of individuals with Autism, which reflect the distributed nature of brain involvement in ASDs (Carper, Moses, Tigue, & Courchesne, 2002; Muller, 2008) and demonstrate areas of dysfunctional cortical activation and atypical cortical specialization in this population (Stigler, McDonald, Anand, Saykin, & McDougle, 2011). Among the structural abnormalities, increased frontal lobe volume has emerged as one of the most consistent findings (Brun, Nicolson, Lepore, Chou, Vidal, et al., 2009; Carper et al., 2002; Hazlett, Poe, Gerig, Smith, & Piven, 2006; Herbert, Ziegler, Makris, Filipek, Kemper, et al., 2004; Palmen, Hulshoff Pol, Kemner, Schnack, et al., 2005). Particularly, regional differences in grey matter volume have been found in the middle and superior frontal gyri, and the medial orbitofrontal cortex (OFC) (Bonilha, Cendes, Rorden, Eckert, Dalgalarrrondo, et al., 2008; Hadjikhani, Joseph, Snyder, & Tager-Flusberg, 2006; Hardan, Libove, Keshavan, Melhem, & Minshew, 2009; Hyde, Samson, Evans, & Mottron, 2010; McAlonan, Cheung, Cheung, Suckling, Lam, et al., 2005; Waiter, Williams, Murray, Gilchrist, Perrett, & Whiten, 2004). Increased grey matter volumes in individuals with ASD have also been reported in the cerebellum (Hardan, Minshew, Mallikarjunn, & Keshavan, 2001; Herbert, Ziegler, Deutsch, O'Brien, Lange, et al., 2003; Palmen et al., 2005), the amygdala and the hippocampus (Schumann, Hamstra, Goodlin-Jones, Lotspeich, Kwon, et al., 2004), the left planum temporale (Herbert,

Harris, Adrien, Ziegler, Makris, et al., 2002; *see also* Knaus, Silver, Dominick, Schuring, Shaffer, et al., 2009 *for a developmental account*) and the caudate (Brambilla, Hardan, di Nemi, Perez, Soares, & Barale, 2003; Hollander, Anagnostou, Chaplin, Esposito, Haznedar, et al., 2005). Also, decreases in brain volumes have been reported in the Vermis (Stanfield, McIntosh, Spencer, Philip, Gaur, & Lawrie, 2008), the right insula (Kosaka, Omori, Munesue, Ishitobi, Matsumura, et al., 2010), the superior temporal sulcus (STS) (Boddaert, Chabane, Gervais, Good, Bourgeois, et al., 2004; Levitt, Blanton, Smalley, Thompson, Guthrie, et al., 2003), the anterior cingulate cortex (ACC) (Haznedar, Buchsbaum, Metzger, Solimando, Spiegel-Cohen, & Hollander, 1997) and the anterior part of the corpus callosum (Frazier & Hardan, 2009).

Task-based functional aberrancies have been reported in the fusiform gyrus in the context of social information processing deficits (Schultz, Gauthier, Klin, Fulbright, Anderson, et al., 2000; Schultz, Grelotti, Klin, Kleinman, Van der Gaag et al., 2003). Particularly, decrease task-based activation has been demonstrated in the fusiform face area (FFA) when individuals with ASD were asked to discriminate between different face stimuli (Humphreys, Hasson, Avidan, Minshew, & Behrmann, 2008). Additionally, it has been noted that individuals with ASD seem to display less recruitment of the FFA in response to faces of strangers in comparison to familiar faces (Pierce & Redcay, 2008). Furthermore, decrease activation of the FFA has been documented in studies of matching facial expression (Wang, Dapretto, Hariri, Sigman, & Bookheimer, 2004) and emotions (Piggot, Kwon, Mobbs, Blasey, Lotspeich, et al., 2004).

Functional activations have been demonstrated to differ even in tasks in which adults with autism show similar behavioural results to control participants. Koshino and colleagues investigated verbal working memory performance using an n-back task with letters in a group of adults with ASD. The first interesting finding was that, on equal behavioural performance, the autism group exploited a visually-oriented strategy, while the control group relied more on verbal strategies. This behavioural effect was supported by imaging data showing that individuals with autism exhibited increased task-related activation in the posterior regions including inferior temporal and occipital regions (Koshino, Carpenter, Minshew, Cherkassky, Keller, & Just, 2005).

Finally, aberrant patterns of neural activation have also been reported in tasks where individuals with Autism show superior abilities compared to typically developing

matched controls. Indeed, superior abilities in visual search tasks – the Embedded Figure Test – (Almeida, Dickinson, Maybery, Badcock, & Badcock, 2010; Ring, Baron-Cohen, Williams, Wheelwright, Bullmore, et al. 1999) were associated with an aberrant neuronal activity (i.e. enhancement) in ventral occipital areas coupled with abnormally low activation in prefrontal and parietal areas. These studies seem to be in line with the theoretical account of superior analytical and systematic abilities in ASD previously described (Baron-Cohen et al., 2001, 2003; Baron-Cohen & Belmonte, 2005). Their theory in fact predicts abnormal high activations in unimodal or low-level processing areas coupled with lower activation in frontal areas and other integrative regions. Hence, the authors discuss the aforementioned neuroimaging studies as evidence that superior abilities in autism seem to possess their distinct neuronal signature (Baron-Cohen & Belmonte, 2005).

The idea of abnormal activation processes in individuals with ASD has been summarized in a very influential theory on the neuronal organization of the autistic brain, which suggests that its main characterization is given by the disruption of long-range connectivity with a relatively intact or even enhanced local connectivity (Just, Cherkassky, Keller, & Minshew, 2004). Even if not explicitly stated, this theory seems to derive from the model of autism firstly described by Uta Frith and summarized in the ‘weak central coherence theory’ (1989), which ultimately proposes how savant skills can emerge in these individuals (Happé & Frith, 2010). Indeed, a recent integrative model of savant syndrome postulates that the disruption of global connectivity in a parallel distributed cortical network is what contributes to impairments in the integrated cognitive processing seen in autism- such as executive functions and social cognition -. On the other hand, this reduced inter-regional ‘collaboration’ could be the process that leads to an enhancement of neural activity and connectivity in local cortical regions, resulting in the specialization and facilitation of low-level cognitive processing, which could be the underlying mechanisms supporting the development of savant skills (Takahata & Kato, 2008, *see also* Baron-Cohen & Belmonte, 2005).

1.8.3. Neural correlates of maths abilities in Autism Spectrum Disorder

Up to now only very few studies have looked at mathematical abilities in Autism Spectrum disorder from a neuroimaging perspective. These have usually focused on

calendrical autistic savants and only report single-cases results. In 2009, Cowan and Frith conducted an fMRI study on two autistic calendrical savants with the intent to answer the question of how these exceptional skills emerge (i.e. either from memory or from extended calculation practice). The authors found increased parietal activation during both mental arithmetic and calendar date problems. Moreover, the activity in this region was modulated by the temporal distance of calendar dates suggesting that the calendar skills observed in savants are the consequence of intensive practice with numbers and calculation.

This finding seems to be supported by a recent anatomical study investigating cortical thickness in a case of ASD with calendrical skills and exceptional drawing abilities. The authors found reduced cortical thickness in regions associated with social cognitions, while increased cortical thickness was found in the bilateral segment of the superior parietal lobe (Wallace, et al., 2009). The authors suggested that increased thickness in the regions of the parietal lobes might have been the result of extensive calculation practice and manipulation of numbers given that these regions are implicated in calculation (Dehaene et al., 2003; Dehaene, Molko, Cohen, & Wilson, 2004; but see **paragraph 1.4** for a more detailed account). Interestingly, this study reports a significantly better performance on a standardized arithmetic task in this individual. Unfortunately, the anatomical results were not correlated with any neuropsychological measure, and structural longitudinal data was not available, making it difficult to comprehensively interpret their findings.

1.9. ‘Measuring’, ‘decoding, and ‘charging’ the brain

In the next two paragraphs I will briefly introduce and describe the principles underling the techniques and the means of analyses that are going to be used in the experimental chapters of this thesis. First, I will describe the principles of functional Magnetic Resonance Imaging (fMRI) as a tool to measure cerebral brain activity (**paragraph 1.9.1**). Moreover, I will look into the latest developments on data analyses of fMRI data and their useful applications especially with clinical and developmental populations (**paragraph 1.9.1.1**). Second, I will introduce the field of brain stimulation and describe the principles of transcranial Direct Current Stimulation (tDCS), the

technique I intend to use in order to ‘modify’ brain activity and associated behavioural responses (**paragraph 1.9.2**).

1.9.1. Functional Magnetic Resonance Imaging (fMRI)

Functional Magnetic Resonance Imaging (fMRI) is a technique for measuring brain activity that has now been extensively used in the field of cognitive neuroscience (*generally attributed to Ogawa & Kwong in the 1990s*; Kwong, Belliveau, Chesler, Goldberg, Weisskoff, et al., 1992; Ogawa, Lee, Kay, & Tank, 1990; Ogawa, Tank, Menon, Ellermann, Kim, et al., 1992). In recent years, fMRI has been applied to other areas of investigation and commerce such as brain-computer interfaces, lie detection, neuro-feedback, and it has been largely used in clinical settings such as communicating with locked-in patients, or pre-surgical planning (*see deCharms, 2008 for a review*). fMRI detects the changes in blood oxygenation and flow that occur in response to neural activity, two phenomenon that are highly coupled (*first proposed by Angelo Mosso, cited in James, 1980*). The logic behind the technique is that when neuronal activity increases there is an increased demand for oxygen. Hence the local response is an increase in blood flow to regions of increased neural activity.

fMRI like MRI is based on the properties of relaxation of hydrogen nuclei, either by itself (MRI), or when coupled with water (fMRI). Particularly, fMRI measures the properties of the molecule of haemoglobin (i.e. the molecule that delivers oxygen to neurons) and its coupling with oxygen. Thus, the molecule of haemoglobin is diamagnetic (zero magnetic moment) when oxygenated, but paramagnetic (magnetic moment) when deoxygenated. This degree of oxygenation modulates the MR signal and it can be measured by the T_2^* gradient (i.e. the rate of the decay measured in terms of transverse relaxation). Moreover since blood oxygenation varies according to the levels of neural activity these differences can be used to detect brain activity by directly measuring what has been referred to as the Blood Oxygenation Level Dependent signal (BOLD). Interestingly, there is a momentary decrease in the BOLD signal after neural activity increases, which has been termed the ‘initial dip’ in the hemodynamic response. This is followed by a period where the BOLD signal increases, not just to a level where oxygen demand is met, but overcompensating for the increased demand. This means the blood oxygenation actually increases following neural activation, and this is what most

studies measure. The first fMRI experiments compared the activity (i.e. measured in terms of BOLD signal) elicited by a very easy task (i.e. finger tapping) to a baseline condition (i.e. rest). Subsequently, experiments got refined in order to be able to answer more detailed theoretical questions in terms of cognitive abilities and structures. Nowadays fMRI studies in the field of cognitive neuroscience mostly compare the cerebral activity during an experimental condition (e.g. a calculation task) to a control condition (e.g. a symbol detection task), which has the same visual and motor components to isolate the neural correlates of that specific cognitive ability (i.e. calculation). Moreover, fMRI studies can investigate the interaction between neural correlates of certain abilities and different clinical and non-clinical populations.

fMRI data are usually analyzed using dedicated software that utilize Univariate methods based on the General Linear Model (GLM) (Friston, Holmes, Worsley, Poline, Frith, & Frackowiak, 1995). The GLM analyzes each voxel's data separately and creates voxel-wise t-statistics maps to identify task-related brain activation (i.e. task versus control) both within (individual subjects analyses) and between subjects (group analyses). Yet, the GLM does not take into account relationships between voxels (i.e. neighboring voxels are likely to activate or inactivate together and sometimes some preprocessing procedures embedded in the GLM model intentionally correlate neighboring voxels). Thus, new statistical methods for the analyses of fMRI data based on multivariate rather than univariate approaches have started to be implemented in research studies (*see next paragraph*).

1.9.1.1. Predictive analyses tools – Support Vector Machine methods

Support Vector Machine methods (SVM) are non-probabilistic binary linear classifiers that can predict, given a set of data for each given input, which of two possible classes the input is a member of. In other words, given a set of training examples, each marked as belonging to one of two categories, an SVM training algorithm builds a model that assigns new examples into one category or the other. Here below we describe two types of analyses that have recently been developed in the field of cognitive neuroscience as they will be the basis of our investigations.

Multivariate pattern analyses (MVPA). Multivariate Pattern analyses (MVPA) is a multi-voxel method that is based on the relationship across voxels of activation. So, MVPA can determine which voxels in the brain are able to discriminate between two groups of interest (or two different conditions), based on the pattern of fMRI activity measured across sets of multiple voxels. Specifically, the algorithm takes the entire voxel data set (i.e. the whole brain) and divides it by the various conditions or the various groups in the experiment. The algorithm then learns a training model – by summarizing the information contained in the voxels – which aims at best discriminating between conditions or groups (or both).

After the learning phase, the test phase compares the different models generated in the learning phase to test data (i.e. new observations that were not included in the initial data set, to prevent ‘over-fitting’ the model to the data) in order to classify which condition (or group), if any, is represented by the brain activity patterns generated. In other words, a classifier is a function that takes the values of various features (i.e. brain density, or functional brain activation) in a sample and predicts which class (e.g. participants group or experimental condition) that sample belongs to (Abrams, Bhatara, Ryali, Balaban, Levitin, & Menon, 2010; Cho et al. 2011; Haynes & Rees, 2006; Kriegeskorte, Goebel, & Bandettini, 2006). Thus, it permits making inferences about patterns of difference, rather than localizing results to individual voxels. Moreover, this method identifies brain regions that provide the greatest information about group membership and it has been proved to be very informative with clinical populations (Coutanche, Thompson-Schill, & Schultz, 2011; Ecker, Marwuand, Mourao-Miranda, Johnston, Daly, et al., 2010; Uddin, Menon, Young, Ryali, Chen, et al., 2011), developmental populations (Cho et al., 2011), and in neurological disorders (Cuingnet, Gerardin, Tessieras, Auzias, Lehericy, et al., 2010; Rizk-Jackson, Stoffers, Sheldon, Kuperman, Dale, et al., 2011; Westman, Simmons, Zhang, Muehlboeck, Tunnard, et al., 2010; Wilson, Ogar, Laluz, Growdon, Jang, et al., 2009).

In summary, MVPA analyses of fMRI data can provide information that is usually overlooked by univariate approaches: while univariate analyses can show which brain regions differ on a relevant dimension (e.g. functional brain activation) between groups, multivariate analyses can reveal which set of brain voxels, in combination, can be used to discriminate between two participant groups.

Support Vector Machine –based algorithms. Another application of Support Vector Machine analyses is to investigate relationships between clinical and behavioural characteristics and differences in functional or structural brain activity patterns.

Recently, some studies have applied this methodology with the aim of looking for biomarkers of neurodevelopmental conditions. This is done in the context of classification, namely assigning a participant to one of two discrete groups to decode the disease state of a participant's brain (Coutanche et al., 2011; Uddin et al., 2011). Particularly, this is usually achieved by computing correlation coefficients between the diagnostic criteria and the distance from the optimal cut-off in 3D space (i.e. the hyperplane) separating the two groups for each region of interest (Ecker et al., 2010). For example, Uddin and colleagues found that subjects with the most severe autism as indexed by a clinical test (i.e. ADI-R scores) were better discriminators between groups on the basis of grey matter densities in certain nodes of the default mode network than subjects with less severe symptomatology. In other words, the most severely affected subjects were located furthest away from the hyperplane separating the two groups in the multivariate classification analyses.

More recently, there has been increasing interest in prediction of continuous values such as neuropsychological (Duchesne, Caroli, Geroldi, Collins, & Frisoni, 2009; Wang, Fan, Bhatt, & Davatzikos, 2010; Rizk-Jackson, Stoffers, Sheldon, Kuperman, Dale, et al., 2011) or cognitive characteristics (Cohen, Asarnow, Sabb, Bilder, Bookheimer, et al., 2010; Kahnt, Heinzle, Park, & Haynes, 2011; Valente, Martino, Esposito, Goebel, & Formisano, 2011). The prediction of continuous values from neuroimaging data (i.e. regression) is achieved through Support Vector Regression (SVR) methods, a subclass of SVM methods (Cohen et al., 2011). Specifically, SVR is a high-dimensional regression method that allows investigators to make real-valued predictions on the data. It can therefore be successfully used to reveal the root of individual differences in cognitive processes, both within the normal range or in impaired individuals (Cohen et al., 2010; Dosenbach, Nardos, Cohen, Fair, Power, 2010).

1.9.2. Transcranial Direct Current Stimulation (tDCS)

Transcranial Direct Current Stimulation (tDCS) is one of the noninvasive methodologies that have been used in order to affect neural activity in the brain and elicit responses at a behavioural level (Wagner, Valero-Cabre, & Pascual-Leone, 2007), the other one being Transcranial Magnetic Stimulation (TMS). Here I will only describe the principles of tDCS as it is going to be the methodological approach used in this thesis.

tDCS applies a constant weak current through the scalp via pairs of electrodes to affect the excitation of neuronal populations, with its maximal effect on the stimulated area beneath the electrodes. This methodology has a long history, and goes back to the pivotal studies of Alessandro Volta and Luigi Galvani at the beginning of the 19th century. Later, it started to be used in clinical settings mainly aimed at reducing depression (Finger, 1994). Only in very recent years tDCS has started to be used as a tool in cognitive and more basic neuroscience studies (Stagg, O'Shea, Kincses, Woolrich, Matthews, & Johansen-Berg, 2009; Utz, Dimova, Oppenländer, & Kerkhoff, 2010).

One of the main reasons tDCS is becoming very popular, particularly in the field of cognitive neuroscience and rehabilitation is that, unlike TMS, it can either enhance (anodal stimulation) or reduce (cathodal stimulation) neuronal excitation. This is achieved by the modification of the transmembrane neural potential (i.e. via its depolarization or hyperpolarization) in order to influence the level of excitability and thereby modulate the firing rate of individual neurons (Wagner et al., 2007). So at the behavioural level performance is enhanced with anodal stimulation and impaired with cathodal stimulation. Interestingly, animal studies found that the long-lasting effects are protein synthesis-dependent and accompanied by modification of intracellular Cyclic adenosine monophosphate (cAMP) and calcium levels, and therefore share some features with long-term potentiation (LTP) and long-term depression (LTD) hence enhancing or reducing synaptic transmission (Nitsche, Cohen, Wassermann, Priori, Lang, et al., 2008). In humans, magnetic resonance spectroscopy has revealed that the effects of tDCS at a molecular level are coupled with the reduction in spontaneous neural activity of GABAergic (anodal stimulation) and glutamatergic activity (cathodal

stimulation) (Stagg, Best, Stephenson, O'Shea, Wylezinska, et al., 2009). Practically, the anodal electrode is the positively charged electrode and the cathodal electrode is the negatively charged one. The current flows (through the scalp) from the anodal electrode to the cathodal one, and creates a circuit to either boost neural activity and enhance performance, or reduce it to impair performance.

Because of its effect, tDCS is a very promising tool for the rehabilitation of psychiatric illnesses (Boggio, Bermanpohl, Vergara, Muniz, Nahas, et al., 2006; Kalu, Sexton, Loo, & Ebmeier, 2012), neurodegenerative disorders (Boggio, Ferrucci, Rigonatti, Covre, Nitsche, et al., 2006; Nardone, Bergmann, Christova, Caleri, Tezzon, et al., 2012), and acquired cognitive conditions, such as cases of aphasia (Baker, Rorden, & Fridriksson, 2010; Holland, Leff, Josephs, Galea, Desika, et al., 2011).

1.10. Theoretical roadmap

In previous paragraphs I have highlighted the behavioural and neurobiological evidence of typical, atypical and exceptional abilities in maths. Specifically, we have seen: (i) how one manifestation of poor numeracy skills (i.e. Developmental Dyscalculia) is considered as a core congenital deficit; that (ii) there can be other forms of atypical development (i.e. low numeracy) that are not the consequence of a core congenitally-based deficit; that (iii) even within the typical population, there are pronounced individual differences; and finally that (iv) cases of exceptional abilities are unlikely to be determined by an exceptional 'starting kit' of core abilities, but rather they seem to be triggered by a great interest for numerical stimuli, which might be coupled with motivational factors, educational context, and possibly superior domain-general skills such as memory or visual abilities that are successfully applied to maths.

As one can see from the literature review proposed here, the challenge of summarizing such evidence within a model that would account for all these phenotypical manifestations of maths abilities, their causes, and the intrinsic heterogeneity of the discipline of mathematics becomes rather apparent.

So far two distinct proposals have been put forward on the development of maths skills: (i) 'the modular view' and (ii) 'the continuum view', which have tried to explain cases of atypical development and individual differences in the population respectively.

1.10.1. The modular view

The first of these proposals is the ‘modular view’, which has been used to explain Developmental Dyscalculia (DD). This view is based on the fact that individuals with DD are fundamentally different from their peers both behaviourally and neurologically. Hence DD has been explained by the ‘*defective number module hypothesis*’ (Butterworth, 1999, 2005; *see also* Wilson & Dehaene, 2007). This hypothesis seems to be borrowed from Fodor’s notion of the modularity of the mind (Fodor, 1983), which states that modular systems must fulfil certain properties: (i) *domain-specificity*, as modules only operate on certain types of inputs and are therefore specialized; (ii) *informational-encapsulation*, as modules do not refer to other psychological systems in order to operate, and are therefore independent; (iii) *obligatory firing*, since modules process in a mandatory manner; (iv) *fast speed*, which would be the consequence of the encapsulation property; (v) *shallow outputs*, as the output of modules is very simple; (vi) *limited accessibility*, also a consequence of encapsulation; (vii) *characteristic ontogeny*, as there are regularities of development and (viii) *fixed neural architecture*. The ‘*defective number module hypothesis*’ implies that the number module encompasses all the aforementioned properties and argues that it is this core system for numbers, involving a single brain area abnormality (Isaacs et al., 2001; Price et al., 2007) that is disrupted in cases of Developmental Dyscalculia (Butterworth, 2005, 2010; Landerl et al., 2004; Piazza et al., 2010; Wilson & Dehaene, 2007). However, it seems that this view can only account for the condition of Developmental Dyscalculia. Yet, as we mentioned earlier (see **paragraph 1.6.2**) there are other types of low numerical skills, intended as a difficulty to develop normal arithmetical abilities and indeed, a recent study has reported that up to 73% of children with low numeracy skills are not dyscalculics (Reigosa-Crespo et al., 2012). The modular view alone does not seem to account for such cases. Moreover, by this model alone it is difficult to explain individual differences in maths in the average population and also cases of exceptional abilities.

1.10.2. The continuum view

The second proposal is the ‘continuum view’ which has mostly intended to explain individual differences. Under this notion, maths abilities are represented along a continuum that goes from low, to average, up to exceptional skills (Dowker, 2005). We can assume that, according to this view, individuals with Developmental Dyscalculia would be at the lowest end of the continuum, followed by people with low numeracy, then average people with their heterogeneous maths abilities, all the way up to exceptional individuals. This is usually the concept that standardized tests are based upon where cut-offs are arbitrarily dividing the scale in order to define typical and atypical development, or performance in general (i.e. lowest quartile of the distribution, lowest 10th percentile, etc.). Of course there is a continuum of performance or ability, but is this the result of an underlying continuum of cognitive functions? And if so, what would this function be? The ‘continuum view’ does not specify this, yet, as it is posed, it could easily be referable to the *g* factor and the general theory of intelligence which assumes that the *g* factor is implicated in all cognitive functions (*first proposed by Spearman, 1904*). However, even if this view might partly explain individual differences (i.e. if maths abilities were intended as ‘general maths intelligence’ they would be normally distributed along a continuum), it cannot account for the condition of Developmental Dyscalculia in its core disability, and the fact that these pupils show average or sometimes higher than average IQ. Moreover, this view could only partly explain exceptional abilities in the sense that external sources of variability, motivational factors or transfer of information between cognitive processes, are not explicitly taken into account in this model.

To sum up, we have discussed how these rather radical theories on maths abilities could only partially explain the heterogeneity of maths skills. Most importantly none of them seems to be able to account for the occurrence of various scenarios in the empirical data such as specific (Developmental Dyscalculia) or general (Low Numeracy) maths impairments, the variability of maths skills in the normal population, and exceptional cases of maths proficiency. Furthermore, none of these proposals has specifically stated how either ‘the number module’ or ‘general maths skills as

distributed along a continuum’ can ultimately interact with other cognitive abilities to generate different profiles of numerical and arithmetic skills.

To conclude, it is hard to think that the human brain starts out modularized for specific cognitive functions. At the same time, it is difficult to think that cognitive abilities are nothing but a ‘cognitive continuum’ with little specialization. More recently, some authors have proposed the ‘neuroconstructivism view’ which poses that neural specializations in the brain are not directly inherited, rather they depend on the interaction with the environment in a process where specializations derive from the coupling between experience and ‘intrinsic biases in neural receptivity’ (Westermann, Thomas, & Karmiloff-Smith, 2010). This thesis aims to collect empirical evidence to elucidate which of these models could better account for typical, atypical and exceptional development of numerical and arithmetic skills.

1.11. Research Aims

In a series of experiments I will investigate the heterogeneous spectrum of mathematical abilities using behavioural and neuro-imaging techniques. Through a convergence of evidence, I hope to characterize typical, atypical and exceptional development of arithmetical skills. The first aim of this thesis will be to establish the ‘nature’ of the endowed system for numerical representation (i.e. ‘the number sense’) and how it interacts with the acquisition of arithmetical skills. Secondly I assess two clinical populations with different levels of symptomatology in order to better characterize the profile of atypical development and differentiate between maths disabilities and difficulties. To this end, I will outline the different developmental trajectories in the population that can lead to poor arithmetical skills (**Chapter II**).

Chapter III explores the interaction between the inhibitory processes of working memory and stimulus-material with arithmetical skills. This aims to clarify whether maths difficulties are the result of an impairment of domain-general rather than domain-specific functions (e.g. Geary, 1993; O’Connor et al., 2000).

Some individuals in the population display highly proficient mathematical abilities. In **Chapter VI**, I explore the behavioural and neural manifestations of

exceptional abilities in a population of children with high functioning autism to experimentally assess whether Autism Spectrum Disorder could be used as a good model of enhanced maths proficiency.

Additionally, this thesis aims at assessing the heterogeneous nature of the discipline of maths and particularly of numerical stimuli. In **Chapters IV & V**, I will investigate the understanding of double-digits numbers, including notation formats such as fractions and decimals in typical and atypical development.

Finally, Chapters **VII & VIII** will explore the benefits of different remediation approaches in cases of atypical development. Particularly, by focusing on the aforementioned differentiation between maths disabilities and difficulties (**Chapter II**), I will assess the advantages of a behavioural intervention program and investigate how behavioural progress can be modulated by symptom's severity (**Chapter VII**). In **Chapter VIII**, I will look at alternative techniques for educational remediation by investigating the potential effects of neural stimulation (tDCS) to enhance numerical proficiency.

In closing I hope to characterize the nature of numerical and arithmetical skills in its different forms (i.e. typical, atypical and exceptional abilities), and thereby challenge classical models of maths development.

II

Nature of the ‘number sense’ in typical and atypical development

“Do not worry about your difficulties in mathematics, I assure you that mine are greater”--Albert Einstein

There are two different concepts for the innate basis of numerical abilities. On the one hand, it is claimed that infants possess a ‘number module’ that enables them to construct concepts of the exact numerosities of sets upon which arithmetic develops. On the other hand, it has been proposed that infants are equipped only with a sense of approximate numerosities, upon which the concepts of exact numerosities are constructed with the aid of language and which forms the basis of arithmetic. Here we test these competing proposals of two core systems of mathematical knowledge by assessing whether performance on approximate numerosity tasks is related to performance on exact numerosity tasks. Moreover, performance on an analogue magnitude task was tested, since it has been claimed that approximate numerosities are represented as analogue magnitudes. In 8–9-year-olds, no relationship was found between exact tasks and either approximate or analogue tasks in normally achieving children, in children with low numeracy or in children with Developmental Dyscalculia. Low numeracy was related not to a poor grasp of exact numerosities, but to a poor understanding of symbolic numerals.

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2.1. Introduction

Current cognitive theories postulate that the human capacity for representing and processing numerical information depends on a specific core numerical capacity, i.e. the ‘number sense’ (Butterworth, 1999; Dehaene, 1997). There is extensive evidence that this capacity is present within the first year of life (Starkey & Cooper, 1980) and it serves as the basis for the development of arithmetical abilities and mathematical concepts (Butterworth, 1999; Dehaene, 1997).

Moreover, brain imaging studies have revealed that specific cortical areas in the parietal lobe are systematically activated whenever an individual manipulates numerical values (Dehaene, Piazza, Pinel, & Cohen, 2003). Furthermore, disruptions within these brain networks have been shown to be associated with deficits of numerical representation and manipulation (Cohen-Kadosh, Cohen-Kadosh, Schuhmann, Kaas, Goebel, et al., 2007; Isaacs, Edmonds, Lucas, & Gadian, 2001; Price, Holloway, Räsänen, Vesterinen & Ansari, 2007). Furthermore, evidence from non-enumerate cultures (Pica, Lemer, Izard, & Dehaene, 2004; Butterworth, Reeve, Reynolds, & Lloyd, 2008) and non-human primates studies (Matsuzawa, 1985), suggest that basic numerical capacities could be innate. Finally, specific mathematical deficits that emerge during children’s development may be rooted within congenital impairments of this core numerical capacity – Developmental Dyscalculia (Butterworth, 1999; Dehaene, 1997).

Although the concept of a core ‘number sense’ has been well established, there is still disagreement about the precise nature of such capacity. Some have described the ‘number sense’ as the capacity to represent the exact numerosities of sets (Butterworth, 1999; Butterworth & Reigosa Crespo, 2007; Zorzi, Stoianov, & Umiltá, 2005); while other authors have referred to it as the ability to represent approximate numerosities (Barth, La Mont, Lipton, & Spelke, 2005; Feigenson, Dehaene, & Spelke, 2004; Halberda, Mazocco, & Feigenson, 2008; Piazza, Facoetti, Trussardi, Berteletti, Conte, et al., 2010). These competing proposals have led to diagnostic variations in characterizing the nature of the underlying impairments in Developmental Dyscalculia (DD). One proposal is that Developmental Dyscalculia is the result of a deficit in the representation of exact numerosities (Butterworth, 1999, 2010; Butterworth & Reigosa Crespo, 2007), another that it is the result of an inability to form approximate

representations of numerical magnitude (Feigenson et al., 2004; Halberda et al., 2008; Piazza et al., 2010). Therefore, the latter proposal postulates that from an approximate innate number sense the child derives an approximate arithmetic system and only from that they build up their exact arithmetic; while the former implies that exact arithmetic derives from an innate exact core number system, which is distinguishable and independent from the approximate systems of numerosity and arithmetic. Here we test this assumption by developing a set of approximate arithmetical tasks (*based on* Barth et al., 2005, 2006) to see how these systems relate to each other and ultimately to the development of arithmetic in the typically developing population (Experiment 1A). Moreover, we use them to investigate whether learners with DD or low numeracy are significantly worse on these tasks (Experiment 1B). We have also developed tasks to assess whether the ability to represent continuous quantity correlates with normal arithmetic performance (Feigenson et al., 2004).

Cognitive studies on DD have reported differential patterns of deficits: some authors have described DD learners to be slower at numerical comparison tasks, but not on non-numerical comparison tasks (e.g. letters) (Landerl, Bevan, & Butterworth, 2004). The same authors also found that DD children show a deficit in subitizing (the rapid apprehension of small quantities); while other researchers reported that DD learners were slow in performing number comparison tasks, but on average on other tasks involving core numerical representation of non-symbolic numerosities (Mejias, Mussolin, Rousselle, Grégoire, & Noël, 2011; Rousselle & Noël, 2007). However, none of these studies have specifically addressed the issue of what accounts for a defective ‘number sense’ and therefore what specific impairments characterize DD or how DD differentiates from more generalized low numerical skills.

Here we intend to hone the understanding of the ‘number sense’ by investigating how it relates to the proposed theoretical accounts (Experiment 1A). Furthermore, we sought to delineate the precise nature of mathematical impairments in DD by identifying DD’s cognitive profile from other forms of general low numeracy (Experiment 1B).

Experiment 1A. The ‘number sense’ in typical development

2.2. Aim 1A

The capacity to represent and mentally manipulate numerosities is the key to learning arithmetic. Notably, usual arithmetical operations of addition, subtraction, multiplication and division can be defined in terms of operations on sets and their numerosities (Giaquinto, 1995). Inasmuch both theories of the ‘number sense’ acknowledge this, their ‘competing’ definitions of the nature of this innate capacity postulate a different developmental trajectory that leads to the mastering of arithmetical computations. On the one hand, the proponents of the approximate system for maths knowledge propose that children get from approximate numerosities to the exact numerosities needed for enumeration and arithmetic by using knowledge of the counting words of the language to ‘bootstrap’ from approximate to exact representations (Carey, 2004). Some have claimed that approximate arithmetic forms the basis of exact arithmetic (Barth, La Mont, Lipton, Dehaene, Kanwisher & Spelke, 2006; Barth et al., 2005; Gilmore, McCarthy & Spelke, 2007), while others have proposed that exact arithmetic derives from an innate exact core number system, which is distinguishable and independent from the approximate systems of numerosity and arithmetic.

Here we attempt to adjudicate between these theories by investigating children of 8 and 9 years of age with no mathematical impairments. By using a set of newly developed approximate and analogue magnitude tasks we ask whether performance on these tasks predicts performance in exact tasks.

2.3 Method 1A

2.3.1. Participants

Twenty-three normal achieving children (NA) in their 4th year of primary school (mean age = 8.961, SD = 4.59, 12 males) were recruited from three different State Middle Schools in the London Borough of Harrow.

Participants were of average mathematical ability. These were assessed using a standardized computerized measure: the *Dyscalculia Screener* (Butterworth, 2003),

which has a mean of 100 and a standard deviation of ± 15 . All children achieved a standardized score > 81 ³ on the three tests of numerical and mathematical abilities (Table 2.1).

Measure	Accuracy	RTs	Inverse Efficiency
<u>Dyscalculia Screener</u>	<i>Mean (SD)</i>	<i>Mean (SD)</i>	<i>Mean (SD)</i>
Simple RTs		417.69 (156.34)	
Dot Enumeration	91.41 (6.44)	1960.91 (704.5)	21.63 (7.57)
Number Comparison	91.90 (6.26)	685.80 (271.05)	7.48 (2.94)
Exact Addition	89.27 (8.46)	3835.43 (2309.5)	42.60 (27.94)

Table 2.1. Dyscalculia Screener scores for the Simple reaction time test and the three numerical and mathematical tasks (Dot Enumeration, Number Comparison and Exact Addition).

2.3.2. Experimental design

2.3.2.1. Standardized tasks

The *Dyscalculia Screener* was chosen as the standardized task to measure mathematical abilities in the NA group. This standardized software comprises three computer-controlled, item-timed tests plus one simple reaction time test. The three tasks are divided into two subscales: a ‘capacity subscale’, which involves a dot enumeration task and a number comparison task, and an ‘achievement subscale’, which comprises an addition task (*see below*). The software diagnoses developmental dyscalculia on the basis of norms calculated for each year group. All the tasks were carried out on all the participating children in the order listed below. Instructions were given on the screen and simultaneously via pre-recorded audio. Pupils were encouraged to perform the tasks as quickly and as accurately as possible. The software recorded both accuracy and reaction times, in milliseconds, for each trial. Given these variables, the software automatically computed an inverse efficiency measure based on the median reaction time for each task which was adjusted for simple reaction time. This measure was then compared with the age-appropriate UK standardization, so that each learner was

³Please note that the standardized score (i.e. > 81) refers to the UK population norms and therefore does not denote either raw accuracy or speed of responses. However, for reasons of clarity and completion, we hereafter report the raw data broken down by the dependent variables of interest (mean accuracy, mean of median RTs and inverse efficiency). The standardized score is calculated taking into account all these variables (plus simple RTs) on the basis of age norms.

compared with similarly aged peers. The data are also stored in raw form for further analysis, if required. In our case the raw data served as a measure of the 'exact system' in order to test the hypothesis of approximate and analogue skills as predictors of exact processes.

Dot Enumeration task. This task is one of the measures of the Capacity subscale of the *Dyscalculia Screener* and assesses the capacity to represent exact numerosities, and the knowledge of the numerosity that each numeral denotes. The pupil is asked to decide using a key press response whether the numerosity of a random array of dots in the left panel of the screen matches the numerosity of a digit in the right panel. The range of numerosity used is between one and ten. Stimuli were dots and Arabic digits. Dots were coloured in black and their diameter was a constant 9 mm. Arabic digits ranged from 1 to 9 inclusive and appeared on the other side of the screen from the dots. Both dots and digits were presented on a white background. There were 68 stimuli presented in a fixed, pseudo-random order. The stimuli remained on the screen until a response occurred and there was no time limit. Accuracy and speed are emphasized.

Number Comparison task. This task forms part of the 'Capacity subscale' of the *Dyscalculia Screener* and assesses the capacity to order numerosities by magnitude and to understand the numerals. The child is presented with two Arabic single-digits –one on the left and one on the right side of the computer screen – and he is asked to select the larger in numerical value as accurately and as quickly as possible, by pressing a key under the chosen number. Stimuli were black Arabic digits. They appeared on two white discs, one on the left side of the screen and one on the right side of the screen. Digit pairs ranged from 1 to 9 inclusive (excluding 5). The physical size of the numbers varied. There were 42 trials presented in a fixed, quasi-random order. The software records both accuracy and reaction times to the order of milliseconds. The stimuli remained on the screen until a response occurred and there was no time limit.

Exact Addition task. The addition task is part of the 'Achievement subscale' and assessed the ability of the participant to do arithmetic. The pupil is asked to verify the results of single-digit addition problems presented in the middle of the computer screen and press a key according to whether the result is correct or not (e.g. $5+3 = 7?$). Stimuli consisted of single-digit problems (results no more than 18). Stimuli were coloured in black and presented on a white background. The size of the stimuli was kept constant

throughout the task. The task comprises a total of twenty-eight trials presented in a fixed, pseudo-random order. Accuracy and speed are both emphasized.

2.3.2.2. Experimental tasks

The experimental tasks comprised three sets of tasks: 1) exact tasks; 2) approximate tasks; 3) non-symbolic discrete and analogue tasks.

The exact tasks used here were the ones from the *Dyscalculia Screener* software (see **paragraph 2.3.2.1** for a detailed description). They were all written in Java; while the other two sets of experimental tasks were written in the Cogent library, running on a Matlab version 6.5 platform. These latter tasks required participants to choose between two possible responses – one displayed on the left and one displayed on the right side of the screen – children were always asked to respond by pressing a key on the side of the keyboard that corresponded to the correct response. Instructions emphasized both speed and accuracy. The task was always to select the larger relevant attribute, as specified by the task’s instructions. Each task was preceded by four training trials.

Approximate tasks. The approximate tasks used in this study were based on Barth and colleagues (Barth et al., 2005, 2006). Yet, we chose to use key presses instead of verbal reports and included an approximate subtraction task as well. In each of the three tasks (comparison, addition and subtraction) the child was asked to compare the non-symbolic numerosity of an animated blue dot array to the numerosity of an animated red dot array. The blue dot and red dot arrays always appeared on opposite sides of the screen with the blue array always appearing first and the red array always last. At probe, children were asked ‘Were there more blue dots or more red dots?’. Stimuli for all three tasks were animated arrays of blue dots and red dots varying in size roughly between 2 and 3 mm in diameter. Dots were drawn within a rectangular area, or ‘enclosure’, in such a way that they did not overlap. Dots of the same color moved together, and did not move in relation to each other. Varying dot sizes and enclosure size allowed us to control for children’s use of strategies based on summed dot surface area or dot density rather than numerosity. The range of numerosities used was from 10 to 58 inclusive, and there were three different ratios between the numerosities of the two compared arrays: 1) 4:7 (or 7:4); 2) 4:6 (or 6:4); 3) 4:5 (or 5:4), and these were counterbalanced across trials in a pseudo-random order within each of the three

approximate tasks. Each child received one block of comparison problems, followed by one block of addition problems and then one block of subtraction problems. Each block consisted of 24 trials presented in a pseudo-random order. For each task, there were two design conditions: 1) the ‘moving dots’ condition where trials began with an occluder presentation then blue dots subsequently moved behind it; and 2) the ‘moving occluder’ condition where trials began with an array of blue dots and an occluder would subsequently move to cover the dots, whose order was counter-balanced between participants.

Approximate comparison task. In this task the child was asked to judge whether the number of blue dots was greater or less than a comparison array of red dots. For half of the trials (‘moving dots’), the occluder appeared on the lower left of the screen (followed by a pause of ≈ 1300 ms), then an array of blue dots appeared above it (followed by a pause of ≈ 1300 ms) and moved down behind the occluder (taking ≈ 650 ms). After a pause (≈ 1300 ms), an array of red dots appeared on the upper right of the screen (followed by a pause of ≈ 1300 ms) and then moved down to rest at the lower right of the screen for ≈ 650 ms. Finally, the screen turned black, prompting for a key response (time limit for response ≈ 5000) (**Figure 2.1A**). For the other half of the trials (‘moving occluder’), a set of blue dots appeared at the lower left side of the screen (followed by a pause of ≈ 1300 ms), then the occluder re-appeared on the lower right (followed by a pause of ≈ 1300 ms), and moved to the left to cover the blue dots (taking ≈ 650 ms). A pause ensued (≈ 1300 ms), and then the array of red dots appeared in the same manner as described above. The screen then turned black, signaling for a key response (time limit ≈ 5000 ms) (**Figure 2.1B**).

Approximate addition task. In this task, two arrays of blue dots were visibly moved behind the occluder in two separate movements and the participant was asked to judge whether the result of this addition was greater or less than a comparison array of red dots. This task was designed so that, after addition, the resultant numerosities matched those of the comparison task. Each addition problem was therefore created by taking a comparison problem and splitting up the first (blue) array into two addend arrays (e.g. ‘20 vs. 25’ could become ‘10 + 10 (=20) vs. 25’). So each addition problem corresponded to a comparison problem. As was the case in the approximate comparison

task, half the trials involved ‘moving dots’ (**Figure 2.1A**) and half involved a ‘moving occluder’ (**Figure 1.2B**).

Approximate subtraction task. In this task, a subset of the blue dots was visibly moved from behind the occluder and the participant was asked to judge whether the result of this subtraction was greater or less than a comparison array of red dots. This task was designed so that, after subtraction, the resultant numerosities matched those of the comparison (and addition) tasks. Each subtraction problem was therefore generated by taking the numerosities of a comparison problem and creating two blue arrays based on the numerosities used in the corresponding addition problem. The difference in the numerosities of the two new blue arrays would equal the numerosity of the blue array in the original comparison problem (e.g. ‘20 vs. 25’ could become ‘30 - 10 (=20) vs. 25’). As was the case in the approximate comparison and addition tasks, half the trials involved ‘moving dots’ (**Figure 2.1A**) and half involved a ‘moving occluder’ (**Figure 1.2B**).

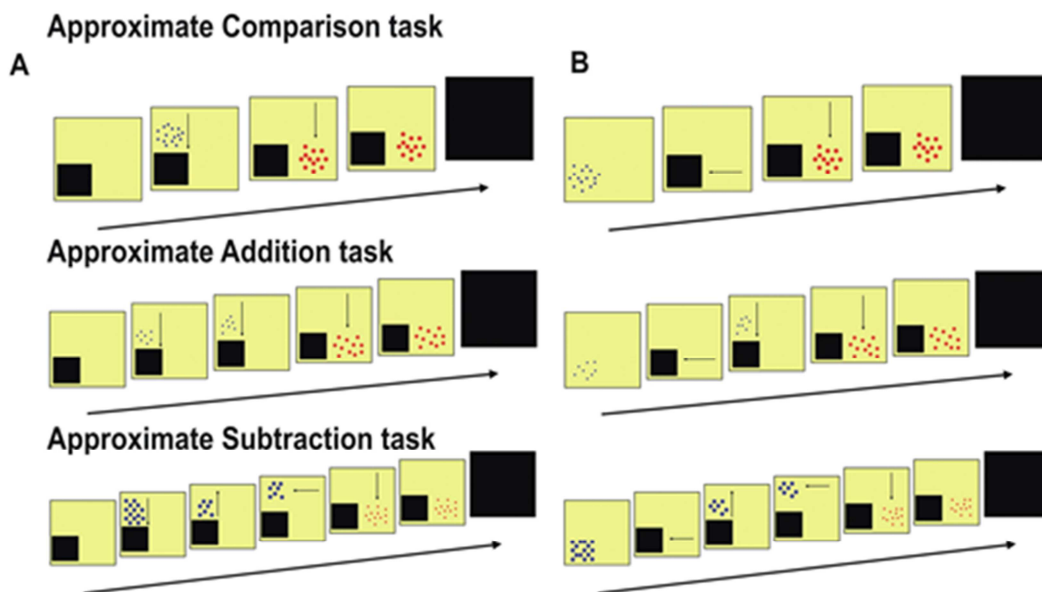


Figure 2.1. Approximate tasks. A) ‘moving dots’ condition; B) ‘moving occluder’ condition.

Non-symbolic discrete numerosity and analogue area tasks. These are two non-symbolic comparison tasks. The child had to select one of either (1) *numerosity* (i.e. select the panel with more squares) and (2) *area* (i.e. select the panel with more blue). Accuracy and speed were emphasized in both tasks. Numerosity and area were varied

orthogonally to create three conditions in a standard Stroop-like manipulation, congruent (more squares and larger area), incongruent (more square and smaller area and visa-versa), and neutral (same number of squares and same area) (**Figure 2.2**). Each of the two panels contained arrays of blue squares drawn in different sizes. The range of numerosities used was 1 to 9 inclusive; this range (1 to 9 in terms of units) was also used for the stimuli in the area task so that the two tasks could be counterbalanced. The ratios between the two panel sets, in terms of numerosity or area units, ranged from 0.20 to 0.83 (where smaller ratios were associated with larger distances in the size of area or numerosity). Each trial presentation lasted a maximum of 4000 ms and a black screen of a fixed duration (800 ms) appeared between trials. There were 48 trials in each task, 16 for each of the three Stroop conditions.

Discrete Numerosity task. Instructions were the following: 'You will see two yellow panels, one on the left, and one on the right side of the screen. These panels contain a set of blue squares. Please press the key (left or right) which corresponds to the panel which contains more squares'.

Analogue area task. Instructions were the following: 'You will see two yellow panels, one on the left, and one on the right side of the screen. These panels contain a set of blue squares. Please press the key (left or right) which corresponds to the panel which contains more blue'.

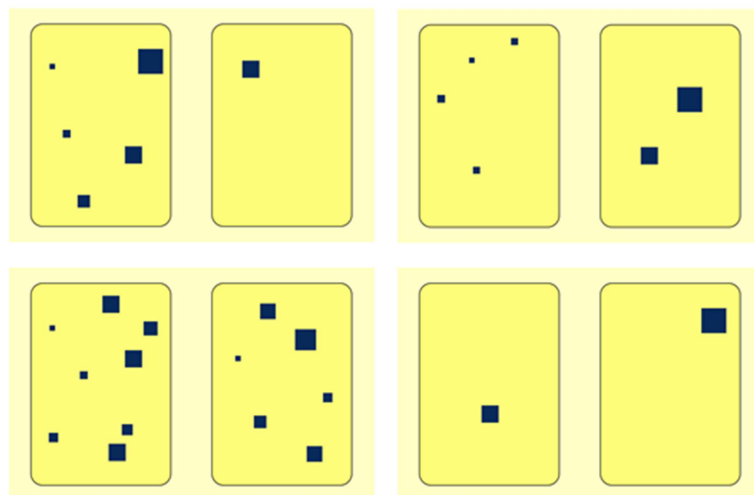


Figure 2.2. Non-symbolic discrete numerosity and analogue area tasks. Top left: congruent trial (more squares and larger area in the left panel); top right: incongruent trial (more squares in the left panel but larger area in the right panel); bottom left: neutral trial in the numerosity task (more squares in the left panel, equated total area between the two panels); bottom right: neutral trial in the area task (larger area in the right panel, equated numerosity between the two panels).

2.4. Results 1A

2.4.1. Correlational analyses

Here we explored whether the cognitive systems under test – the approximate, the analogue and the exact systems – are distinct from one other.

The main dependent variable used in these analyses was inverse efficiency, since it combines accuracy and speed, and it is known that both are important when looking at performance of children. This measure was calculated for each subject for all the experimental tasks by dividing the median reaction times by the proportion of correct responses. Given that all data were normally distributed (K-S p values all $> .05$), parametric correlational analyses were carried out on the dependent variables of interest for the experimental tasks and for the *Dyscalculia Screener* tasks which served as the measures for tapping the exact system.

2.4.1.1. The approximate system

Here we correlated performance on the three approximate tasks (comparison, addition and subtraction). This analysis showed a significant positive correlation between all approximate tasks: *Approximate Comparison* and *Approximate Addition*: ($r = 0.693$, $p < .001$); *Approximate Comparison* and *Approximate Subtraction*: ($r = 0.78$, $p < .001$); *Approximate Addition* and *Approximate Subtraction* ($r = 0.942$, $p < .001$)⁴ (Figure 2.3).

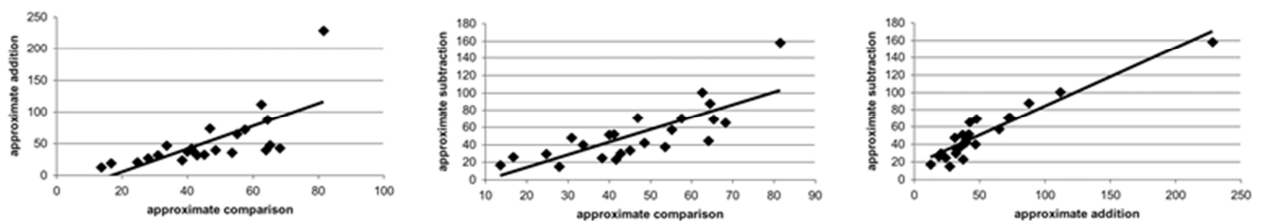


Figure 2.3. Correlation of inverse efficiency scores for approximate tasks. Left: correlation between performance on *Approximate Comparison* and *Approximate Addition*; middle: correlation between performance on *Approximate Comparison* and *Approximate Subtraction*; right: correlation between performance on *Approximate Addition* and *Approximate Subtraction*.

⁴Please note that all correlations remained significant after removing the one outlier (see Figure 2.3). *Approximate Comparison* and *Approximate Addition*: ($r = 0.671$, $p = 0.001$); *Approximate Comparison* and *Approximate Subtraction*: ($r = 0.74$, $p < .001$); *Approximate Addition* and *Approximate Subtraction* ($r = 0.906$, $p < .001$).

2.4.1.2. The approximate system in relation to the discrete numerosity and analogue area tasks.

Here we looked at the correlation between performance on the approximate comparison task with the discrete and analogue area tasks respectively. Results showed no significant correlations between these tasks: *Approximate Comparison* and *Discrete Numerosity* ($r = 0.24$, $p = .270$); *Approximate Comparison* and *Analogue area* ($r = 0.78$, $p = .216$).

2.4.1.3. The approximate system in relation to the exact system

Here we looked at the correlation between performances on the approximate tasks with the measures of the exact system (subtests of the *Dyscalculia Screener*). Specifically, we correlated the approximate comparison task with the two numerosity tasks of the *Dyscalculia Screener* (dot enumeration and number comparison). We then correlated performance on the approximate addition task with the arithmetic measure of the *Dyscalculia Screener* (exact addition). None of the correlations were significant: *Approximate Comparison* and *Dot Enumeration* ($r = 0.101$, $p = .647$); *Approximate Comparison* and *Number Comparison*: ($r = -0.063$, $p = .774$) and *Approximate Addition* and *Exact Addition*: ($r = 0.168$, $p = .445$).

2.4.1.4. What predicts exact arithmetic?

In this analysis we took performance on the exact addition task and correlated it to the approximate, exact and analogue measures in order to closely investigate significant predictors of the former. The results showed a significant positive correlation between *Discrete Numerosity* and *Exact Addition* ($r = 0.486$, $p < .01$) (**Figure 2.4**). Moreover, there was a significant positive correlation between *Dot Enumeration* and *Exact Addition* ($r = 0.576$, $p < .005$) (**Figure 2.4**). None of the other variables tested revealed a significant correlation with exact addition: *Approximate Comparison* and *Exact Addition* ($r = 0.092$, $p = .667$); *Analogue Area* and *Exact Addition* ($r = 0.116$, $p = .600$); *Number Comparison* and *Exact Addition* ($r = 0.349$, $p = .103$).

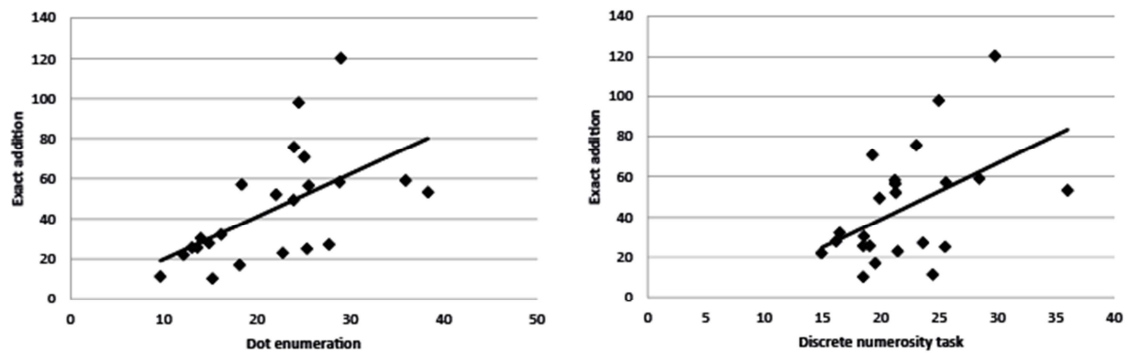


Figure 2.4. Correlation of inverse efficiency scores for numerosity tasks and exact arithmetic. Left: correlation between performance on dot enumeration task and performance on exact addition; right: correlation between performance on discrete numerosity task and exact addition.

2.5. Interim Discussion 1A

Performance in approximate comparison, addition and subtraction were highly correlated (**Figure 2.3**), supporting the claim for the existence of an approximate system, which is responsible for the comparison of numerosities (quantities), but also for their mental computation in an approximate manner (Carey, 2004; Dehaene, 1997; Feigenson et al., 2004).

On the contrary, performance in the approximate tasks did not correlate with performance in the analogue area task, suggesting that the system for approximation cannot be reduced to the system for analogue magnitude, as proposed by Feigenson and colleagues (2004).

Correlation analyses between approximate and exact tasks (both symbolic and non-symbolic) also proved to be non-significant, suggesting that the approximate system is not used to carry out exact numerical tasks. Moreover it suggests that this system does not necessarily form the basis of exact arithmetic, as previously proposed (Barth et al., 2005, 2006; Gilmore et al., 2007, Gilmore, McCarthy, & Spelke, 2010; Halberda et al., 2008).

On the other hand, performance on exact numerical tasks was positively correlated with performance on the exact arithmetic task (**Figure 2.4**), supporting the claim for the ‘exact’ number system as a building block on the basis of which exact arithmetical computations can be constructed (Butterworth, 1999, 2005).

Experiment 1B. The ‘number sense’ in Developmental Dyscalculia and Low Numeracy

2.6. Aim 1B

The exact number sense theory has a simple account of Developmental Dyscalculia (DD), which is termed the ‘defective number module hypothesis’ (Butterworth, 2005): the cognitive deficit in DD is a disability to deal with exact numerosities. By implication, sufferers may therefore be entirely normal on tests of approximate numerosity and on tests of analogue magnitude. In a recent proposal, Rousselle and Noël argued that DD arises from a disconnection between concepts of numerosity, which are intact, and the symbols that denote them (Rousselle & Noël, 2007). However none of these studies have specifically addressed the issue of what accounts for a defective ‘number sense’ and therefore what specific impairments characterize DD. Although proponents of the approximate number sense’s approach have not addressed the issue of DD or LN, it is reasonable to infer that DD, a congenital condition, will be a deficit in the approximate system, which could lie in the ability to extract approximate numerosities from the stimulus, or in the ability to represent it as an analogue magnitude. This may manifest itself in one of two ways: either in the ability to estimate numerosities or in the ability to estimate analogue quantities.

In this experiment we intended to delineate the precise nature of mathematical impairments in DD by identifying DD’s profile from other forms of general low numeracy (LN).

Moreover, we aimed at better characterizing the cognitive profile of children with low numeracy by asking whether this type of maths impairment is the result of poor ability in the core numerical capacities mentioned above: the system for exact numerosities, or the system for approximate numerosities, or in their representations as analogue magnitudes.

2.7 Method 1B

2.7.1. Participants

A total of thirty-six children in their 4th year of primary school (8-9 year olds) took part in this experiment. Participants were recruited from three different State Middle Schools in the London Borough of Harrow.

Participants in the experimental groups (LN and DD) were initially selected on the basis of teachers’ assessment: teachers were asked to nominate children who they felt were of average general ability but had serious difficulties during numeracy lessons. The children defined as normal achievers (NA) tested in Experiment 1A served as the control group for Experiment 1B. All children in experiments 1A and 1B were from the same schools in the London Borough of Harrow, and were of the same age (Year 4). Moreover, they were from the same classes as the children in the experimental groups. Therefore, age and education were perfectly equated between the experimental and the control groups. As in Experiment 1A, the standardized numerical assessments used the *Dyscalculia Screener* software (Butterworth, 2003) which diagnoses Developmental Dyscalculia on the basis of norms (test average of the nationally standardized score = 100, SD = 15). IQ was examined only in the two experimental groups, LN and DD, using the WISC-III full protocol (Wechsler, 1996). After pro-rating scores for the arithmetic subtest, only one subject from the LN group had to be discarded from the study because his Full Scale IQ (FSIQ = 67) did not fall in the average range (FSIQ > 70). All other participants in the two experimental groups fell within the average IQ range according to their age group (mean FSIQ = 99.67, SD = 16.1; mean VIQ = 41.62, SD = 12.36; mean PIQ = 45.37, SD = 13.08).

To be classified as dyscalculic (DD), children had to obtain: (1) a standardized score below 81 on at least one of the two tasks of the ‘capacity subscale’ (either Dot Enumeration or Number Comparison) of the *Dyscalculia Screener* (see **Table 2.2**); (2) an IQ score within the normal range for their age (FSIQ score > 70); and (3) a reading aloud score within the normal range from the British Ability Scales Second Edition – BAS-II (Elliott, Smith & McCulloch, 1996). The *Dyscalculia Screener* identified two DD learners. Both DD pupils performed within the average range on the reading test

(standard scores of 95 and 99 respectively). This result allowed us to discard the possibility of them having Dyslexia (Lewis, Hitch & Walker, 1994).

To be assigned to the Low Numeracy (LN) group, participants had to obtain: (1) a standardized score below 81 on the ‘achievement subscale’ of the *Dyscalculia Screener* (see **Table 2.2**); (2) a standardized score within the normal range (> 81) in both tasks of the ‘capacity subscale’; and (3) an IQ score within the normal range for their age group (FSIQ score > 70).

In the Normal Achievement (NA) group, participants scored > 81 on all three tasks of the *Dyscalculia Screener* (see **Table 2.1**).

	LN (N = 10)			DD1			DD2		
Measure	Accuracy	RTs	Inverse Efficiency	Accuracy	RTs	Inverse Efficiency	Accuracy	RTs	Inverse Efficiency
	Mean (SD)	Mean (SD)	Mean (SD)	Mean	Mean	Mean	Mean	Mean	Mean
<i>Dyscalculia Screener</i>									
Simple RTs	453.3 (211.9)			371			341		
Dot Enumeration	85.4 (4.33)	2278.1 (803.4)	26.68 (9.73)	66.18	1105	16.70	80.88	2522	31.18
Number Comparison	89.7 (4.83)	1092.4 (366.5)	12.27 (4.09)	93.33	1168	12.51	76.67	1327	17.3
Exact Addition	58.6 (9.09)	3783.4 (2099.2)	65.25 (33.14)	35.71	1427	39.97	75.71	3975	46.37

Table 2.2. Dyscalculia Screener scores for the Simple reaction time test and the three numerical and mathematical tasks (Dot Enumeration, Number Comparison and Exact Addition) for the LN group and the two DD cases.

2.7.2. Experimental design

All standardized and experimental tasks were the same as the ones described in Experiment 1A. Stimuli, task parameters and timing also were identical to those described in Experiment 1A.

2.8. Results 1B

2.8.1. Differences between the Normal Achievers and the Low Numeracy group

Here we wished to investigate whether performance in the approximate, analogue and exact tasks was poorer in the LN group than in the NA group. We also

tested the hypothesis that the problem with the LN group could lie in an inability to deal with the symbolic representations of numerosity (Arabic digits) (see Rousselle & Noël, 2007).

Finally, we investigate Stroop effects in the non-symbolic discrete and analogue magnitude tasks within and between groups.

Analogous to Experiment 1A, the main dependent variable used for most of these analyses was inverse efficiency, since it combines accuracy and speed (see **section 2.4.1**), and it is known that both are equally important in the diagnosis of arithmetical learning disabilities (Butterworth, 2005; Landerl et al., 2004). The more conventional variables (accuracy and speed of responses) were instead used for the analyses of the Stroop effects. Between-group analyses compared the performance of the LN group to the NA group in all experimental tasks and in the two Capacity tasks of the *Dyscalculia Screener* (Dot Enumeration and Number Comparison). Where data were normally distributed, One-way ANOVAs were used to compare between groups’ performance. In those cases where the assumptions for the normal distribution were violated, a Mann-Whitney test was used.

2.8.1.1. Approximate tasks

One-Way ANOVAs showed no significant differences between the two groups in any of the approximate tasks. Specifically: *Approximate Comparison*: [$F(1,32) = 0.515$; $p = .478$]; *Approximate Addition*: [$F(1,32) = 0.203$; $p = .656$]; *Approximate Subtraction* [$F(1,32) = 1.232$, $p = .275$].

2.8.1.2. Analogue Area task

The One-Way ANOVA revealed no significant differences between the two groups in the *Analogue Area* task: [$F(1,32) = 1.50$; $p = .230$].

2.8.1.3. Symbolic and non-symbolic numerosity tasks

The dependent variable measured in the non-symbolic *Discrete Numerosity* task was not normally distributed according to the Kolmogorov Smirnov test [$K-S\ p\ value < .05$]. A Mann-Whitney test was therefore used to analyse performance on this

task. One-way ANOVAs were performed on the two capacity tasks from the *Dyscalculia Screener*: symbolic *Number Comparison* task and *Dot Enumeration* task.

The Mann-Whitney test showed no significant difference between the two groups in the non-symbolic *Discrete Numerosity* task ($U = 85.5$; $Z = -1.16$; $p = .253$). One-way ANOVAs revealed a significant difference between the two groups in the symbolic *Number Comparison* task [$F(1,32) = 14.556$; $p < .001$] (**Figure 2.5**), while no significant difference was found between the two groups in the *Dot Enumeration* task [$F(1,32) = 3.466$, $p = .072$].

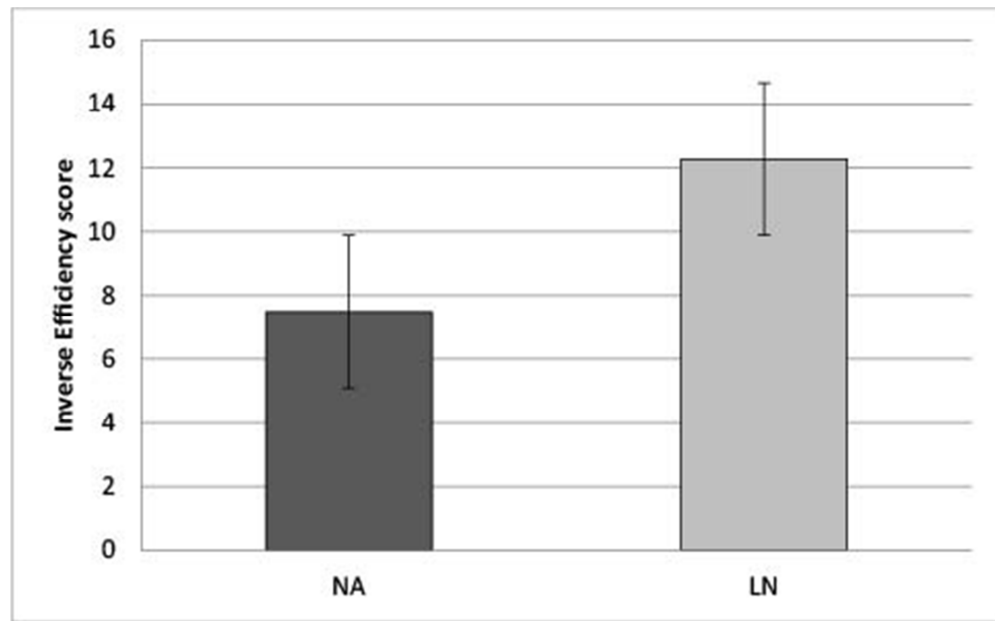


Figure 2.5. Symbolic Number Comparison task. Inverse efficiency scores on the Number Comparison task plotted against group. Error bars indicate 1 standard error of the mean.

2.8.1.4. Non-symbolic Stroop effects

A 2x3x2 Repeated Measures ANOVA was conducted on accuracy and on median reaction times for correct responses only, with task (*Discrete Numerosity* or *Analogue Area*) and condition (congruent, neutral, incongruent) as the within subjects factors. Group (NA or LN) was the between subjects factor.

Accuracy. The ANOVA revealed a main effect of task [$F(1, 31) = 35.928$; $p < .001$; $\eta^2 = .537$]: the *Discrete Numerosity* task was less error prone than the *Analogue Area* task (**Figure 2.6**). There was also a main effect of group [$F(1,31) = 8.062$; $p < .02$; $\eta^2 = .206$] and a main effect of condition [$F(2, 31) = 35.520$; $p < .001$; $\eta^2 = .534$]

(Figure 2.6). Pairwise comparisons (adjusted for multiple comparisons with Bonferroni correction) revealed significant differences between the congruent condition and the incongruent condition (mean difference = -3.067; $p < .001$) and between the neutral condition and the incongruent condition (mean difference = -2.455; $p < .001$). There was a task \times condition interaction [$F(2, 31) = 4.682$; $p < .02$; $\eta^2 = .131$] suggesting that the congruency effects were more pronounced in the *Analogue Area* task than in the *Discrete Numerosity* task; the significant task \times group interaction [$F(1, 31) = 7.425$; $p < .01$; $\eta^2 = .193$] suggests that the LN group was worse than the NA group mainly in the *Analogue Area* task; while the condition \times group interaction [$F(2, 31) = 4.396$; $p < .02$; $\eta^2 = .124$] seems to reveal that poorer performance only occurs in the neutral and incongruent conditions, but not in the congruent one (Figure 2.6). The task \times condition \times group interaction was not significant [$F(2, 31) = 1.151$; $p = .323$].

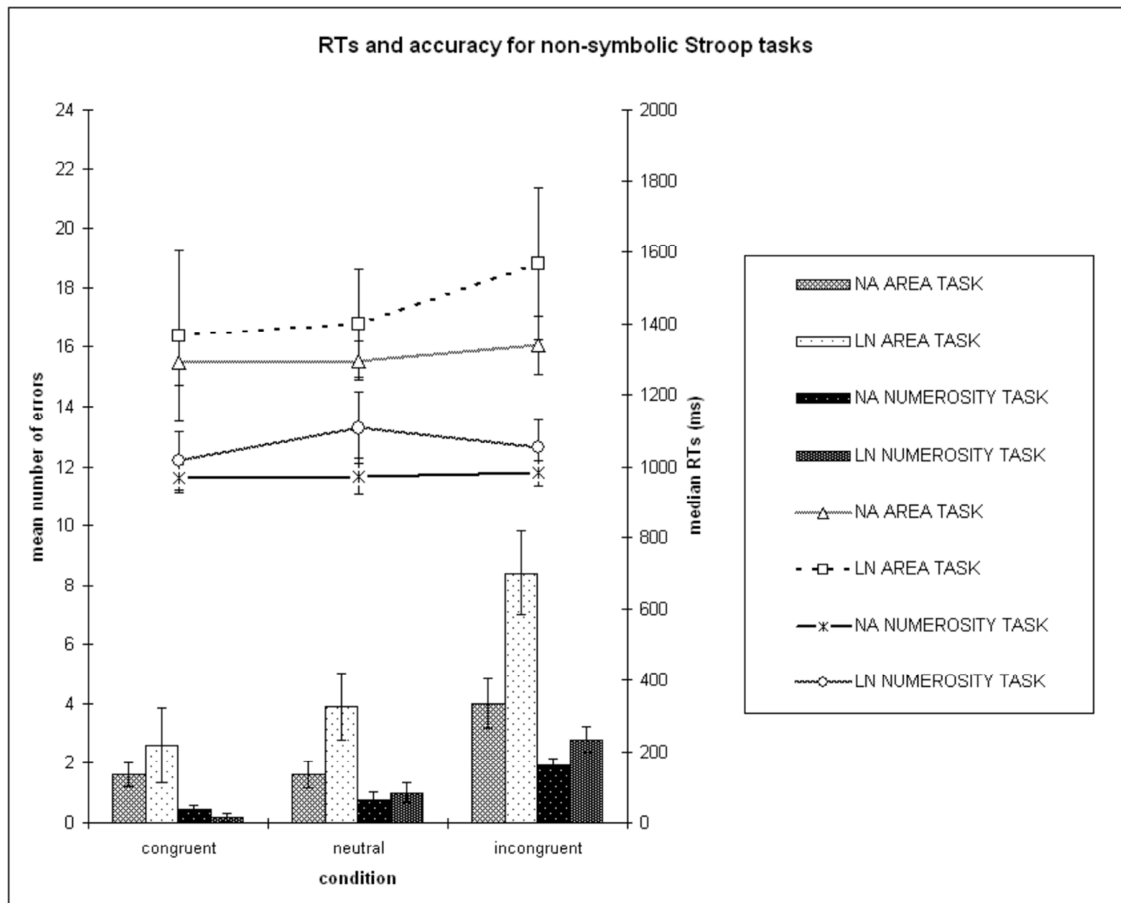


Figure 2.6. Congruency effects in the *Discrete Numerosity* and *Analogue Area* tasks. Bars show mean of number of errors and lines show reaction times. Error bars indicate 1 standard error of the mean.

Reaction Times. The ANOVA revealed a main effect of task [$F(1, 31) = 41.934$; $p < .001$; $\eta^2 = .575$]: median reaction times were 1375.05 and 1016.35 for the *Analogue Area* and *Discrete Numerosity* tasks respectively (**Figure 2.6**). There was no main effect of condition [$F(2, 31) = 3.195$; $p = .09$] and no main effect of group [$F(1, 31) = 1.150$, $p = .292$]. Additionally, none of the interactions were significant: task \times condition [$F(2, 31) = 1.401$; $p = .515$]; task \times group [$F(1, 31) = 0.213$; $p = .882$]; condition \times group [$F(2, 31) = 1.059$; $p = .416$]; and task \times condition \times group [$F(2, 31) = 0.605$; $p = .76$].

2.8.2. Single cases with Developmental Dyscalculia

Here we sought to investigate whether performance in the approximate, analogue and exact tasks was poorer in the two cases of Developmental Dyscalculia (DD) than in the NA group. The aim of these analyses was to delineate the precise nature of mathematical impairments in DD and to identify the cognitive profile of this developmental condition compared to age and IQ matched controls. By comparing performance of the two dyscalculic subjects to the control group we were able to test three hypotheses: i) does the condition of Developmental Dyscalculia show an impairment in dealing and manipulating approximate or analogue numerosities; ii) do DD participants show difficulties in dealing with exact numerosities; and finally iii) could the impairment that characterize DD lie in an inability to link an intact representation of numerosity with their symbolic expressions (Arabic digits).

Given the nature of this sample (see **paragraph 2.8.1**), the dependent variable chosen for these analyses was also inverse efficiency. To compare each of the two single DD cases with the NA sample, instead of the conventional z-test, we used the more conservative non-central t-statistic for single case analyses proposed by Crawford and colleagues (Crawford & Howell, 1998; Crawford, Garthwaite, Azzalini, Howell & Laws, 2006).

2.8.2.1. Subject 1 (DD1)

Approximate tasks. No significant difference was found in any of the approximate tasks for the inverse efficiency variable between DD1 and the control group: *Approximate Comparison*: ($t = -0.840$; $p = .410$.); *Approximate Addition*: ($t = -0.029$, $p = .978$); *Approximate Subtraction*: ($t = 1.145$, $p = .265$).

Discrete Numerosity and Analogue Area tasks. No significant difference between DD1 and the control sample was found in the *Analogue Area* task ($t = -1.427$, $p = .167$), nor in the *Discrete Numerosity* task ($t = 0.277$, $p = .784$).

Exact addition task. Performance of DD1 in the exact addition task of the *Dyscalculia Screener* was in the lowest 17% of the age group.

2.8.2.2. Subject 2 (DD2)

Approximate tasks. A significant difference was found on the *Approximate Comparison* task: ($t = 5.189$; $p < .001$) (**Figure 2.7A**). No significant differences appeared in the two approximate calculation tasks: *Approximate Addition*: ($t = 0.740$; $p = .467$); *Approximate Subtraction*: ($t = 0.782$, $p = .443$).

Discrete Numerosity and Analogue Area tasks. A marginal significant difference between DD2 and the NA group was evident in the *Analogue Area* task ($t = 2.156$; $p = .052$), while a significant difference was found in the *Discrete Numerosity* task ($t = 4.002$; $p = .001$) (**Figure 2.7B**).

Exact addition task. Performance of DD2 in the exact addition task was classified as defective by the *Dyscalculia Screener* (the bottom 7% of his age group).

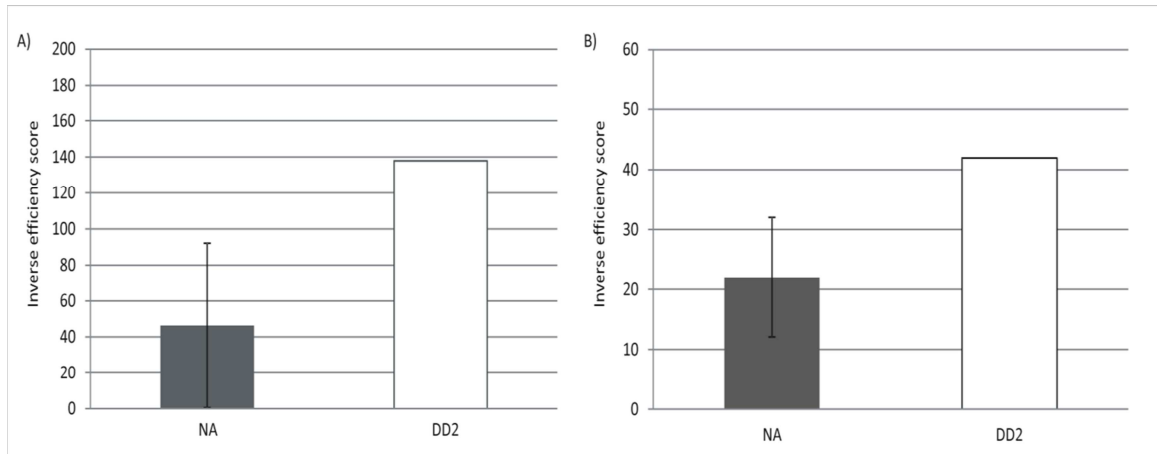


Figure 2.7. Impaired performance in a single DD case. A) *Approximate Comparison* task. B) *Discrete Numerosity* task. Error bars indicate 1 standard error of the mean.

2.9. Interim Discussion 1B

All in all, the results of Experiment 1B represent the first attempt to delineate a clear distinction between different types of numerical impairments. Here below results for each group will be discussed separately.

2.9.1. Low Numeracy group (LN)

This group was defined by poor performance in exact arithmetic. Essentially, their performance on the experimental tasks was comparable to the NA group. Notably, there were no differences from NA in any of the approximate tasks, suggesting that this cannot account for their mathematical deficits. There were also no differences in the discrete numerosity or analogue area tasks, implying that they had no particular difficulty representing and comparing either exact non-symbolic numerosity, or in comparing analogue quantities. However, LN participants performed significantly worse than the NA group in the symbolic number comparison task from the *Dyscalculia Screener*, suggesting that they may struggle interpreting symbolic numbers, though the non-symbolic numerosity comparison task indicated that they had no particular difficulty in representing exact numerosities as such. This supports previous findings (Mazzocco, Feigenson, & Halberda, 2011; Mejias et al., 2011; Rousselle & Noël, 2007) suggesting that disconnections between numerosity concepts and their symbols can lead to arithmetical impairments.

The analogue area task proved significantly more difficult than the discrete numerosity task as measured by both reaction times for correct responses and accuracy. It is therefore unlikely that children of 8 to 9 years use analogue quantities as a proxy for numerosity. This is in contrast with the sample of 3-year-olds tested by Rousselle, Palmers and Noël (2004), who did use area in preference to numerosity. Their subjects were much younger than ours, and may have been relying on a different strategy. Moreover, their stimuli varied the length but not the width of the bars, which may have made differences in area easier to compute.

2.9.2. Developmental Dyscalculia (DD) single cases

DD1 was diagnosed as DD on the basis of his performance on the symbolic number comparison task from the *Dyscalculia Screener*. Nevertheless, his performance on non-symbolic comparison tasks was at normal levels for approximate comparison, as well as analogue area and non-symbolic discrete numerosity judgements. Although his exact (symbolic) addition was poor, he performed at normal levels on the approximate addition task. These findings suggest that his disability cannot be interpreted in terms of deficits in either the approximate or the analogue systems. Moreover, he appears to have a normal non-symbolic exact numerosity system. He would therefore fit well into the disconnection account of DD proposed by Rousselle and Noël (2007), where the core deficit lies in linking the symbols to the concepts. Thus, his overall performance is similar to that of the LN group, with the only difference that DD1 is much more severely impaired compared to LNs as indicated by his performance on the symbolic comparison task. Moreover, this result suggests that previous authors might have included children with low numerical abilities in their sample, not just DDs, as a consequence of less stringent cut-off criteria (Mejias et al., 2011; Rousselle & Noël, 2007).

DD2, by contrast, failed the dot enumeration task from the *Dyscalculia Screener*, as well as being abnormally poor on timed (symbolic) addition. In the approximate tasks, he performed at normal levels on addition and subtraction. He also performed at normal levels on the analogue area task. Taken together, his DD condition cannot be explained as a core deficit in his approximate or analogue systems. On the other hand, he performed significantly worse than normal on the non-symbolic numerosity task, suggesting a more deep-rooted deficit in the capacity to represent exact numerosities (Butterworth, 2005).

Altogether, the results of the single case analyses also suggest that of the two capacity tasks from the *Dyscalculia Screener* - *Dot Enumeration* and *Number Comparison*, the former may be a better predictor of profound deficits in the capacity to understand numerosities.

2.10. Final Discussion

In this Chapter we attempted to adjudicate between two theories of the specific inherited basis of our ability to represent numbers (Butterworth, 1999; Dehaene, 1997). Two approaches were used to determine this. First, we asked whether exact numerical performance in typically developing children was predicted by performance in approximate and analogue tasks, or in exact/discrete tasks. Second, we asked whether children with low numeracy (LN) or DD individuals differed from their control group in either the approximate tasks, the analogue tasks, or solely in the exact tasks. Moreover, we had tasks which used both symbolic and non-symbolic material. This allowed us to test the hypothesis of a specific relationship between the ability to translate from symbolic to non-symbolic information (and *viceversa*) and mathematics achievement.

In brief, the approximate tasks correlated with each other (**Figure 2.3**), but not with the analogue area task; and neither tasks discriminated between the groups. On the contrary, the exact tasks positively correlated with the ability to perform exact arithmetic (**Figure 2.4**). Moreover, the exact tasks appeared to be the best measures in order to discriminate the DD individuals from the NA group (**Figures 2.5 & 2.7B**).

The capacity to represent and mentally manipulate numerosities is the key to learning arithmetic. In fact, the usual arithmetical operations can be defined in terms of manipulations on sets and their numerosities (Giaquinto, 1995). However, the literature presents a huge disagreement on what accounts for the ability to operate on numerical sets. Such disagreement ultimately results in a theoretical clash on what are the building blocks upon which cognitive development and education construct arithmetic and ultimately mathematics. The main distinction that has been drawn finds two distinct systems for core number knowledge: a system for exact numerosities (Butterworth, 1999, 2005; Butterworth & Reigosa-Crespo, 2007; Zorzi & Butterworth, 1999; Zorzi et al., 2005) and a system for approximate numerosities (Dehaene, 1997). These competing theories have different implications on the development of arithmetical abilities. On the one hand, the proponents of the exact system for numbers postulate that the ability to perform exact computations on numbers derives from the exact system for manipulating numbers and numerosities. On the contrary, it has been claimed that approximate arithmetic forms the basis of exact arithmetic (Barth et al., 2005, 2006;

Gilmore et al., 2007). Such dichotomy has important theoretical, but more importantly practical implications for education and also for the diagnostics of developmental learning disabilities such as Developmental Dyscalculia (DD) and Low Numeracy (LN). The number module theory has a simple account for DD, which is termed the ‘defective number module hypothesis’ (Butterworth, 2005). In this case the cognitive deficit is an inability to deal with exact numerosities. By implication, individuals with DD may therefore be entirely normal on tests of approximate numerosity and on tests of analogue magnitude. Our data from Experiment 1B supports such hypothesis: the two DD cases showed deficits in the exact numerosity tasks (**Figure 2.5 & 2.7B**), but not in the approximate or analogue tasks, with the exception of case DD2 whose performance seems to be deficient even in one of the approximate tasks (**Figure 2.7A**). Yet, it is important to note that his performance on the approximate arithmetic tasks (addition and subtraction) did not differ from the control group, arguing against the notion of approximate arithmetic being a precursor for the ability to perform exact arithmetic (Barth et al., 2005, 2006; Gilmore et al., 2007). Moreover, Experiment 1A shows that even in normal development, there is no relationship between the ability to perform exact arithmetic and the ability to perform computations on sets of approximate numerosities. Interestingly, a significant relation was found between the ability to perform exact computations on sets (using both symbolic and non-symbolic material) and the ability to do exact addition (**Figure 2.4**), again supporting the idea of an ‘exact core system for numbers’ as a precursor for exact arithmetic.

Yet, it is important to note that our data does not speak against the existence of a system that deals with approximate numerosities (Dehaene, 1997). Instead, in this Chapter we show that such system is well developed in typically developing children and can be fundamental to solve approximate operations on sets (**Figure 2.3**). However, we believe this system is not sufficient for performing exact computations on sets (exact arithmetic). Finally, by demonstrating that even children suffering from Developmental Dyscalculia can display spared numerical abilities, such as approximate arithmetic and knowledge of analogue magnitudes (Experiment 1B), we hope to provide teachers, educators and ultimately politicians with a tool to help this population. We suggest that these intact abilities could represent a valid learning strategy upon which to construct targeted teaching and rehabilitation programs. As we have seen in Experiment 1A, the

ability to perform computation on approximate and analogue quantities might not directly relate to the ability to solve exact arithmetic, but could still represent a critical aid for this population to rely on.

The data from this Chapter also suggests that developmental mathematical difficulties can have multiple origins as it has been previously suggested (Dowker, 2005; Jordan et al., 2002; Mazzocco & Myers, 2003; Temple, 1994). Interestingly, one of the earliest theories of Developmental Dyscalculia also proposed multiple origins (Kosc, 1974). However, most of these studies have not drawn a sharp distinction between Developmental Dyscalculia and other mathematical difficulties. Our data provides the first empirical evidence of a clear and theoretically based distinction between Developmental Dyscalculia and more general – and less severe – forms of low numeracy. Most of the studies investigating Developmental Dyscalculia tend to include children with low numeracy in their sample (e.g. Rousselle & Noël, 2007). While acknowledging the fact that this could lead to potential benefits in terms of sample size and statistical power, we believe that it is important to draw a distinction between Developmental Dyscalculia, a congenital condition with a specific neurological signature (Isaacs et al., 2001; Price et al., 2007; Rykhlevskaia, Uddin, Kondos, & Menon, 2009) and more general low numerical abilities.

In Experiment 1B we define Developmental Dyscalculia from a theoretically driven diagnosis (Butterworth, 2003, 2005). On the basis of this, we theorize that Low Numeracy (LN), which is not the result of DD, will have different causes from DD itself, though apparently leading to the same scholastic/educational outcome – ‘bad at arithmetic’ -. There have been many proposals as to what these causes might be including: poor working memory (Geary, 1993; Geary & Hoard, 2005; Hitch & McAuley, 1991) (see **Chapter III** for a more detailed account), poor long-term memory (Geary, 1993; Geary & Hoard, 2005), poor language skills (Donlan, Bishop & Hitch, 1998; *but see* Gelman & Butterworth, 2005) and poor cognitive skills (Kovas, Harlaar, Petrill, & Plomin, 2006; O’Connor, Cowan & Samella, 2000). In this Chapter we wanted to systematically test whether LN is the result of poor abilities in core numerical (either approximate, exact, or analogue) capacities or the result of a more subtle deficit which does not imply an impaired number system per se. Interestingly, our data shows that children with low numerical skills have no difficulties with approximate, analogue

or exact tasks. Instead, they seem to have a problem when dealing with symbolic numerical stimuli (Arabic digits). Our data shows that the LN group is less accurate and slower than the control group in the symbolic number comparison task (**Figure 2.5**). This result supports Rousselle and Noël’s hypothesis (2007) who claim that mathematical disabilities are the consequence of a deficient ability to link intact numerical representations with the symbolic meaning of their expressions (*see also* Mejias, Mussolin, Rousselle, Grègoire, & Noël, 2011). In our view, the results of these authors suggest that they might have been including children with low numeracy in their sample. Contrary to other studies, which use the lowest quartile as their cut-off (Koontz & Berch, 1996; Rousselle & Noël, 2007; Siegel & Ryan, 1989), the *Dyscalculia Screener* identifies as DDs only the pupils who score below the 7th percentile on the tests of capacity. Moreover, our cut-off for Developmental Dyscalculia is based on its reported prevalence (Butterworth & Reigosa-Crespo, 2007; Shalev, 2007; *see also* Reigosa-Crespo, Valdés-Sosa, Butterworth, Estévez, Rodríguez, et al., 2012). Notably, given an estimated prevalence of 3.5-7% for the condition (Butterworth & Reigosa-Crespo, 2007; Reigosa-Crespo et al., 2012; Shalev, 2007), we find two DD children amongst a total of thirty-six children tested (5.5%). Altogether our results suggest that there can be multiple types of numerical impairments, and whatnot, also of Developmental Dyscalculia. This is not surprising given the heterogenic nature of the discipline of mathematics. Moreover, our findings highlight the critical role of developing a firm link between ‘sets and symbols’ as one of the skills for succeeding in maths (Ansari, 2008; Holloway & Ansari, 2009; Mazzocco et al., 2011).

All in all, the results of Chapter II provide the first comprehensive account of the typical and atypical development of numerical representations and the ability to consequently perform computations on these representations. Moreover, the results of this Chapter delineate a clear distinction between different clinical profiles of mathematical disabilities by better characterizing the conditions of Developmental Dyscalculia but also the one of general low numeracy (LN). Given this distinction, in the next Chapter we aim to further explore the condition of low numeracy, which has so far been neglected in the literature or often inter-mingled with DD. In particular, we look at the working memory abilities of this group, since this cognitive skill has

periodically been linked with the ability to perform calculation (Ashcraft, 1992, 1995; Ashcraft & Kirk, 2001; Hayes, 1973; Heathcote, 1994; Hitch, 1978).

2.11. Summary

The results of Experiments 1A&B confirm the existence of a system for approximate numerosities, but also suggest that by the time children reach the age of eight or nine, it has no functional relationship with the capacity for exact numerosities. Moreover, our results do not support claims of approximate arithmetic as a precondition or precursor of exact arithmetic (i.e. Barth et al., 2005, 2006; Gilmore et al., 2007; Halberda et al., 2008). Second, our results question the theory of approximate numerosities being represented as analogue quantities (Feigenson et al., 2004). Importantly, children with low numeracy (LN) performed at normal levels on all non-symbolic tasks, implying that basic numerical concepts are intact. Yet, their ability to utilize numerical symbols is significantly impaired thus supporting previous theories (Mazzocco et al., 2011; Rousselle & Noël, 2007). Experiment 1B elucidates one of the underlying impairments in LN and has important implications for possible learning interventions in these children, which should stress the linkage of symbols to concepts (but see also **Chapter III** for a more comprehensive characterization of the LN profile).

Overall, our results seem to validate the proposal that we are born with a ‘core number sense’, which can construct exact numerosities on the basis of sets, and whose deficit can lead to Developmental Dyscalculia (see Butterworth, 2005). However, the two cases, defined by the *Dyscalculia Screener*, had different cognitive profiles. One case presented with very poor dot enumeration and lower IQ, he therefore might have had additional cognitive problems that contributed to his poor arithmetic. The other DD was, by contrast, defective in symbolic number comparison, but his IQ was higher. He showed much better arithmetic, so either his better general cognitive ability helped him compensate for low basic numerical capacity, or his number comparison test performance was unrepresentative of his ability to compare numbers. Nevertheless, the literature contains many reports of severe arithmetical difficulties in the presence of normal or superior IQ, and normal or superior working memory (e.g. Landerl et al., 2004).

It has been noted that the presenting characteristics of Developmental Dyscalculia can be modulated by co-occurring cognitive difficulties (Geary, 2011b; Rubinsten & Henik, 2009), and it has also been observed that Developmental Dyscalculia occurs more often than expected with other neurodevelopmental disabilities (Lewis et al., 1994; Shalev, 2007), again contributing to the presenting picture. At the same time, the data reported in this Chapter show that children can have severe low numeracy without being dyscalculic in the sense of having a deficit in core number capacities (Reigosa et al, 2012; and see **Chapter VII**). Thus, the mere presence of severe arithmetical difficulties without further cognitive investigation will not be sufficient for a classification of Developmental Dyscalculia.

III

‘Updating’ the cognitive profile of low numeracy: Working Memory abilities

“The existence of forgetting has never been proved: We only know that some things don't come to mind when we want them” -- Friedrich Nietzsche

Here we wished to determine how the sub-components of Working Memory (Phonological Loop and Central Executive) influence the arithmetical performance of children with low numeracy. Specifically, we aimed at distinguishing between Working Memory inhibition and updating processes within the Central Executive, and the domain-specificity (words and numbers) of both subcomponents in a population of children with low numeracy and their age matched typically-developing controls. We show that both groups were similar for phonological loop abilities, while Working Memory updating demonstrated a domain-specific modulation related to the level of children’s arithmetical performance. Moreover, inhibition processes interacted with domain-specificity and arithmetical attainment. These results are important for a better characterization of the clinical profile of low numeracy. Finally, our findings are particularly relevant for the diagnostic assessment of arithmetical ability and low numeracy and should be considered in existing tests of arithmetical development as well as in the educational practice.

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3.1. Introduction

As described in Chapter II, there have been many proposals for the possible causes of low numeracy. These include: poor working memory (Geary, 1993; Geary & Hoard, 2005; Hitch & McAuley, 1991), poor long-term memory (Geary, 1993; Geary & Hoard, 2005), poor language skills (Donlan, Bishop & Hitch, 1998; *but see* Gelman & Butterworth, 2005) and poor cognitive skills (Kovas, Harlaar, Petrill, & Plomin, 2006; O’Connor, Cowan & Samella, 2000). In **Chapter II** we demonstrated that children with low numeracy (LN) were completely normal on tests tapping the approximate system for numbers, as well as in tasks of analogue magnitude and discrete numerosities of non-symbolic material. However, our LN population seemed to have a problem when dealing with symbolic numerical stimuli (Arabic digits). Thus, the question that arises is whether the condition of low numeracy is only a problem of linking numerosities to the numbers that represent them, or whether the cognitive profile of this population might be characterized by other difficulties (or strengths) outside the maths domain.

It is widely assumed that Working Memory (WM) is important in calculation (Baddeley & Hitch, 1977; Hitch, 1978), and in particular, during the early development of arithmetical skills (Gathercole & Pickering, 2000; Geary, 1990, 1993; Geary & Hoard, 2001; Ginsburg, 1997; Jordan & Montani, 1997; Kirby & Becker, 1988; Russell & Ginsburg, 1984; Shalev & Gross-Tsur, 2001). A key distinction made in studies investigating the relationship between WM and arithmetical abilities has focused on the role of the different subsystems of WM as originally proposed by Baddeley and Hitch (1977). A primary distinction has been made between the Phonological Loop (PL) and the visuo-spatial sketchpad (VSSP). The former has been associated with solving single-digit addition problems (Hecht, 2002; Seyler, Kirk, & Ashcraft, 2003); while the latter has been linked with the encoding of visually presented problems (Logie, Gilhooly, & Wynn, 1994). Ultimately, the third component, the Central Executive (CE) system has been thought to play a key role in aspects of calculation that require the storage and manipulation of intermediate results online, by updating the results of operations such as carrying and borrowing (Baddeley, 2000). Given the distinctions proposed by this model, it has become important to draw a distinction between the different components of WM and their relationship to arithmetical abilities. This has become particularly

crucial especially given the mixed results on the role of the different sub-systems in supporting calculation. For example, De Rammelaere and colleagues found that articulatory suppression, which should interfere with PL but not with CE did not affect calculation; while random interval generation, thought to be the responsibility of CE, did reduce arithmetical performance (De Rammelaere, Stuyven, & Vandierendonck, 1999, 2001). Moreover, it has been found that children with specific difficulties in arithmetic (compared to both age-matched and ability-matched controls) do not differ on tasks that rely primarily on PL, such as immediate serial recall (e.g. digit span), but perform worse on tasks tapping the Central Executive component (McLean & Hitch, 1999; Passolunghi & Siegel, 2001; Siegel & Ryan, 1989). On the other hand, it is the PL component of Working Memory which has specifically been associated with arithmetical impairments (Hecht, Torgensen, Wagner, & Rashotte, 2001; Hitch & McCauley, 1991; Swanson & Sachse-Lee, 2001).

The most convincing evidence relating the sub-components of WM to mathematical achievement comes from a longitudinal study investigating children from kindergarten to third grade, which found that different systems of WM can discriminate between different levels of mathematical impairments as well as levels of maths proficiency (Geary, Bailey, Littlefield, Wood, Hoard, & Nugent, 2009). Specifically, these authors underlined the importance of the CE component for the correct retrieval of simple addition facts, which is of particular relevance to the present study.

According to our reasoning, to better evaluate the role of CE, it is important to have a measure that directly taps the critical function of CE in arithmetic — selecting and maintaining task-relevant numerical information. Of particular relevance to the current study, are processes of inhibiting and updating the contents of memory (Miyake, Friedman, Emerson, Witzki, Howerter, & Wager, 2000). Moreover, it has also been suggested that there is a domain-specific coding within the WM system. In particular, number representations may be maintained in WM differently than words (Butterworth, Cipolotti, & Warrington, 1996). Notably, a recent neuroimaging study on healthy adults claims for a special role for numbers in the PL component of Working Memory (Knops, Nuerk, Fimm, Vohn, & Willmes, 2006) by demonstrating stimulus specific modulation of numerical stimuli, but not word stimuli, in the intraparietal sulcus, a region often

associated with numerical processing (*see* Dehaene, Piazza, Pinel, & Cohen, 2003; *see* also **Chapter I, paragraph 1.4**).

Given the aforementioned evidence, it becomes important to distinguish number from word stimuli, and to directly address the issue of selecting and maintaining task-relevant information.

The present study addresses this issue in a group of children who presented a selective impairment in arithmetic. Specifically we look at their ability to update relevant information and the ability to inhibit irrelevant information. In addition to established tests to assess the capacity of the PL (digit and word span tests), a novel test was developed based on an Updating task previously used by Palladino, Cornoldi, De Beni, and Pazzaglia (2001). WM updating is defined as the amount of information recalled after being held and subsequently manipulated. This concept is similar to what Broadbent (1958) called ‘channel capacity’, and Cowan (1995) called the ‘capacity of the focus of attention’. Inhibition in the CE was defined as the amount of information to be suppressed according to a pre-specified criterion (*see* **paragraph 3.3.2.2** for details). This concept is similar to the ‘selective filter’ defined by Broadbent (1958) and to ‘controlling the direction of attentional focus’ as proposed by Cowan (1995). The advantage of using this Updating task is that it can pose variable demands on maintenance and inhibition processes separately (Moro, 2008). Particularly, our task requires participants to recall information that is relevant according to a given criterion, while the irrelevant information is being inhibited.

Experiment 2. WM Maintenance and Updating of different stimulus material in Low Numeracy

3.2. Aim 2

In this study we tested a group of children with low numeracy who exhibited a selective impairment in exact calculation (addition) (see **Chapter II**) and their matched control peers in two canonical span tasks and two novel updating tasks using numbers and words as stimuli. Our first aim was to determine whether selective impairments in arithmetical abilities could be attributed to a deficit in WM. Moreover we wanted to investigate which subcomponent of WM might be impaired or spared in this population of children. Specifically, whether impairments could be seen in maintenance processes (PL subcomponent) as measured by the span tasks, or in updating and inhibition processes (CE subcomponent). Furthermore, by manipulating WM load and inhibition levels in the updating tasks, we aimed at better differentiating the updating and inhibition processes of the CE component of WM. Finally, we aimed to determine whether different stimulus categories could discriminate between the two groups of children’s WM abilities.

3.3. Method 2

3.3.1. Participants

Thirty-three children in their 4th year of primary school (mean age = 8.919, SD = 4.57, 11 males) took part in this experiment. They were recruited from three different State Middle Schools in the London Borough of Harrow.

The sample was divided into two groups: 1) normally achieving (NA) children (N = 22) and 2) low numeracy (LN) children (N = 11) on the basis of their scores on the *Dyscalculia Screener* (see **Chapter II, paragraph 2.7.1**). Particularly, the LN group was defined as being in the bottom 7% of the population on the ‘Achievement test’ of the *Dyscalculia Screener* (i.e. test of exact addition). Additionally, the LN group had to present an average score on the ‘Capacity tests’ of the *Dyscalculia Screener* and an IQ

within the normal range⁵. It is important to note that these two groups were the same as the ones described in **Chapter II**, with the exception of one participant in the NA group who was no longer available at the time of testing. Furthermore, the behavioural profile described in detailed in **Chapter II** applies to all the subjects of Experiment 2. Demographics and scores on approximate, analogue and exact tasks are summarized in **Table 3.1** for each of the two groups. It is important to note that the LN group differed from their control peers only on the symbolic number comparison task of the *Dyscalculia Screener*; while differences between LN and NA were not evident in any of the other tasks: approximate comparison and approximate arithmetic, manipulation of analogue and discrete non-symbolic quantities and enumeration of dots (see **Chapter II**, paragraph 2.8.1).

Measure	NA (N = 22)			LN (N = 10)		
	Accuracy	RTs	Inverse Efficiency	Accuracy	RTs	Inverse Efficiency
	Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)
Dyscalculia Screener						
Simple RTs	414.24 (159.12)			453.25 (211.9)		
Dot Enumeration	90.17 (6.49)	1971.39 (719.3)	21.79 (7.7)	85.38 (4.33)	2278.05 (803.4)	26.68 (9.73)
Number Comparison	91.69 (6.31)	699.0 (269.76)	7.63 (2.91)	89.67 (4.83)	1092.35 (366.5)	12.27 (4.09)
Exact Addition	87.17 (5.65)	3941.43 (2306.2)	45.86 (24.92)	58.57 (9.09)	3783.4 (2099.2)	65.25 (33.14)
Number tasks						
Approximate Comparison	67.42(13.1)	710.75(239.93)	45.88 (17.56)	60.0 (13.06)	703.0 (355.23)	52.13 (29.3)
Approximate Addition	70.45(16.1)	761.02(432.08)	51.80 (45.97)	76.67(8.38)	784.5(532.8)	45.17 (34.3)
Approximate Subtraction	61.74(11.8)	699.23(302.89)	51.60 (32.85)	52.92 (8.57)	824.1 (521.67)	67.39 (46.76)
Discrete Numerosity	93.46 (5.27)	987.52 (191.71)	22.16 (4.95)	84.79 (21.3)	1175.9 (496.04)	41.39 (56.18)
Analogue Area	84.75 (11.2)	1269.89 (243.56)	31.67 (6.97)	76.25 (13.3)	1263.8 (399.26)	36.4 (15.8)

Table 3.1. Dyscalculia Screener and Number tasks’ scores for NA and LN. Cells in Bold indicate significant differences between the groups in tasks where LN’s performance was worse than NA.

⁵Please note that, as mentioned in Chapter II (paragraph 2.7.1), one of the participants from the LN group had to be removed from the analyses as his IQ was < 70.

3.3.2. Experimental design

3.3.2.1. Standardized tasks

The standardized tasks used to allocate participants to the groups were the same as the ones described in Experiment 1A (**Chapter II, paragraph 2.3.2.1**). The *Dyscalculia Screener* (Butterworth, 2003) was used to assess maths abilities and the WISC-III full protocol (Wechsler, 1996) was used as a measure of participants’ IQ in the experimental group only.

3.3.2.2. Experimental tasks

The experimental tasks comprised two types of Working Memory tasks: 1) *Span* tasks, as a measure of the Phonological Loop component of WM; 2) *Updating* tasks, as a measure of the Central Executive component of WM. The order of administering tasks was counterbalanced among participants. The dependent variable used was accuracy which was defined as the percentage of correct recalls from a given list.

Span tasks. Two verbal span tasks were used: a Digit Span and a Word Span task, both forward and backward. The stimuli used in the Digit Span forward and backward were identical to those used in the WAIS-III scale (Wechsler, 1996). Stimuli in the Word span task were of two semantic categories – Animals and Objects – and were selected in order to match the digit stimuli in number of syllables (one to five) and word lengths (number of letters). Lists were of increasing complexity from two to nine items. Stimuli were verbally presented by the candidate at the rate of one item per second.

The task instructions were the following for both forward tasks (Digit and Word): “I am going to say some numbers (or words). Listen carefully, and when I am through, I want you to say them right after me. Just say what I say”. Instructions for the backward tasks were: “Now I am going to say some more numbers (or words). But this time when I stop, I want you to say them backward, from the last one that I saw to the first one. For example, if I say 7-1-9, what would you say?” If the child was correct, the task will begin with two practice trials otherwise the experimenter will give the correct answer of the problem (9-1-7) and assure the child had understood the task, before administering the practice trials.

Updating tasks. A task of Working Memory was devised to test participants’ ability to update relevant information and inhibit irrelevant information during a task of free recall with a semantic criterion (Moro, 2008). It was adapted from Palladino and colleagues (2001). Sixteen lists of words (eight lists of names of animals and eight lists of names of objects) and sixteen lists of two-digit numbers (odd in half of the lists even in the other half) were presented to the children who were required to retain the relevant stimuli based on an on-going semantic criterion (magnitude of the stimuli). The words were bi- or tri-syllabic, highly familiar and imaginable nouns initially selected from a list by Burani and colleagues (Burani, Barca, & Arduino, 2001). This word-set was also piloted in 23 adult participants who were asked to judge, on a scale from 1 (very small) to 9 (very big), the dimensions of 53 animals and 100 objects. The order of presentation of all types of stimuli was randomized within and between participants. Only the items with a clearly discriminable size were used. A second pilot study (with 20 adult participants) was later conducted in order to check the discriminability of the selected items within the lists. The lists were balanced for number of syllables and length of the word (number of letters).

Stimuli in the Number Updating task were double-digit numbers ranging from 22 to 99. They were associated to the animals and objects according to the size judgement from the pilot study (i.e. the smallest animal/object used in the list would correspond to number 22, which was the smallest number in the list of numbers). In this way the lists were similarly constructed so that each number corresponded to an object or animal. The numbers excluded were teens, multiples of ten, and numbers containing 1 as a unit. Since numbers with ‘1’ as decade (10-19) were removed, numbers with ‘1’ as the unit (e.g. 21, 31, ... and 91) were also removed, so that each digit (2-9) would have an equal rate of occurrence. Moreover, in this way there was an equal number of odd and even numbers. A possible source of confusion could have been that the number of syllables composing the double-digit number stimuli is bigger (3 to 5 syllables) than single-digit numbers, but the use of two-digit numbers was necessary in order to have a large number of different items and match the two tasks.

Four practice trials (two for each task) preceded the actual experiment to ensure that the child understood the task requirements. The stimuli were verbally presented by the candidate at the rate of one item per second and the child was required to recall a

predefined number of the smallest items presented. This procedure required the child to constantly update the incoming information and inhibit or suppress the items that were no longer relevant.

Working Memory load was manipulated by having four recall conditions with participants having to recall one (WML1), two (WML2), three (WML3), or four (WML4) of the smallest items presented. From now on we will refer to this factor as Working Memory Load. Inhibition was a two level factor with the participants having to ignore (or initially recall and then inhibit) one item (condition of Low Inhibition, LI) or three items (condition of High Inhibition, HI). As a result, the list length varied between two and seven items. An example of a list with three items to maintain and three items to inhibit is: “giraffe – pelican – tortoise – tiger – chicken – dolphin”. Here, the participant had to remember the three smallest animals in the list (i.e. pelican, tortoise, and chicken) while inhibiting the recall of the other three. An example of a list with numbers where two items had to be recalled and three had to be inhibited is: “26-68-92-66-35”. Here the items to recall are 26 and 35 (i.e. the two smallest numbers in the list). In order to perform correctly on this task the child, while listening to the presented items, had to constantly update the information with the new item presented and to inhibit one of the previously recalled items that was no longer fulfilling the criterion.

3.4. Results 2

We assessed differences between groups on the WM assessments by repeated measures ANOVA. The model for the Span tasks had Task (Word task and Number task) and Condition (Forward and Backward) as the within subjects factors and Group (LN and NA) as the between subjects factor. The Updating task used a 2×4×2×2 mixed design with Task (Word and Number), WM Load (one, two, three and four items to recall) and Inhibition (Low – one item to inhibit and High – three items to inhibit) as within subject factors. Group (LN and NA) was the between subjects factor. Accuracy was defined as the percentage of correct recalls. Greenhouse–Geisser corrections were applied to all factors with more than two levels to correct for violations of sphericity (Keselman& Rogan, 1980).

3.4.1. Span tasks

Repeated measures ANOVA revealed a main effect of Task ($F(1,30)=4.8$, $p=.036$, $\eta^2=.13$): the Number task was significantly better than the Word task; and a main effect of Condition ($F(1,30)=154.6$, $p<.001$, $\eta^2=.84$): the Forward condition was significantly better maintained than the Backward condition.

There was no main effect of Group ($F(1,30)=.078$, $p=.78$, $\eta^2=.003$) (**Figure 3.1**). All the interactions were not significant Task by Group ($F(1,30)=3.6$, $p=.068$, $\eta^2=.1$); Condition by Group ($F(1,30)=.03$, $p=.85$, $\eta^2=.001$); Task by Condition ($F(1,30)=.89$, $p=.35$, $\eta^2=.03$) and Task by Condition by Group ($F(1,30)=.29$, $p=.59$, $\eta^2=.009$).

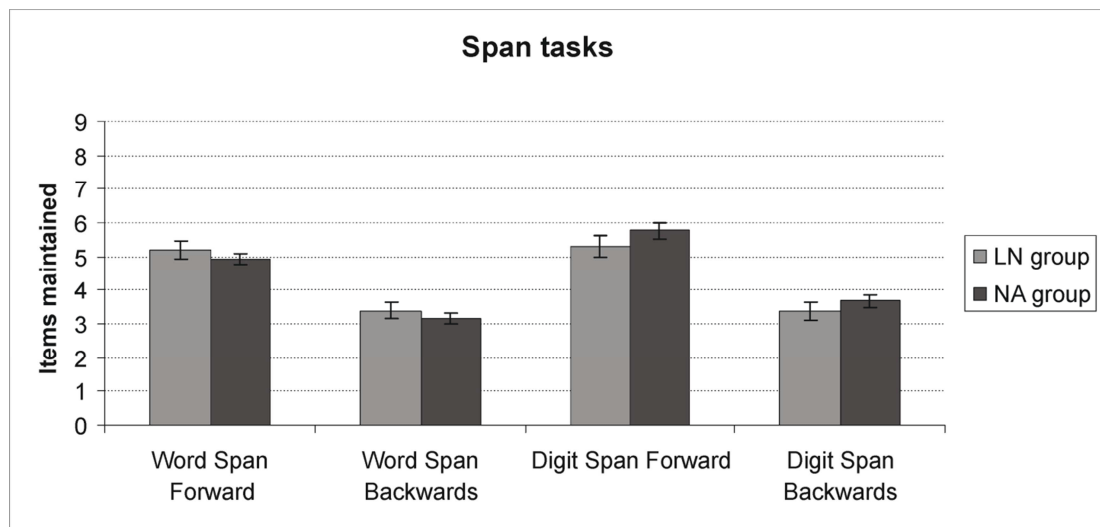


Figure 3.1. Mean of maintained items for the LN group (light grey) and for the NA group (dark grey) for each of the Span tasks. Error bars indicate 1 standard error of the mean.

3.4.2. Updating tasks

A repeated measures ANOVA revealed a main effect of Task: the Word task was more accurate than the Number task [$F(1,30)=53.87$, $p<.001$, $\eta^2=.64$]. There was a main effect of Working Memory Load [$F(2.33, 69.79)=190.799$, $p<.001$, $\eta^2=.86$] and a main effect of Inhibition [$F(1,30)=117.99$; $p<.001$, $\eta^2=.79$]: in both tasks, the proportion of correct recalls decreased with increased WM load and it was modulated by increased inhibition of irrelevant information. There was also a Task by WML interaction [$F(2.75, 82.61)=7.51$; $p<.001$, $\eta^2=.2$]: the two tasks only differed in the

WML3 and WML4 conditions where the accuracy of the Number task was lower than the Word task ($p < .001$).

The Task by Inhibition interaction was also significant [$F(1,30)=7.42$; $p < .05$, $\eta^2=.2$]: the two tasks only differed in the Low Inhibition condition ($p < .05$). Moreover, the Task by Inhibition by WML interaction was also significant [$F(2.74,82.32)=3.79$; $p < .05$, $\eta^2=.11$]. Post-hoc pair-wise comparisons (paired sample t-tests) showed that the two tasks (Words and Numbers) differed on both levels of Inhibition only in the WML3 condition ($p < .005$) (**Figure 3.2**).

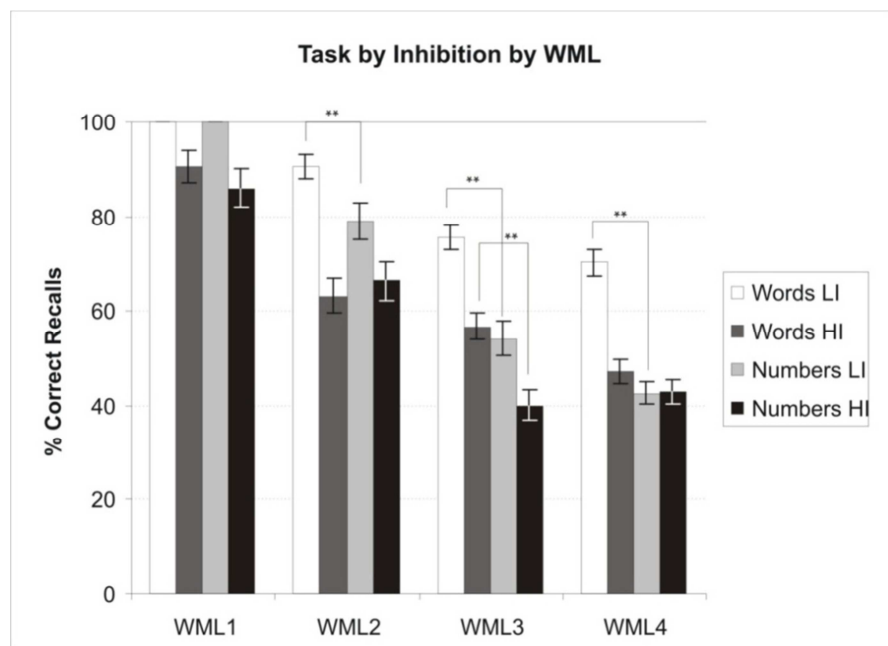


Figure 3.2. Percentage of correct recalls for the four WM Load conditions (x-axis), on the two inhibition conditions for both tasks. ** $p < .005$. Error bars indicate 1 standard error of the mean.

The interaction of WML by Inhibition was not significant ($F(2.73, 81.9)=2.24$, $p=.095$, $\eta^2=.07$). There was no main effect of Group [$F(1,30)=.008$, $p=.93$, $\eta^2=.00$]. However, a Task by Group interaction was found [$F(1,30)=6.72$, $p < .05$, $\eta^2=.18$] (**Figure 3.3**). The three-way interaction Task by Inhibition by Group was also significant [$F(1,30)=8.34$, $p < .01$, $\eta^2=.22$] (**Figure 3.4A**).

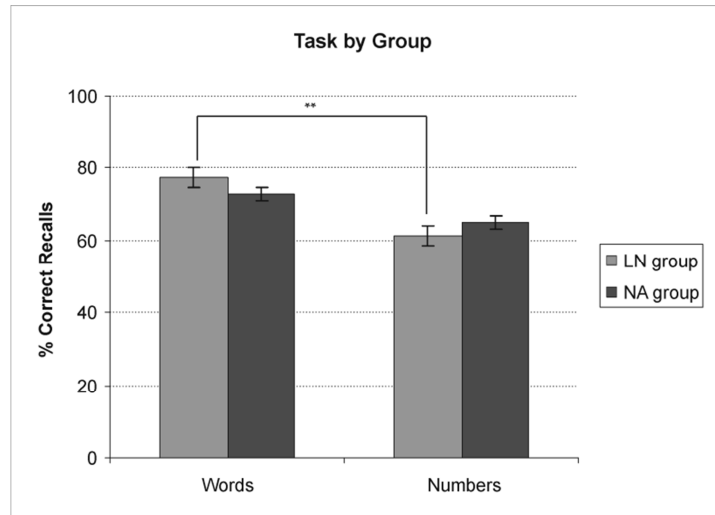


Figure 3.3. Percentage of correct recalls for the Word task and the Number task in the LN and NA groups. ** $p < .005$. Error bars indicate 1 standard error of the mean.

Post-hoc analyses (independent sample t-tests) revealed that the LN group performed significantly better than the NA group in the HI condition of the Word task ($p < .05$) (**Figure 3.4B**). The other interactions were not significant: Inhibition by Group ($F(1,30)=1.7$, $p=.202$, $\eta^2=.05$); WML by Group ($F(2.33, 69.79)=.57$, $p=.64$, $\eta^2=.02$); Task by WML by Group ($F(2.75, 82.61)=1.04$, $p=.37$, $\eta^2=.03$); WML by Inhibition by Group ($F(2.73, 81.9)=.36$, $p=.78$, $\eta^2=.01$); and Task by WML by Inhibition by Group ($F(2.74, 82.32)=.093$, $p=.95$, $\eta^2=.003$).

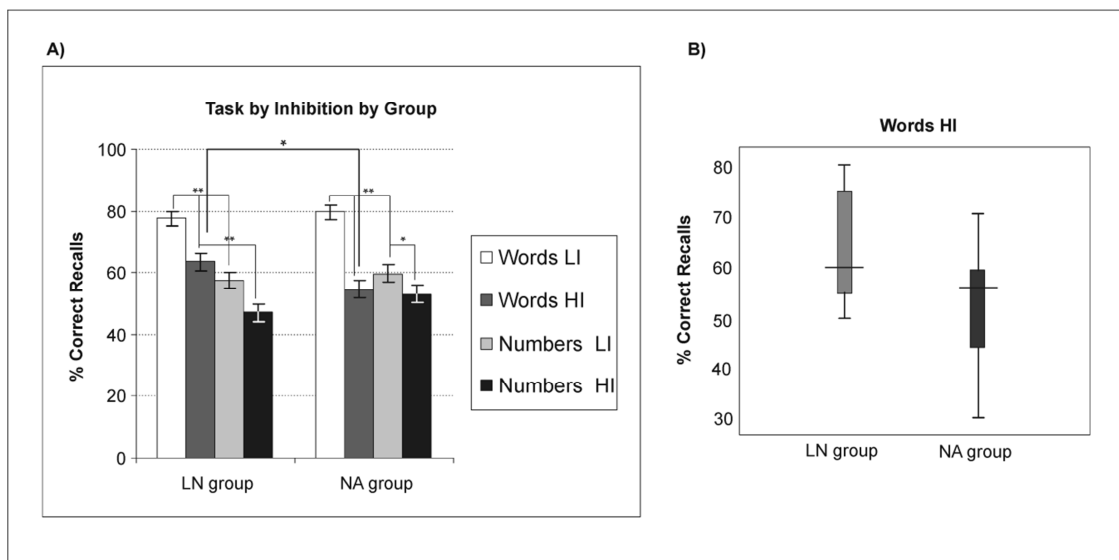


Figure 3.4. A) Percentage of correct recalls for the Word and the Number tasks with levels of Inhibition in the LN and the NA groups (* $< .05$; ** $< .005$). B) The LN group performed significantly better than the NA group on the Word task at High Inhibition level. Error bars indicate 1 standard error of the mean.

3.5. Final Discussion

In this chapter we employed a new updating task to assess the contribution of Working Memory components to the processes of calculation in a population of 8-9 year old children with low numeracy, compared to a group of typically developing peers. The groups were distinguished on the basis of their speed and accuracy on a standardized test of numerical capacity and attainment in curriculum exact arithmetic (Butterworth, 2003; but see also **Chapter II**). Additionally, they were tested on their ability to carry out approximate addition and subtraction with non-symbolic (dot array) stimuli since these are held to be foundation for exact arithmetic (Gilmore, McCarthy & Spelke, 2007; Halberda, Mazocco, & Feigenson, 2008; but see also **Chapter II** for a more detailed account). Importantly, the groups were readily distinguishable on the basis of the arithmetical attainment of exact addition, but not on their ability to carry out the capacity task of dot enumeration (**Table 3.1**). Moreover, the groups did not differ on any of the approximate arithmetic tasks (**Table 3.1**).

Comparing the two groups (NA and LN) on measures of WM, our findings support a complex relationship between the components of WM and calculation. Not only both span tasks forward, which are robust assessments of the Phonological Loop (PL) component of WM, but also both span tasks backward, which also rely strongly on PL, were performed at the same level in each group, suggesting that these WM processes were not critical for distinguishing typical from low attainment in arithmetic. As found previously by McLean and Hitch (1999), forward span did not predict calculation ability, and likewise in this study it did not distinguish LN from NA. This is also in line with a study of Developmental Dyscalculia in 8-9 year olds, which found no differences on a digit span task between dyscalculics and typical learners (Landerl, Bevan, & Butterworth, 2004). This suggests that the immediate serial recall does not tap the processes critical to arithmetical performance. Yet, it is necessary to transiently maintain information to carry out many arithmetical tasks. This function seems to be supported and mediated by the Central Executive component of WM (Noël, Seron, & Trovarelli, 2004). Indeed our data shows that differences between the groups were only evident in the Updating tasks (**Figure 3.3 & 3.4**).

Generally, the results on the Updating task demonstrate that updating was easier in the Word task than in the Number task. Second, it showed clear and significant effects of Working Memory load (number of items to recall) and Inhibition (number of items to inhibit) in both groups: LN and NA handled increasing Working Memory Load and the effects of Inhibition equally satisfactorily (**Figure 3.2**). Yet, a Task by Group interaction was found suggesting that the LN group performed better on Word than Number stimuli, while the contrary was true for the NA group (**Figure 3.3**). In our view, this finding stresses a need to specify the stimulus type in WM tasks (*see also* Knops et al., 2006), especially when investigating maths learning disabilities. Moreover, this finding is consistent with the proposal for domain- or material- specific capacities in Working Memory (Butterworth et al., 1996; Semenza, Miceli & Girelli, 1997). Additionally, the three-way interaction of Task \times Inhibition \times Group, suggests that High Inhibition had a greater effect on Words than Numbers in the NA group while it had a greater impact on Numbers than Words in the LN group. Surprisingly, the LN group performed better on Words than Numbers while no significant effect was found in the other direction (**Figure 3.4B**). One explanation for this could be the fact that Words elicited a bigger effect on semantic processes and were a better discriminator of group performance. Under this notion, two-digit numbers display greater semantic and syntactic complexity compared to long nouns. Furthermore, words can be remembered through verbal encoding and image encoding, while numbers cannot (Paivio, 1991; Paivio, Walsh, & Bons, 1994). This could explain why, in general, highly imaginable concrete nouns are more easily remembered than numbers and other word categories (i.e. abstract nouns or verbs). On the other hand, one digit numbers in the span tasks were better recalled than nouns of the same length for both groups. However, another possibility is that we were unable to detect any specific differences in number stimuli due to our relatively small sample size in the LN group. Yet, beyond the general effect of difficulty on the stimulus material, our three-way interaction seems to suggest a better capacity of the LN group to update information on high inhibition levels for words but not for numbers. The latter result could be of particular relevance for the field of education: teachers could help these pupils to make use of their ability to update information of imaginable concrete nouns to facilitate the learning of more difficult material (i.e. arithmetical facts), a great struggle for this population (see **Table 3.1**).

Finally, given our results, we propose that existing batteries of neuropsychological assessments should be included in tasks which tap the specific subsystems of WM (in this case the CE component) and that discriminate between the stimulus materials used.

3.6. Summary

Working memory is the ability to hold transient information in mind and is critical for higher cognitive functions, such as language and mathematical abilities. The results of Experiment 2 confirm the existence of a link between working memory abilities and arithmetical performance. In particular, our findings support a complex relationship between the distinct components of WM and calculation. Using a cross-sectional design we demonstrated that Span tasks, which assess the Phonological component of Working Memory were performed at the same level in each group, suggesting that this WM subsystem was not critical in distinguishing typical from low attainers (**Figure 3.1**). On the other hand, we found that the capacity to update relevant information and inhibit irrelevant information, a key signature of the Central Executive component (CE) of WM, did discriminate between the groups. Particularly, the LN group performed worse on Number than on Word stimuli, while the NA group performed better on Numbers than Words (**Figure 3.3**). Additionally, our three-way interaction seems to suggest that high Inhibition had a greater effect on Words than Numbers in the NA group while the opposite was true for the LN group (**Figure 3.4**). We therefore propose that poor arithmetical performance could be associated with a deficit in WM updating that seems to be material-specific (*see* Knops et al., 2006).

To conclude, the results of Experiment 2 acknowledge a close relationship between the CE component of WM and the ability to perform or under-perform exact arithmetic (*see* Geary, Bailey, Littlefield, Wood, Hoard, & Nugent, 2009). However, the effects differ by semantic category: i.e. words *versus* numbers. Notably, it is necessary to maintain information to carry out many arithmetical tasks, and our results are consistent with the evidence from neurological patients that this process could be mediated by a domain- or material- specific capacity of working memory (Butterworth et al., 1996; Semenza et al., 1997).

Finally this study, together with Experiment 1B is of particular relevance for the diagnostic assessment of arithmetical disabilities, specifically the characterization of

low numeracy. Furthermore, as in the case of the two DD learners described in **Chapter II**, the present results could be an aid for teachers and educators to develop a better, and more targeted educational approach to help these pupils during maths lessons. Finally, we propose that these domain-specific updating tasks should be implemented in the currently existing testing batteries for assessing the educational development of children.

IV

The development of rational numbers: fractions and decimals

*"Not everything that counts can be counted.
Not everything that can be counted counts"*
-- Albert Einstein

Understanding fractions and decimals is a difficult concept to learn because whole numbers are the most frequently and earliest experienced type of number, and learners must avoid conceptualizing fractions and decimals in terms of their whole-number components ("the whole-number bias"). Here we use a computerized version of number line tasks on adults and children in order to explore the understanding of four types of number notations: fractions, integers, decimals, and money. In the *Number to Position* task, participants were instructed to place a mark on a line that corresponds to the target number (e.g. 25, .25, 1/4, or 25p). Alternatively, in the *Position to Number* task, participants had to report the number corresponding to a pre-placed mark on the line. Results were very similar for decimals, integers, and money in both tasks and for both groups, demonstrating that the linear representation previously shown for integers is also evident for decimals already by the age of 10. By contrast, fractions seemed to be 'task-dependent' so that when asked to place a fractional value on a line, both adults and children displayed a linear representation, while this pattern did not occur in the reverse task. Moreover, our findings suggest a developmental trajectory in the mastering of fractional stimuli as adults were more accurate than 10 to 11 year old children. Yet, both adults and children were slower to make judgments about fractions, speaking in favour of the hypothesis that additional mental demands are required when processing these types of numerical stimuli.

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4.1. Introduction

Most studies of mathematical cognition have focused on the processing of whole numbers, both from behavioural and neuro-imaging perspectives. Yet, as we have seen in previous chapters, the discipline of mathematics is very heterogeneous and most importantly, comprises many different types of number notations (see **Chapter I, paragraph 1.2**). Moreover, certain areas of mathematics are more difficult to grasp than others (Hasemann, 1981). For example, fractions have long been recognized as a difficult topic in mathematics. Yet, the understanding of this numerical notation represents a fundamental aspect of the majority of school's curriculum. Particularly, it has been noted how numerical notations such as fractions are the critical foundations of algebra (National Mathematics Advisory Panel (NMAP), 2008). From a general perspective, the understanding of proportion is critical for mastering simple aspects of everyday life (e.g. calculate the appropriate food quantities for a recipe; or whether a dress on sale is worth buying), and sometimes even in more critical situations (e.g. calculating drug doses on the basis of body weight). Yet, most existing research in this area comes from the perspective of education or educational psychology. In this Chapter instead we take a cognitive approach in order to investigate the processing and representation of rational numbers in adults and children.

The few studies investigating the processing of rational numbers -fractions and decimals-have found that learners have great difficulty understanding these types of notations (Bright, Behr, Post, & Wachsmuth, 1988; Hartnett & Gelman, 1998; Mack, 1995). One of the most striking difficulties for the learner is to avoid treating fractions and decimals in terms of their whole number components, sometimes termed the “whole number bias” (Ni & Zhou, 2005). This is not surprising since learners have to make a large conceptual leap from thinking of numbers as integers (Smith, Solomon, & Carey, 2005). In a recent study of 8 to 11 year olds, 38% denied there were any numbers between 0 and 1; similarly, 46% thought that $1/75$ was larger than $1/56$ because 75 is larger than 56; and 43% could not say why a fraction is composed of two numbers (Smith, et al., 2005). Hence, acknowledging the existence of these types of numbers is particularly difficult because it challenges the fundamental concept of number as counting number (e.g. ‘1’ and ‘ $1/2$ ’ are different kinds of entities as ‘1’ is a number that

occurs in the count list and '1/2' is not) (Smith et al., 2005). Furthermore, the transition from a system where numbers are used for counting to one that is reasoned proportionally can be problematic in late adolescence (Hoyles, Noss, & Pozzi, 2001) and even for educated adults (Bonato, Fabbri, Umiltà, & Zorzi, 2007). In the latter study, when subjects were asked to compare the real values of two fractions – e.g. $1/8$ versus $1/7$ – they typically compared the integer values of the numerator or the denominator rather than computing the real value of the fraction, thus supporting the idea that this bias can still persist even into adulthood.

The same “whole number bias” has been shown for decimals. A recent educational study reports that only half of the 8-11 year olds tested could select the larger from these two pairs: .65 vs. .8 and 2.09 vs. 2.9 (Smith et al., 2005). Finally, it has been reported that even fractions represented non-symbolically (i.e. proportions) lead to errors showing a whole number bias. In Fabbri, Tang, Zorzi & Butterworth (2009), subjects were required to compare the proportion of white dots in an array of white and black dots with a reference array of white and black dots. Subjects showed striking congruity effects: that is, if both numerosity and the proportion of white dots were greater (e.g. 6:3 vs 5:5), subjects were faster and more accurate than if proportion was greater but numerosity was smaller (e.g. 4:2 vs 5:5).

To summarize, behavioural studies so far have suggested that fractions –and to some extent also decimals– are processed in a componential manner without accessing the denoted numerical value (Bonato et al., 2007). However, all studies to date have been using number comparison paradigms which may impose additional cognitive demands (e.g. on working memory resources) and may induce a componential process (e.g. it is easier to decide that 35 is larger than 24 than to decide that 35 is larger than 19) (Nuerk, Weger, & Willmes, 2001, *but see* Dehaene, Dupoux, & Mehler, 1990; Hinrichs, Yurko, & Hu, 1981).

In Experiments 3A, B and C, we aim to control for these potential confounds by implementing a number line task in which participants have to place a mark on a physical line to denote the magnitude of the number presented and thereby controlling for processes of comparison. This is a variant of the classic ‘thermometer task’ used for the quick clinical evaluation of numerical competence of neuropsychological patients

(e.g. van Harskamp, Rudge, & Cipolotti, 2002). It has also been used in the educational and developmental literature, usually with subdivisions marked and often in a forced-choice task where the participant has to select one among several candidate positions (Bright et al., 1988; Rittle-Johnson, Siegler, & Alibali, 2001). In addition to this we also use the opposite manipulation where participants see a hatch mark on a line and need to report the numerical value (*see* Siegler & Opfer, 2003).

Finally, it has recently been suggested that two separate representations for single and double-digit numbers may exist for adults (Nuerk et al., 2001) and young children (Moeller, Pixner, Kaufmann, & Nuerk, 2009). Yet, these studies only used natural numbers from 0 or 1 upwards and therefore neglected the representation of other forms of two digits numbers such as fractions and decimals. Here we aim to test the nature of the representation of these types of numbers using the two aforementioned number line tasks.

Experiment 3A. Rational number representations in adults – The mental number line tasks

4.2. Aim 3A

Although many studies have investigated the representation of numbers –and their mapping to space - through the metaphor of the mental number line, how this mapping is modulated by different numerical notations has not yet been explored in adults or children. Moreover, most of the study investigating the representation of numbers had relied on comparison tasks where the dependent variables measured were two well established behavioural effects: the Spatial Numerical Association of Response Codes –SNARC - and the size effect. Recently, it has been proposed that the nature of the representation of double-digit numbers is different than the representation of single-digit numbers as componential strategies have been reported when making comparison judgments on integers (Nuerk et al., 2001).

In Experiment 3A we wished to assess the nature of the numerical representation of fractional and decimal stimuli compared to other forms of numerical stimuli in a group of well-educated adults using two explicit number to space mapping tasks:

Number to Position and *Position to Number* (adapted from Siegler & Opfer, 2003 – see **paragraph 4.3.2.1**).

4.3. Method 3A

4.3.1. Participants

A total of eighteen university students - BA or MA/MSc- (7 males, mean age = 24.3 years, SD = 6.74) from a variety of academic backgrounds took part in Experiment 3A. Participants' level of mathematical education was taken into account in the following way. Participants were divided into three subgroups: 1) no maths A-level, 2) maths A-level, and 3) maths at University level (see **paragraph 4.4.1**).

All participants reported normal or corrected-to-normal vision, and all except one were right-handed.

4.3.2. Experimental design

4.3.2.1. Experimental tasks

Two complementary estimation tasks were used, Number to Position (NP) and Position to Number (PN) (*see* Siegler & Opfer, 2003).

In the NP task, participants were instructed to move the cursor to the desired position using a mouse on a fixed- length line presented on the computer screen. The number-line coordinates for participant responses were recorded in terms of pixel count along the length of the line. In the PN task participants were required to type numbers into onscreen boxes.

Each problem involved a line with the left end labeled “0” and the right end labeled “1” or “100” or “£1” depending on the notation condition. The right label changed to conceptually match the notation session, whilst preserving the scaling. The left to right orientation is similar to previous studies (Siegler & Opfer, 2003; van Harskamp et al., 2002) and is thought to reflect the orientation of the mental number line in Western subjects (Dehaene, Bossini, & Giraux, 1993). The size of the images containing the stimuli was kept constant across all trials and notational conditions (726 x 483). Stimuli were randomized within testing blocks and displayed on a 1024 x 768

pixels monitor. All participants were tested using the same computer (model: Asus m6800n wide-screen) which has a dot pitch of 0.282 mm.

Four stimulus *notation* conditions were used in the NP task: **fractions** (e.g. $1/4$), **integers** (e.g. 25)⁶, **decimals** (e.g. 0.25) and **money** (e.g. 25p). The numerical values of the fractions were all <1 and were evenly distributed on the number line: $1/20$, $1/9$, $1/6$, $1/5$, $2/9$, $1/4$, $2/7$, $1/3$, $2/5$, $4/9$, $1/2$, $4/7$, $3/5$, $13/20$, $5/7$, $3/4$, $7/9$, $5/6$, $6/7$, $19/20$. The stimuli for the decimals and money notation conditions were selected so that they matched to an approximation of the numerical values of the fraction stimuli (e.g. 0.1 for $1/9$). Stimuli for the integers condition matched in scale to the numerical values of the fraction stimuli (e.g. 25) (**Figure 4.1A**).

In the PN task, the same four *notation* conditions were used with marks on the line corresponding to the numerical values in the NP task. For instance, in the **fractions** condition, participants would have to assess the fractional value of a mark corresponding to $1/4$ by freely choosing their estimate from the infinite spectrum of possible fractions (**Figure 4.1B**).

⁶By “integer” we mean here non-negative whole numbers.

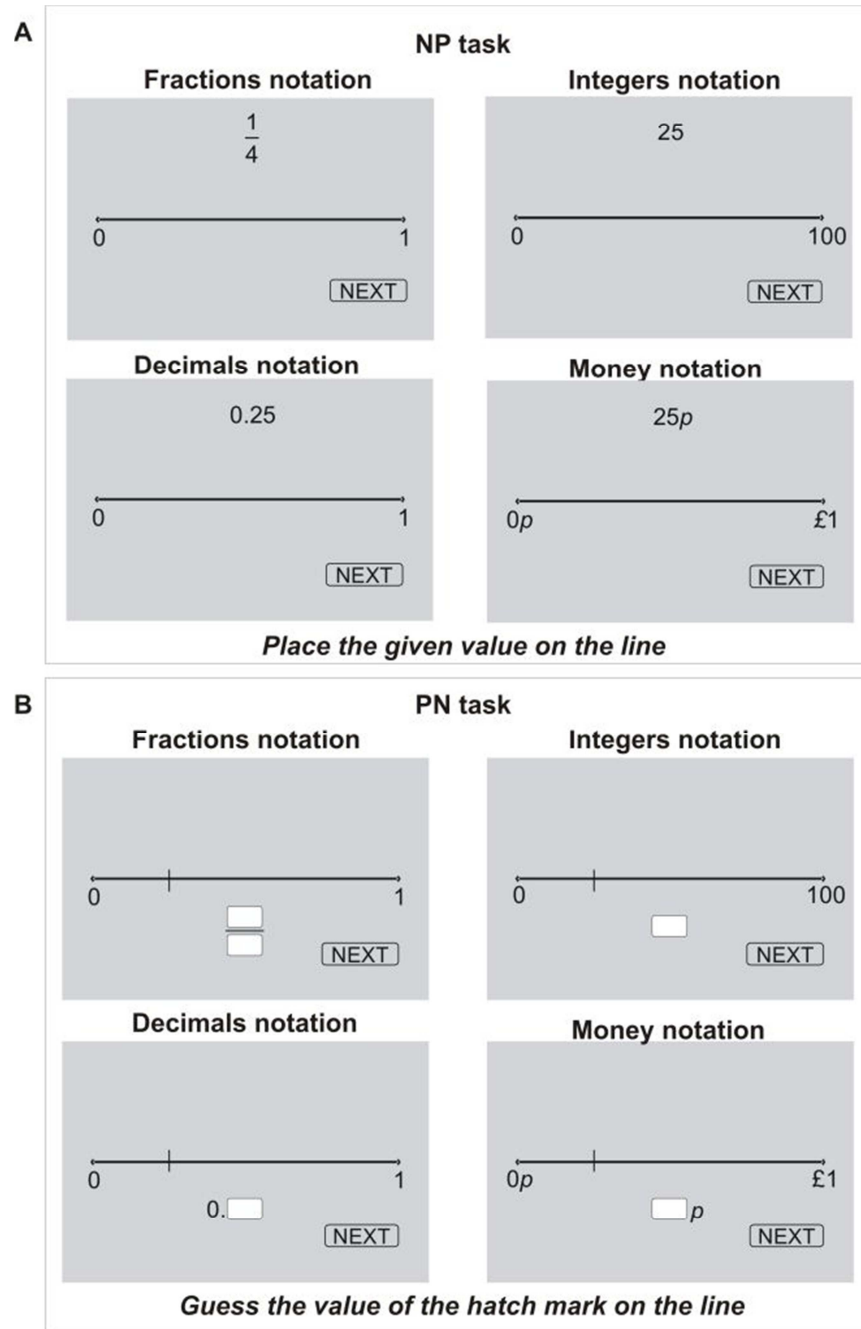


Figure 4.1. Example of stimuli for both tasks: A) Number to Position (NP), and B) Position to Number (PN).

In the NP task a line marked with the minimum and maximum values at either end was presented in the centre of the screen simultaneously with a target, centrally positioned on the top of the line. Participants were required to indicate the corresponding position of the target on the line. Participants responded by selecting the appropriate position using a mouse whose cursor starting point was not fixed.

In the PN task participants were presented with the same number-line as in the NP task with a hatch mark indicator bisecting the line at locations which corresponded to the numerical values used in the NP task. Participants were asked to estimate the value corresponding to the marked position on the line expressed as a **fraction, integer, decimal, or money** amount (**Figure 4.1B**). Participants responded by typing their answer into a box on the screen.

Participants responded at their own pace and were allowed to correct their responses. This study was composed of a single block for each of the four notation conditions (20 stimuli each). Stimulus presentation was randomized within each set; order of tasks and notation conditions were counterbalanced across participants.

4.4. Results 3A

4.4.1. Education

The slope (β_1 value) of the regression equation $Y = \beta_0 + \beta_1 x$ of the linear model was taken as an index of representational acuity (*see* Siegler & Opfer, 2003; *see also Chapter VIII*). The more the β_1 value deviates from 1, the less accurate the estimate. A 4 (notation condition) by 3 (education group) mixed model analysis of variance was performed. Due to violations of sphericity, Greenhouse-Geisser corrections were applied to all factors with more than two levels (Keselman & Rogan, 1980).

In the Number to Position (NP) task, there was no main effect of notation condition [$F(2.017, 30.256) = .796$; $p = .461$; $\eta^2 = .05$], no main effect of group [$F(2,15) = 2.553$, $p = .111$, $\eta^2 = .25$], and no significant interaction [$F(4.034, 30.256) = .340$; $p = .851$; $\eta^2 = .043$]. Similarly, in the Position to Number (PN) task, there was no main effect of notation condition [$F(1.018, 14.252) = .225$; $p = .647$; $\eta^2 = .016$], no main effect of group [$F(2,14) = 0.893$, $p = .432$, $\eta^2 = .113$], and no interaction [$F(2.036, 14.252) = 1.22$; $p = .325$; $\eta^2 = .148$]. The three groups were therefore merged into one group for subsequent analyses.

4.4.2. Regression analyses

Regression analyses were performed on participants mean estimates⁷ plotted against the actual values of the target stimuli separately for each task. To identify the best fitting model, paired sample t-tests were applied on the residuals of the regression models of interest: linear and logarithmic for the NP task; linear and exponential for the PN task (Siegler & Opfer, 2003). In the NP task, the linear model was significantly the best model for all notation conditions: Fractions ($t(19) = -2.74, p < .05$); Integers ($t(19) = -4.6, p < .001$); Decimals ($t(19) = -5.55, p < .001$) and Money ($t(19) = -5.00, p < .001$) (**Figure 4.2A**). In the PN task, the linear model was the best fitting model for Integers ($t(19) = -4.24, p < .001$); Decimals ($t(19) = -2.61, p < .05$) and Money ($t(19) = -4.05, p < .01$), but not for Fractions ($t(19) = -.62, p = .54$) (**Figure 4.2B**).

⁷Individual estimates displayed identical results as the group mean (e.g. Moeller, Pixner, Kaufmann, & Nuerk, 2009).

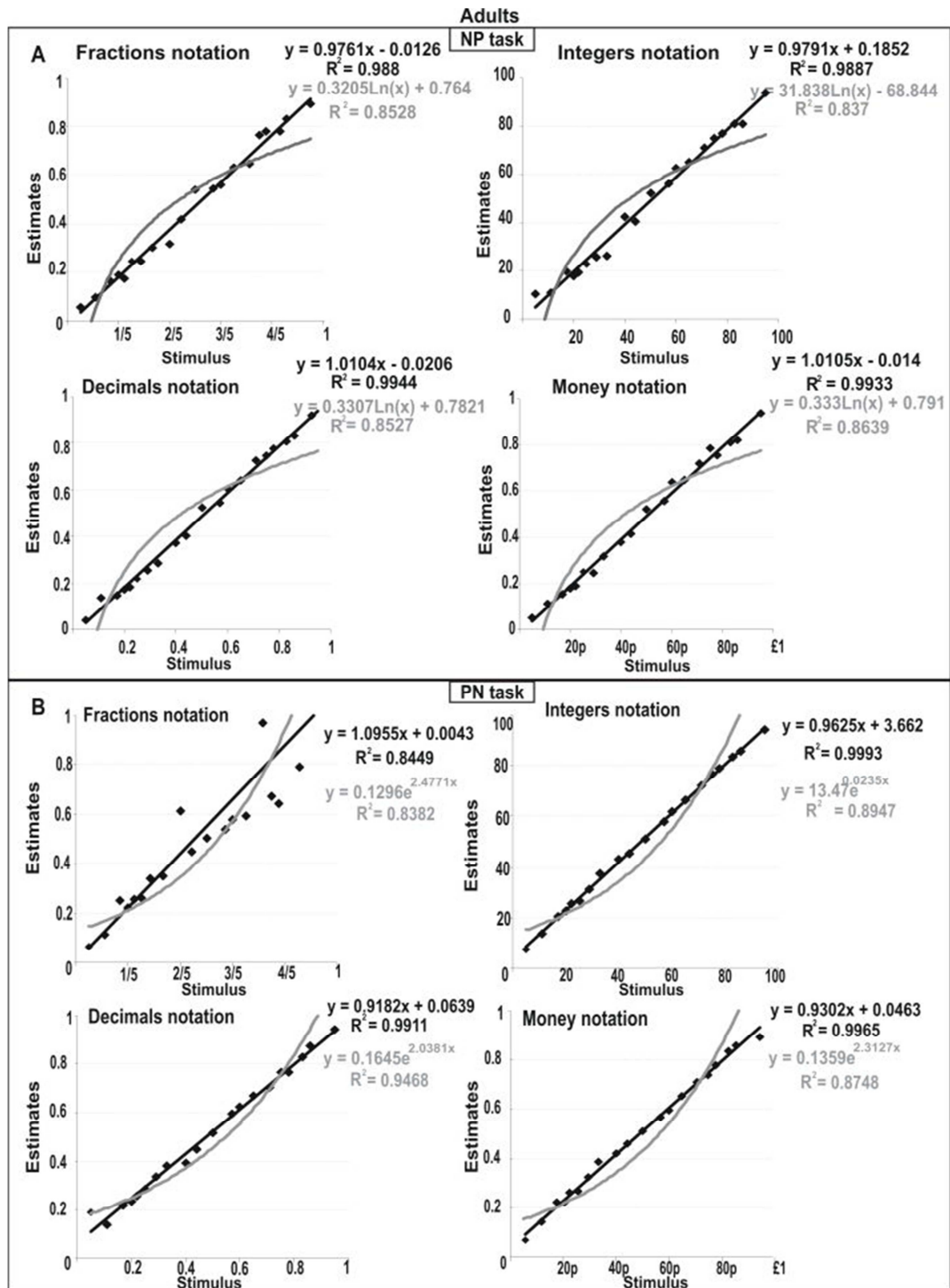


Figure 4.2. Average location of estimates regressed against value of the stimulus for each of the four notation conditions for both tasks. A) Number to Position (NP); B) Position to Number (PN). In black: equation and best fitting line of the linear model. In grey: A) equation and best fitting line of the logarithmic model; B) equation and best fitting line of the exponential model. A) Top left: in the fractions notation condition, all stimulus estimates fit well on the linear function $y = x$ with no distinction between familiar and unfamiliar fractions (e.g. $1/2$ vs $3/5$).

4.5. Interim Discussion 3A

Experiment 3A tested the possible models of number line representations – linear, logarithmic and exponential – in two number line estimation tasks which used four numerical notations: fractions, decimals, integers and money. Moreover, in order to better assess the mapping of fractional stimuli we used a variety of numerators and denominators. Previous research has found that fractions are difficult even for well-educated adults (Bonato et al., 2007). Yet, the study by Bonato and colleagues used a comparison task which might have easily biased subject towards the use of a componential strategy to solve the problem. Therefore, their data do not necessarily reveal the nature of the ‘internal’ representation of this type of numerical notation. Moreover, no studies have used the number line paradigm to investigate the representation of decimal stimuli in adults. Only a recent study by Cohen (2010) has assessed the nature of the representation of decimal stimuli in a population of adults using a series of number comparison tasks. By measuring the distance effect (Moyer & Landauer, 1967) and the physical similarity of numerals (Cohen, 2009), he found a fundamental difference in the representation of integers compared to decimals (Cohen, 2010). In particular, he claimed that the representation associated with a particular numerical notation is specific for that notation, rather than universal. Yet, these interpretations are still based on a magnitude comparison judgment which might not be reflecting the nature of the internal representation compared to the more explicit tasks used here.

In Experiment 3A we used a computerized version of the standard number-line tasks used by Siegler and Opfer (2003) -Number to Position (NP) and Position to Number (PN) - to investigate the internal representation of different types of double-digit numbers including rational numbers (fractions and decimals).

In the NP task, the best-fitting model was linear, and never logarithmic. Moreover, the data did not fit a two-step function (**Figure 4.2A**). This result demonstrates the existence of a common internal representation for magnitudes in its different means of expressions hinting at the idea of magnitude as an abstract entity (i.e. this remains a largely debated issue in the field of mathematical cognition, though it is beyond the scope of this thesis). Moreover, our results show for the first time that this

common representation (i.e. linear) encompasses rational numbers, suggesting that given the appropriate task - which directly examines the mapping of numbers to space - the representation of these types of notations is as good as the one of integers. Furthermore, our results challenge the idea that the difficulties with fractions and decimals are due to a fundamentally different internal representation of these types of numbers as previously proposed (Bonato et al., 2007).

However, the interpretation of the PN task was more complex. Nevertheless, the linear model was better than the other model under test (i.e. exponential) for all numerical notations except fractions, where there was still a trend but not a significant one (**Figure 4.2B**). The results of the PN task seem to suggest that the internal representation of fractions, which is shared with other types of double-digits numbers (**Figure 4.2A**), might be more difficult to access and perhaps could only be triggered by the appropriate task.

Thus, the results of Experiment 3A suggest that, with the appropriate task, adults will represent the real value of fractions and decimals in a linear manner (i.e. accurately). Therefore, in this particular population these two types of double-digit numbers do not seem to have a separate representation from double-digit integers, as previously suggested using magnitude comparison tasks (Bonato et al., 2007; Cohen, 2010). Thus, our experiment shows that double-digit numbers are linearly represented suggesting that, given the appropriate task, their processing is holistic rather than componential as previously proposed (Nuerk et al., 2001).

Finally, the tasks used in our study seem to offer a more direct way of tapping into the internal representation of the real value of two-digit numbers. Moreover, they demonstrate that although adults might prefer to rely on the comparison of the single components, the real value of rational numbers can be accessed (Meert, Grégoire, & Noël, 2009) as demonstrated by two recent neuro-imaging studies showing that specific populations of neurons in the parietal and frontal cortices are tuned to the real value of fractional (Ischebeck, Schocke, & Delazer, 2007) and proportional (Jacob & Nieder, 2009) stimuli.

Experiment 3B. Rational number representations in children – The mental number line tasks

4.6. Aim 3B

In Experiment 3A we demonstrated that well-educated adults display a linear representation of rational numbers. So when the task does not encourage a componential strategy, well-educated adults' performance on fraction and decimal stimuli is not affected by the 'whole number bias'. However, it is possible that the many years of education and experience with rational numbers (e.g. checking sales in a store, dealing with recipes, etc.) could have led to a comprehensive acquisition of these representations subsequently reducing the effects of the 'whole number bias'. On the other hand, some authors (Opfer & DeVries, 2008) have proposed that children could be facilitated in their representation and understanding of rational numbers because they have had less experience with integers and therefore their representation of the latter notation is not sufficiently established to affect the representation of rational numbers. Given the evidence for a developmental trajectory of integers' representation, which is reflected in its refinement from logarithmic – *approximate* – to linear – *exact* (see Siegler & Opfer, 2003), Opfer and DeVries have proposed that a logarithmic representation could in fact facilitate the understanding of rational numbers. On the other hand, according to a recent study, computational skills of rational numbers are highly predicted by language abilities (Seethaler, Fuchs, Star, & Bryant, 2011)⁸. This leads us to assume that children, who have less advanced language skills compared with adults, would show a less precise representation of rational numbers. Unfortunately, the literature is very sparse and inconsistent on the topic. In Experiment 3B we therefore aim to investigate the nature of the representation of rational numbers in 10-11 year olds. This age window was chosen because these children would have already been introduced to the concept of these types of numbers in school, yet their experience with these numbers would be significantly less than well-educated adults, allowing us to test the hypotheses outlined above. Moreover, we used the same number line tasks described

⁸Even though this is quite difficult to conceptualize as language is strongly linked with counting which presupposes equal size intervals. Evidently for the case of fractions and decimals this does not apply, as these types of numbers do not get counted.

in Experiment 1A (paragraph 4.3.2.1) in order to discourage a decomposition strategy. More importantly, we use fractions without common components, and we define the end points of the number line by 0 and 1 for fractions and decimals, rather than by fractional stimuli (0/3 and 3/3 as per Opfer & DeVries, 2008).

4.7. Method 3B

4.7.1. Participants

A total of nineteen normal-achieving children in their sixth year of schooling recruited from two different middle schools in London (7 males, mean age = 10.83 years, SD = 0.23) took part in Experiment 3B. All children were previously screened for maths learning disabilities (see **Chapter II**). Please note that the present group of nineteen children is a sub-sample of the NA group previously investigated (see **Chapter II**). Four children from the initial NA group (Experiment 1A) were no longer available at the time of testing for Experiment 3B. All remaining nineteen participants scored > 81 on all three tasks of the *Dyscalculia Screener*. Their demographics and scores on approximate, analogue and exact tasks are summarized in **Table 4.1**. The studies in Experiment 3B were approved by the UCL Ethics Committee.

NA (N = 19)			
Measure	Accuracy	RTs	Inverse Efficiency
	Mean (SD)	Mean (SD)	Mean (SD)
<u>Dyscalculia Screener</u>			
Simple RTs		426.08 (172.45)	
Dot Enumeration	90.20 (6.82)	2082.05 (734.05)	23.01 (7.85)
Number Comparison	92.22 (6.36)	695.67 (277.62)	7.56 (3.02)
Exact Addition	88.49 (5.44)	4177.25 (2448.7)	48.28 (30.08)
<u>Number tasks</u>			
Approximate Comparison	68.05 (13.9)	715.19 (252.49)	45.89 (18.18)
Approximate Addition	69.21 (16.8)	795.39 (470.53)	55.69 (50.06)
Approximate Subtraction	60.65 (12.3)	711.08 (330.21)	53.99 (35.83)
Discrete Numerosity	92.94 (5.68)	1015.97 (197.97)	22.93 (5.12)
Analogue Area	83.56 (12.1)	1307.30 (245.23)	33.05 (6.80)

Table 4.1. Dyscalculia Screener and Number tasks' scores for the 10-11 year olds. Accuracy indicates % of correct responses.

4.7.2. Experimental design

All experimental tasks were the same as the ones described in Experiment 3A (see **Figure 4.1**). Stimuli, task parameters and timing were also identical to those described in Experiment 3A (see **paragraph 4.3.2.1**).

4.8. Results 3B

4.8.1. Regression analyses

As in Experiment 3A, regression analyses were performed on participants mean estimates plotted against the actual values of the target stimuli separately for each task. To identify the best fitting model, paired sample t-tests were applied on the residuals of the regression models of interest: linear and logarithmic for the NP task; linear and exponential for the PN task (Siegler & Opfer, 2003). In the NP task, the linear model was significantly better than the logarithmic model for all notation conditions: Fractions ($t(19) = -3.6, p < .01$), Integers ($t(19) = -4.9, p < .001$), Decimals ($t(19) = -4.4, p < .001$) and Money ($t(19) = -4.67, p < .001$) – **Figure 4.3A**. In the PN task, the linear model was the best fitting model for all notation conditions: Integers ($t(19) = -4.24, p < .001$); Decimals ($t(19) = -2.65, p < .05$) and Money ($t(19) = -4.52, p < .001$), except Fractions ($t(19) = -1.24, p = .23$) – **Figure 4.3B**. Please note that even in this case (Experiment 3B), the individual estimates displayed identical results to the group mean.

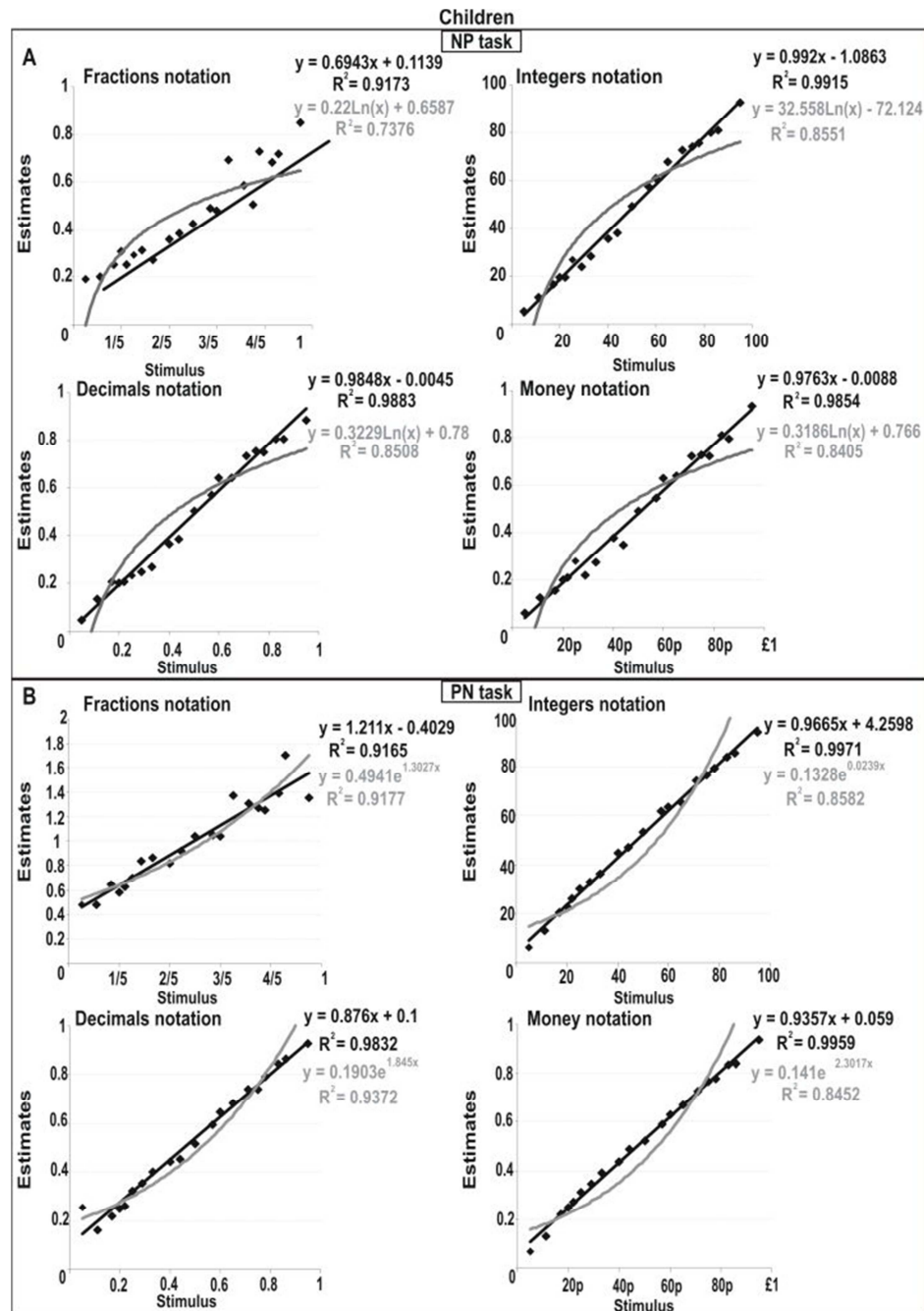


Figure 4.3. Average location of estimates regressed against value of the stimulus for each of the four notation conditions for both tasks. A) Number to Position (NP); B) Position to Number (PN). In black: equation and best fitting line of the linear model. In grey: A) equation and best fitting line of the logarithmic model; B) equation and best fitting line of the exponential model. A) Top left: in the fractions notation condition, all stimulus estimates fit well on the linear function $y = x$ with no distinction between familiar and unfamiliar fractions (e.g. $1/2$ vs $3/5$).

4.9. Interim Discussion 3B

In Experiment 3B we investigated how well children understood the real value of four types of number notations: integers, fractions, decimals and money. It has been

proposed that over the course of development, individuals start with a logarithmic mental representation of integers and progress to a linear representation (Siegler & Opfer, 2003), though these studies have only involved the natural numbers from 0 or 1 upwards. Here we asked whether this transition applies to new ranges and types of numbers. It has been frequently observed that fractions and decimals seem particularly difficult for children to understand (Mack, 1995; Ni & Zhou, 2005; Smith et al., 2005). Therefore it may be that although children show linear representations for integers they may still represent the value of decimals and fractions in a non-linear way. Indeed, it has been shown that 8 year olds displaying a logarithmic representation for integers are facilitated in estimation tasks using fractions with a fixed numerator or denominator (Opfer & DeVries, 2008). No studies thus far have investigated the development of fraction and decimal representations in children who display a linear representation for integers and were formally introduced to the concept of fractions in school.

The results of Experiment 3B show that in the NP task, children displayed a linear representation for all number notations, including fractions and decimals (**Figure 4.3A**), suggesting that if there is a developmental trend from a logarithmic to a linear representation of numbers as has been proposed for integers (Siegler & Opfer, 2003), it has already taken place for all these notations by the age of 10 to 11 years. Yet, when asked to estimate the value of a hatch-mark on a given number line (the PN task), participants' estimates were linearly mapped for integers, decimals and money notations, but not for fractions (**Figure 4.3B**).

As for the adults' population, the results of this experiment seem to suggest that, given the appropriate task, children as young as 10 can accurately represent the real value of these types of numerical notations. Accessing this linear representation could then result in greater difficulties and cognitive demands depending on the task (i.e. the *Number to Position* versus the *Position to Number* task).

Finally, the results of Experiment 3B seem to suggest that there is an internal linear representation for all these types of numbers including rational numbers, even in children. Moreover, our results seem to suggest that this representation has nothing or little to do with schooling or everyday exposure to these types of 'paradoxical' notations. Moreover, our data does not speak in favor of the hypothesis that children

who had less exposure with rational numbers show facilitation with these types of notations because of a less anchored integers' representation (Opfer & DeVries, 2008).

Experiment 3C. Rational number processing across development

4.10. Aim 3C

In Experiments 3A&B we presented the evidence of a linear representation for integers, but also for rational numbers in adults and 10 year old children. However, the results of these two experiments only partially address the question of whether 10 to 11 year old children, who have just been introduced to the concept of fractions in their school curricula, would experience more difficulties in the comprehension of fractions and decimals' real values than educated adults. Moreover, the results of the previous experiments only address the issue of the representation of these types of number notations. Thus, the question of how these notations are processed remains unsolved. Furthermore, even if we demonstrated an accurate representation for rational numbers in adults and children which suggests that experience has little or nothing to do with the development of such representation we do not know whether greater exposure to these types of numbers would affect their processing in terms of accuracy and speed of responses.

In Experiment 3C we therefore aimed to investigate differences in performance between adults and children in the processing of four different notation conditions in terms of both accuracy and speed of responses.

Second, we wanted to determine if a number line task revealed a greater level of difficulty, either in terms of accuracy or in terms of speed, in processing fractions and decimals when compared to numerically and spatially equivalent stimuli.

4.11. Method 3C

4.11.1. Participants

The same two groups of participants described in Experiments 3A and B (see paragraphs 4.3.1 and 4.7.1) took part in Experiment 3C.

4.11.2. Experimental design

All experimental tasks were the same as the ones described in Experiment 3A (see **Figure 4.1**). Stimuli, task parameters and timing were also identical to those described in Experiment 3A (see **paragraph 4.3.2.1**).

4.12. Results 3C

4.12.1. Accuracy analyses

As an index of accuracy, a deviant was defined as the absolute value of the subtraction between the participants' response and the target value (in pixels on the line). A 4 (notation condition) x 2 (age-group) mixed model ANOVA was performed. Pair-wise comparison analyses were adjusted for multiple comparisons (Bonferroni corrected). Due to violations of sphericity, Greenhouse-Geisser corrections were applied to all factors with more than two levels (Keselman & Rogan, 1980).

In the NP task, the analyses revealed a main effect of notation condition [$F(1.628,61.86) = 89.113, p < .001, \eta^2 = .701$]: the Fractions notation was significantly less accurate than the other notations ($p < .001$). There was also a main effect of group [$F(1,38) = 57.194; p < .001, \eta^2 = .601$], and a significant interaction [$F(1.628,61.86) = 30.81; p < .001, \eta^2 = .448$]: the children group displayed lower accuracy than the adults group in the Fractions and Decimals notations ($p < .005$) (**Figure 4.4A**).

In the PN task, both main effects were also significant in the same direction as in the NP task: notation condition [$F(1.065,40.47) = 119.57, p < .001, \eta^2 = .759$], and group [$F(1,38) = 89.682, p < .001, \eta^2 = .702$] (**Figure 4.4B**). The interaction was also significant [$F(1.065,40.47) = 56.734; p < .001, \eta^2 = .599$]: the children group was significantly less accurate than the adults group in all notation conditions, except Decimals ($p = .117$) – (**Figure 4.4B**).

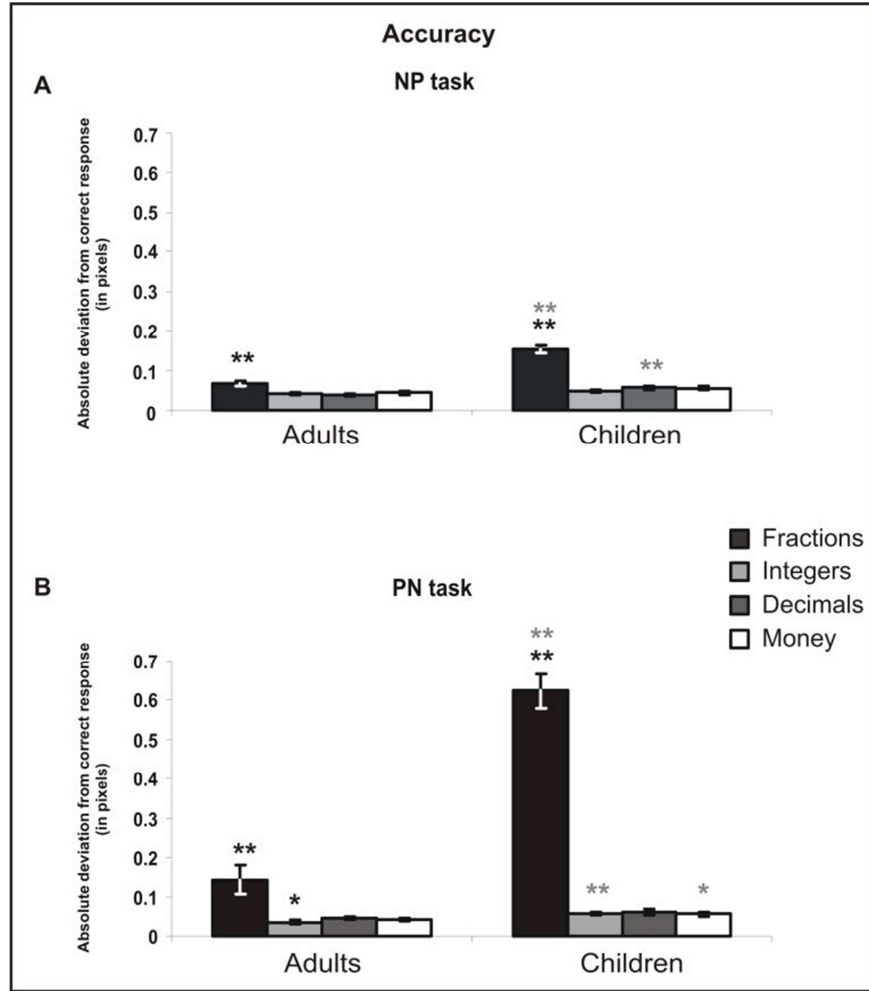


Figure 4.4. Accuracy in terms of the absolute deviation from correct responses (in pixels) plotted against age group for both tasks. A) Number to Position (NP); B) Position to Number (PN). Black asterisks indicate significant differences between notation conditions * $p < .05$, ** $p < .005$. Grey asterisk indicate significant differences between groups * $p < .05$, ** $p < .005$. Error bars indicate 1 standard error of the mean.

4.12.2. Latency analyses

These analyses were performed on median RTs. As for the analyses of accuracy, we performed a 4 x 2 mixed model ANOVA with notation condition as the within subjects factor (four levels) and group as the between subjects factor (two levels).

In the NP task, both main effects were significant: notation condition [$F(2.036, 77.36) = 60.813$, $p < .001$, $\eta^2 = .615$] – the Fractions notation was significantly slower than the other notations ($p < .001$) – and group [$F(1, 38) = 8.011$, $p < .01$, $\eta^2 = .174$]. The interaction was also significant [$F(2.036, 77.36) = 10.291$, $p < .001$, $\eta^2 = .174$].

.213]: the two groups differed in their speed of responses in all notation conditions, except Fractions ($p = .146$) (**Figure 4.5A**).

In the PN task, both main effects were also significant: notation condition [$F(1.79,68.01) = 145.35$, $p < .001$, $\eta^2 = .793$]– the Fractions notation was performed significantly slower than the other notation conditions ($p < .001$) – and group [$F(1,38) = 45.297$, $p < .001$, $\eta^2 = .544$]; while the interaction was not significant [$F(1.79,68.01) = 2.881$, $p = .069$, $\eta^2 = .07$] (**Figure 4.5B**).

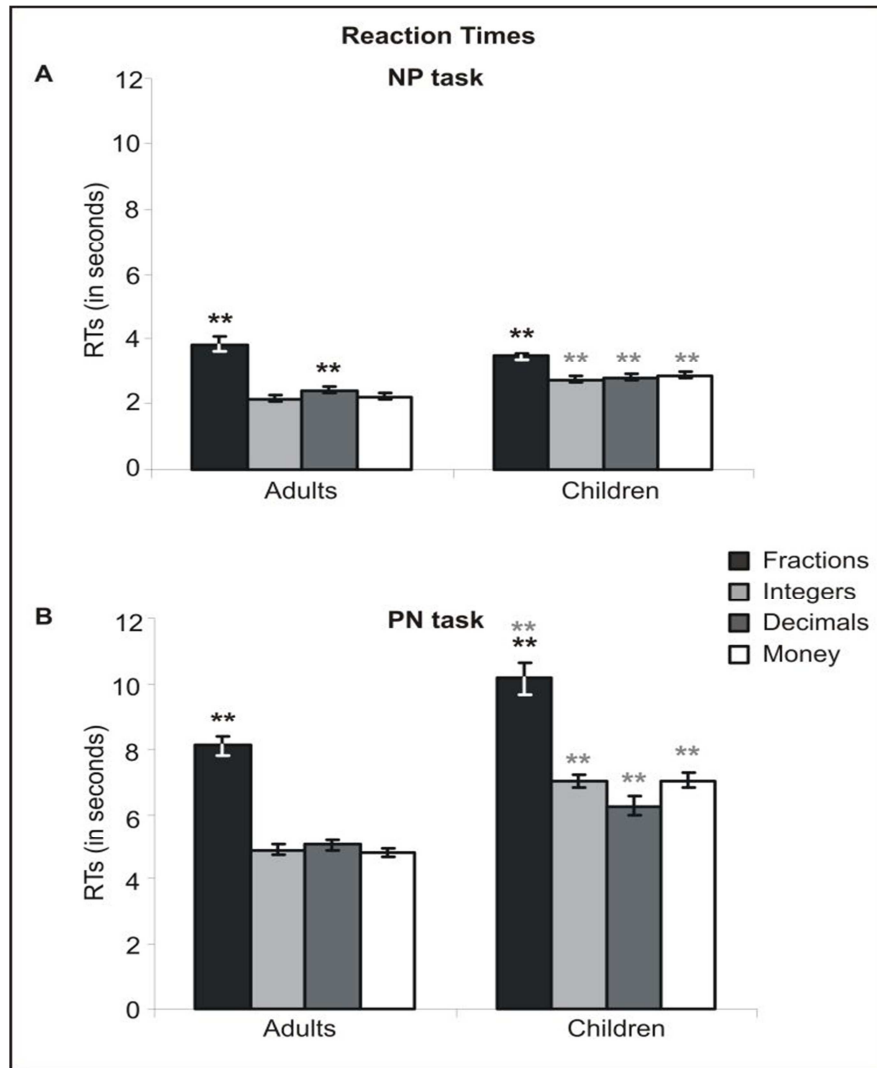


Figure 4.5. Reaction times (seconds) plotted against age group for both tasks. A) Number to Position (NP); B) Position to Number (PN). Black asterisks indicate significant differences between notation conditions * $p < .05$, ** $p < .005$. Grey asterisk indicate significant differences between groups * $p < .05$, ** $p < .005$. Error bars indicate 1 standard error of the mean.

4.13. Interim Discussion 3C

In Experiment 3C we investigated whether rational numbers show a lower degree of proficiency (in terms of accuracy and reaction times) compared to integers. Moreover, we wished to see if children displayed worse performance than adults in processing these types of notations. This allowed us to test for the role of experience and exposure to rational numbers in terms of their development.

As we have seen in Experiment 3B (see **paragraph 4.6**), one hypothesis is that experience ease the processing of rational numbers, while the other is that the more developed the system for integers, the worse the mastering and processing of rational numbers (Ofper & DeVries, 2008).

The data of Experiment 3C revealed an interesting, yet different pattern of results in terms of the two dependent variables of interest: accuracy and latencies.

In the NP task we showed that the mapping of fractional stimuli led to significantly more errors than the other notations in both groups (**Figure 4.4A**). This result could be interpreted either purely in favor of the ‘whole number bias’ hypothesis (Bonato et al., 2007) or, as we would like to propose, in terms of a greater difficulty in accessing the accurate (and holistic) representation of rational numbers quite possibly due to a failure to inhibit the information on whole numbers. Moreover, the accuracy results of the NP task seem to suggest that, at least for the mastering⁹ of fractions, and decimals, the role of experience could be to important.

The picture seems to be slightly different when we look at speed of responses in the NP task. Particularly, the processing of fractional stimuli took twice as long compared to integers, money and decimals (**Figure 4.4B**). Furthermore, the group effect evident in the accuracy data (i.e. children being worse at accurately mapping fractions compared to adults) disappears when looking at reaction times. This result seems to suggest that when examining the processing of fractions (in terms of speed of responses) the difficulties in accessing the holistic representation of these stimuli are equally prominent in the two age groups. On the other hand, decimal stimuli seem to be accessed slower than integers and money stimuli only in the group of well-educated

⁹We deliberately suggest that accuracy is a measure of the mastering of numbers, while reaction times are a measure of processing numbers.

adults, which in turn could reflect a better processing of the latter notations in this group.

The situation in the PN task was more complex, perhaps because the task itself was more demanding as can be seen in both the accuracy measures (**Figure 4.4B**) and in the reaction times which were around twice as long (**Figure 4.5B**). However, it is quite possible that some of the additional time could have been the consequence of the purely motor action of typing the chosen numbers into the response boxes. Moreover, this could explain children being systematically slower than adults in all notation conditions. Yet, the pattern of results on accuracy is comparable to the one of the NP task, at least for the fractions notation (**Figure 4.4.B**), while performance on decimal stimuli was accurate in both groups.

All in all the results of Experiment 3C seem to suggest that, at least for the NP task, fractions, and to some extent also decimals, are harder to master compared to whole numbers. Moreover, our data led us to propose that experience might play a role in the mastering of rational numbers (both fractions and decimals), but not in their processing times.

4.14. Final Discussion

In this Chapter we investigated how children and adults understand and represent the real value of four different types of multi-digit number notations: fractions, integers, decimals and money. It has been proposed that children are characterized by heterogeneous mental representations of numbers (Berteletti, Lucangeli, Dehaene, Piazza, & Zorzi, 2010), and that the ‘representational starting kit’ is characterized by a logarithmic mental representation of integers that progresses to a linear representation over the course of development (Siegler & Opfer, 2003). Since previous investigations have only used natural numbers from 0 or 1 upwards, we asked whether this transition applies to alternative ranges and types of numbers. Our experiments used a computerized version of the standard number-line tasks used by Siegler and Opfer (2003). This enabled a more precise measure of the accuracy of response, as well as, the possibility to measure the time taken to respond (reaction time).

Experiments 3A&B tested the possible models of number line representations – linear, logarithmic and exponential - in two number line estimation tasks which used a variety of numerators and denominators. For the NP task, the best-fitting model for both adults and children was linear, and never logarithmic for any notation including fractions. Thus, even if a developmental transition exists from a compressed (i.e. logarithmic) to a linear representation of numerical values, it is already implemented by the age of 10 for all number notations.

Additionally, we investigated the acuity of numerical estimation by measuring actual deviations from correct responses within and between groups. In the NP task we showed high accuracy in all notations (deviation from correct responses range between 0.039 and 0.15 pixel counts). However, the mapping of fractions led to significantly more errors than the other notations, even for adults, whilst accuracy for decimals was nearly the same as for the integers and money notations (**Figure 4.4A**). As would be expected, children displayed significantly lower accuracy with rational numbers compared with adults, whilst no significant differences were found between the groups for integers and money (**Figure 4.4A**). This result speaks in favor of the role of experience for the development of a good (or at least better) mastering of these types of notations.

Latency analyses showed that the processing of fractional stimuli took twice as long compared to the other formats (**Figure 4.4B**) with no difference for processing times between adults and children. Yet, children were significantly slower than adults with integers, decimals and money stimuli.

The PN task was considerably more demanding, as shown by both accuracy (**Figure 4.5A**) and reaction times that were approximately twice as long (**Figure 4.5B**). Some of the additional time could have been the result of choosing the numbers to type in, or the action of typing into two separate cells (fractions condition). Moreover, this could also explain the main effect of group (i.e. children were significantly slower than adults in all numerical notations). Nevertheless, the linear model was better than the logarithmic model for both adults and children for integers, money and decimals, but not fractions.

These results shed new light on the understanding the real value of rational numbers. It has been previously observed in number comparison tasks that when processing fractions (Bonato et al., 2007; Ni & Zhou, 2005) and decimals (Rittle-Johnson et al., 2001) both adults and children are subject to a ‘whole number bias’ (Ni & Zhou, 2005) where they treat fractions in a componential manner. By using number line tasks and a stimulus design that does not encourage componential strategies, we show that, at least when they are given a numerical stimulus to map onto a defined physical space, both adults and children can access and correctly represent (i.e. linearly) the real value of fractions and decimals and process them in a holistic way. However, when a spatial cue has to be translated into a numerical value, fractional but not decimal stimuli require greater effort and the mapping is not linear for either of the groups. This is compatible with the idea that when allowed or encouraged by the task, the use of componential strategies might be preferred to the access of the numerical value of the fraction (Kallay & Tzelgov, 2009).

Finally, it has been proposed that what is meant to drive the change from a logarithmic to a linear representation¹⁰ of integers is counting with equal size intervals (Carey, 2004; Le Corre & Carey, 2007). However, in the case of fractions and decimals this could not be applied as we do not count rational numbers. The data in this Chapter suggest instead that the representation could be intrinsically linear (Zorzi & Butterworth, 1999; Zorzi, Stoianov, & Umiltá, 2005), supporting the proposal of the exact nature of the ‘number sense’ (see **Chapter II**). Moreover, this internal representation seems to be shared with rational numbers, as it has been suggested by neuroimaging studies (Ischebeck et al., 2007; Jacob & Nieder, 2009). Essentially, the data from this Chapter could be taken as indirect evidence that language does not necessarily play a role in the ‘linearization’ process of number representation as previously proposed (Carey, 2004).

In conclusion, the timed number line tasks used here offer a precise method for assessing number understanding. They demonstrate that fractions, but not decimals are more difficult to mentally manipulate than other number notations, as reflected by the accuracy and reaction time data. Yet, we show for the first time that, when asked to

¹⁰Or more generally from an approximate, compressed representation to an exact one.

place a numerical value on the number line, a linear representation is evident already in children by the age of 10 for both fractional and decimal stimuli. The same is not entirely true for the position to number task, where fractional stimuli do not yield linear responses and never seem to make the transition from an exponential to a linear representation even with experience.

Finally, the current design of these tasks offer a novel and simple approach for assessing another aspect of number understanding in children and adults who are suspected of suffering from arithmetical learning disabilities, such as Developmental Dyscalculia and Low Numeracy.

4.15. Summary

The results presented in this Chapter shed new light on the understanding of the real value of rational numbers. It has been found that children and even adults have great difficulty understanding fractions and decimals (e.g. Bonato et al., 2007; Mack, 1995; Ni & Zhou, 2005; Rittle-Johnson et al., 2001; Smith et al., 2005). Particularly, it has been observed that both adults and children are subject to a “whole number bias” (Ni & Zhou, 2005) where they treat fractions in a componential manner (Bonato et al., 2007). The tasks used in these studies required participants to compare two numbers (either fractions or decimals), which was found to be particularly difficult even for 11 year olds. Our results with number line tasks suggest, by contrast, that adults and children do understand the real value of fractions and decimals if the task affords tapping directly into the representation rather than requiring a comparison.

V

Double-digit number representations in children with low numeracy

“Perfection is attained by slow degrees; it requires the hand of time”

--Voltaire

The representation and processing of double-digit numbers has been reported to be less precise in children with mathematical disabilities. Specifically, children with this condition seem to show a logarithmic representation for integers, which according to some theories could be one of the reasons for their difficulties with numbers and later arithmetic. On the other hand, it has been proposed that a ‘deficient’ representation of integers could become beneficial when dealing with other types of number notations such as fractions and decimals. Here we wished to test these competing proposals by exploring the understanding of double-digit numbers in children with Low Numeracy. We used four types of number notations which included integers, fractions, decimals, and money in two number line tasks: *Number to Position* and *Position to Number*. The results of the *Number to Position* task show that the linear model was the best to account for the data in all number notations, including fractions and decimals, suggesting that the mapping of double-digit numbers in space is ‘intact’ even in children with maths difficulties. Yet, in the *Position to Number* task a linear representation was evident only for integers and decimals, but not fractions. Finally, children with low numeracy performed more accurately, yet slower than their typically developing peers in all notations suggesting an intact, yet harder to access representation for double-digit numbers.

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5.1. Introduction

In the previous Chapter we have seen how, despite the often reported difficulties in manipulating rational numbers, adults and typically developing children show a comprehensive understanding of these types of number notations. Yet, a question that arises is how these types of double digit numbers are dealt with by populations who have numerical difficulties such as Developmental Dyscalculia or Low Numeracy. It is important to assess the numerical representation of double-digit numbers (including rational numbers), because it has been demonstrated that children's learning of a linear, mathematical number line correlates with their mathematical achievement (Booth & Siegler, 2006). As we mention in **Chapter IV**, the number line tasks developed in this thesis could serve as a valuable tool to assess numerical representations of different types of numerical notations in atypical populations. Notably, it has recently been shown that children experiencing mathematical learning disabilities tend to be less accurate in their number line representations (Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Geary, Hoard, Nugent, & Byrd-Craven, 2008). These results suggest that these children might have not yet acquired a linear model of numerical representation resulting in less precise performance while solving these tasks. It follows that the processing of proportional numerical representation (fractions and decimals) which is thought to gain precision later in life (see **Chapter IV**), will most likely have not developed into a learned linear function. However, a recent study has shown that 8 year olds displaying a logarithmic (i.e. less precise) representation for integers are facilitated in estimation tasks using fractional stimuli with a fixed numerator or denominator, but not decimals stimuli (Opfer & DeVries, 2008). This might suggest that there is a cost in refining our numerical representation of integers (Thompson & Opfer, 2008), or that an inaccurate representation of numbers (i.e. integers), could actually become beneficial when asked to process other types of (paradoxical) notations such as fractions and decimals. Yet, a recent study has claimed that difficulties in naming decimals are an indicator of mathematical learning disabilities, and that children with low numeracy (i.e. defined by the authors as having a milder impairment in maths) failed to rank notations such as fractions and decimals (Mazzocco & Devlin, 2008).

Thus far, the literature has been controversial on the issue of the representation of double-digit numbers in atypical development. It has been proposed that a less precise (i.e. logarithmic) representation characterizes children with mathematical disabilities and, to a lesser extent, children with low numeracy (Geary et al., 2007, 2008; Opfer & DeVries, 2008). Yet, no studies thus far have investigated the development of fractional and decimal representations in children who display a linear representation for integers and were formally introduced to the concept of fractions in school. Moreover, no studies have used number-line mapping tasks of fractional stimuli with an unfixed numerator or denominator.

In this Chapter we assess the understanding of double-digit numbers, including notations such as fractions and decimals in a population with low numerical skills in order to address the question of whether an intact (i.e. linear) representation of double-digit integers is in place in 10 year olds with low numeracy. Moreover, we want to ask whether, given a linear representation of integers, these children will experience more difficulties than typically developing children in representing and processing rational numbers.

Experiment 4A. Double-digit number representations in children with low numeracy– The mental number line tasks

5.2. Aim 4A

Although few developmental studies have assessed the representation of single and double-digit numbers in children with mathematical disabilities (Geary et al., 2007, 2008; *see also* Opfer & DeVries, 2008) there is still no definite answer on the nature of this representation. Moreover, these studies often use different terminology to describe the nature of the impairment (i.e. mathematical learning disabilities (MLD), children with low numeracy (LN)¹¹) making the interpretation and most importantly the generalization of the results very difficult. Few studies have claimed that given a less precise (i.e. logarithmic) representation of double-digit integers, the representation of

¹¹Children with Developmental Dyscalculia have not been included in these types of studies.

other types of notations such as fractions and decimals could become easier (Opfer & DeVries, 2008). However, the latter study has addressed this issue through number line tasks that use fixed numerator and denominator for fractional stimuli, thus biasing the responses. On the contrary, other studies suggest that an inability to deal with fractions and decimals could be a diagnostic marker of mathematical disabilities and low numeracy (Mazzocco & Devlin, 2008). Yet, this study only collected nominal and ordinal data which does not allow any assumptions on the representation (or cognitive processing) of these types of stimuli. Moreover, these authors did not draw a sharp distinction between their experimental populations as they used an arbitrary cut-off criteria (which has been a typical procedure – *see also* Geary et al., 2008), rather than a theoretically-based criteria. In other words, it is unclear from their study what types of maths impairments these children experienced. Here we take a different approach by studying a population of children with low numeracy (LN) whose performance is in the bottom 7% of the population¹², but with a specific deficit in arithmetic (see **Chapter II**). The aim of our study was to assess the nature of the numerical representation of double-digit numbers in LN. First, we wished to see whether these children present a precise (i.e. linear) representation for double-digit integers. Second, we wanted to test their representation of fractions and decimals to assess whether it differs from their representation of integers. Moreover, given that our stimulus set uses unfixed numerator and denominator, we expect that if the aforementioned representations overlap, they should be either linear or logarithmic for all notation conditions. On the contrary, if it is true that the representation of fractions and decimals is different from the representation of integers, we would expect different models (i.e. linear or logarithmic) to be predictive of the data, suggesting either facilitation or a cost in one direction or the other. In order to test these hypotheses we use the same explicit number line tasks described in **Chapter IV** (see **paragraph 4.3.2**).

¹²This roughly corresponds to the prevalence criteria for mathematical disabilities in Geary et al., 2008 and Mazzocco & Devlin, 2008.

5.3. Method 4A

5.3.1. Participants

A total of ten children with low numeracy took part in Experiment 4A. Criteria for inclusion were based on their scores on the *Dyscalculia Screener* (see **Chapter II, paragraph 2.7.1**). As mentioned earlier, the LN group was defined as the bottom 7% of the population on the ‘Achievement test’ of the *Dyscalculia Screener* (i.e. test of exact addition). Additionally, they had to score within the average range on the ‘Capacity tests’ of the *Dyscalculia Screener* and have an IQ within the normal range. It is important to note that this group was the same as the one described in **Chapter III** (see **paragraph 3.3.1**). Demographics and experimental scores on numerical tests are reported below (**Table 5.1**) to facilitate visualization without going back to previous Chapters. This study was approved by the UCL Ethics Committee.

	LN (N = 10)		
Measure	Accuracy	RTs	Inverse Efficiency
	Mean (SD)	Mean (SD)	Mean (SD)
<u>Dyscalculia Screener</u>			
Simple RTs		453.25 (211.9)	
Dot Enumeration	85.38 (4.33)	2278.05 (803.4)	26.68 (9.73)
Number Comparison	89.67 (4.83)	1092.35 (366.5)	12.27 (4.09)
Exact Addition	58.57 (9.09)	3783.4 (2099.2)	65.25 (33.14)
<u>Number tasks</u>			
Approximate Comparison	60.0 (13.06)	703.0 (355.23)	52.13 (29.3)
Approximate Addition	76.67(8.38)	784.5(532.8)	45.17 (34.3)
Approximate Subtraction	52.92 (8.57)	824.1 (521.67)	67.39 (46.76)
Discrete Numerosity	84.79 (21.3)	1175.9 (496.04)	41.39 (56.18)
Analogue Area	76.25 (13.3)	1263.8 (399.26)	36.4 (15.8)

Table 5.1. Dyscalculia Screener and Number tasks’ scores for the LN. Accuracy indicates % of correct responses.

5.3.2. Experimental design

All experimental tasks were the same as the ones described in **Chapter IV** (see **Figure 4.1**). Stimuli, task parameters and timing were also identical to those described in **Chapter IV**, Experiment 3A, B, & C (see **paragraph 4.3.2.1**).

5.4. Results 4A

5.4.1. Regression analyses

Regression analyses were performed on participants mean estimates¹³ plotted against the actual values of the target stimuli separately for each task. To identify the best fitting model, paired sample t-tests were applied on the residuals of the regression models of interest: linear and logarithmic for the NP task; linear and exponential for the PN task (Siegler & Opfer, 2003).

In the NP task, the linear model was significantly better than the logarithmic model in all four notation conditions: Integers ($t(19) = -4.255$, $p < .001$); Fractions ($t(19) = -2.311$, $p < .05$); Decimals ($t(19) = -4.572$, $p < .001$), and Money ($t(19) = -4.239$, $p < .001$) (**Figure 5.1A**).

In the PN task, the linear model was significantly better than the logarithmic model in the Integers notation condition ($t(19) = -5.213$, $p < .001$) and in the Decimals notation condition ($t(19) = -4.983$, $p < .001$); but not in the Fractions ($t(19) = -1.123$, $p = .275$) and Money notation conditions ($t(19) = -1.888$, $p = .074$) (**Figure 5.1B**).

¹³Individual estimates displayed identical results as the group mean (e.g. Moeller, Pixner, Kaufmann, & Nuerk, 2009).

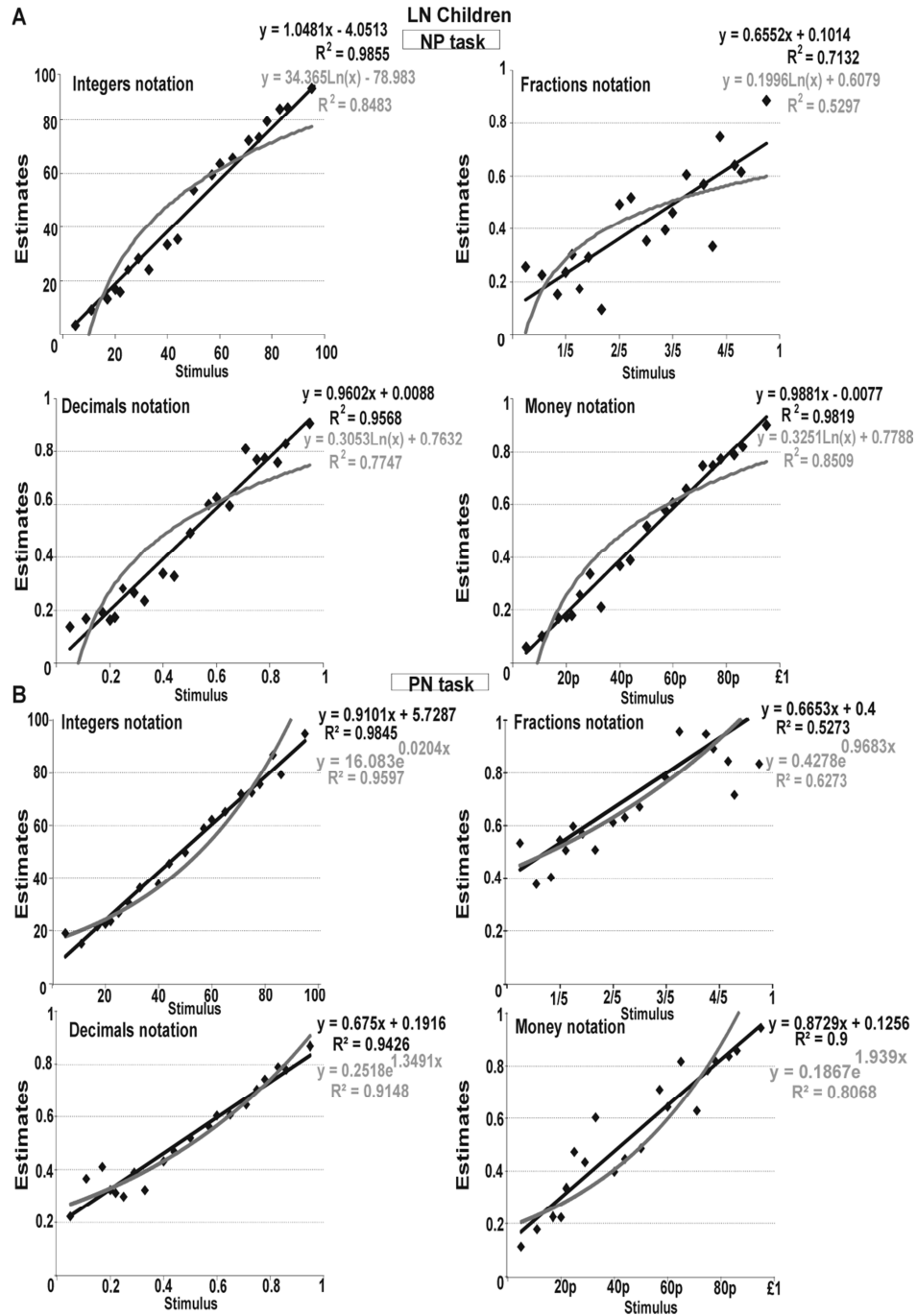


Figure 5.1. Average location of estimates regressed against value of the stimulus for each of the four notation conditions for both tasks. **A)** Number to Position (NP); **B)** Position to number (PN). In black: equation and best fitting line of the linear model. In grey: **A)** equation and best fitting line of the logarithmic model; **B)** equation and best fitting line of the exponential model. **A)** Top left: in the fractions notation condition, all stimulus estimates fit well on the linear function $y = x$ with no distinction between familiar and unfamiliar fractions (e.g. $1/2$ vs $3/5$).

5.4.2. Correlational analyses

In order to directly test the hypothesis proposed by Mazzocco and Devlin (2008), we performed binary correlation analyses on the three dependent variables of the *Addition task* from the *Dyscalculia Screener*- accuracy, RTs and inverse efficiency - (see **Chapter II** and **Table 5.1**) and the slopes (β_1 value) of the regression equation $Y = \beta_0 + \beta_1 x$ of the linear model for each subject. The slope of the regression equation is often taken as an index of representational acuity (see Siegler & Opfer, 2003; see also **Chapter VIII**). The more the β_1 value deviates from 1, the less accurate the estimate.

In the NP task, the results indicated a positive correlation between the accuracy variable of the *Addition task* and the indices of representational acuity (see above) of the Fractions, Decimals and Integers notation conditions (see **Table 5.2**). All other correlations were not significant.

	Fractions notation	Decimals notation	Integers notation
Accuracy Addition task			
Pearson r	.697	.708	.861
p-value	.025	.022	.001

Table 5.2. Significant correlation coefficients (and relative p-values) for the Addition task and the β_1 values of the Fractions, Decimals and Integers notation conditions.

5.5. Interim Discussion 4A

Experiment 4A tested the possible models of number line representations (i.e. linear, logarithmic and exponential) of double-digit numbers in a population of children with low numeracy (LN). We used two number line estimation tasks with different numerical notations: fractions, decimals, integers and money (see **Chapter IV, paragraph 4.3.2.1**). Moreover, in order to better assess the mapping of fractional stimuli we used a variety of numerators and denominators. Previous research has suggested that children might be facilitated in their processing of rational numbers because of a poorly anchored representation of whole numbers (i.e. integers) (Opfer & DeVries, 2008). Hence, children experiencing maths difficulties make an ideal candidate population in order to test this hypothesis. In this account, the potentially less automatic processing of integers would result in less interference when processing other types of more ‘paradoxical’ notations (i.e. children with maths difficulties might be less

exposed to the ‘whole number biased’). Yet, these studies have not made a clear prediction on the intrinsic nature of the representation (rather than the processing) of rational number stimuli. On the other hand, few studies have proposed that a poor understanding of rational numbers is highly correlated with mathematical disabilities (Mazzocco & Devlin, 2008). In Experiment 4A we investigated the nature of the representation of double-digit numerical stimuli in a population with a selective impairment in arithmetic. Moreover, we aimed to test the hypothesis that there is a strong link between mathematical disabilities and the representational acuity of double-digit number representations.

In the NP task, regression analyses revealed that the linear model was the best to account for the data in all notation conditions, including fractions and decimals (**Figure 5.1A**). This suggests that even if this group experiences difficulties in performing arithmetic, the mapping of all types of double-digit positive numbers into space (i.e. number line representation) is accurate and ‘intact’.

In the PN task, the LN group showed a linear representation for integers and decimals, but not for fraction and money stimuli (**Figure 5.1B**). Altogether these results are comparable to the ones in **Chapter IV** on the representations of double digit numbers in adults and typically developing 10 year olds. Moreover, the interpretation of the results from Experiment 4A is in accordance with the account proposed in **Chapter IV**: the internal representation of fractions, which is shared with other types of double-digit number notations (**Figure 5.1A**) might be more difficult to access given certain tasks (e.g estimating the rational value of a hatch mark on the line). Furthermore, the results of Experiment 4A strengthen the notion that a common representation (i.e. linear) encompasses rational numbers even in children with low numeracy.

The results of our correlational analyses support the idea of a tight link between the representation of rational numbers and good arithmetic performance (**Table 5.2**). Moreover, our results extend the findings reported in Mazzocco & Devlin (2008) who only used tasks that tapped into the nominal and ordinal understanding of these types of notations. In addition, our findings suggest that the better the internal representation of double-digit numbers, the better the ability to perform exact calculation.

Finally, the results of Experiment 4A extend our previous findings on the internal representation of double-digit numbers (including fractions and decimals) in well-educated adults and typically developing 10 year olds to a population with low numeracy. We therefore propose that given the appropriate task – which directly tests the mapping of numbers to space – the representation of these types of notation is as good as that of integers, even in children with low numeracy.

Experiment 4B. Double-digit number processing in typical and atypical development

5.6. Aim 4B

In Experiment 4A we demonstrated that similar to well-educated adults and typically developing children, children with low numeracy display a linear representation of double-digit numbers (including rational numbers) when given the appropriate task (i.e. mapping of numbers onto physical lines). Yet, the results of Experiment 4A only partially address the question of whether children with low numeracy might or might not have more difficulties with these types of numbers. Moreover, they leave open the question on the mastering and processing of double-digit numbers in this population.

In Experiment 4B we therefore aimed to investigate differences in performance between typically and atypically developing children in processing the numerically identical notation conditions (fractions, decimals, integers and money) in terms of both accuracy and reaction times. Second, we wanted to ask the question of whether number line tasks could reveal a greater level of difficulty in processing rational numbers compared to integers.

5.7. Method 4B

5.7.1. Participants

The low numeracy group (N = 10) was the same as that described in Experiment 4A. The control group (N = 19) consisted of the same normal-achieving children described previously in Experiment 3B (see **Chapter IV**) (7 males, mean age = 10.83 years, SD = 0.23). The demographics and numerical test scores of both groups are displayed in **Table 5.3** to facilitate visualization without going back to previous Chapters.

The studies in Experiment 4B were approved by the UCL Ethics Committee.

Measure	NA (N = 19)			LN (N = 10)		
	Accuracy	RTs	Inverse Efficiency	Accuracy	RTs	Inverse Efficiency
	Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)
Dyscalculia Screener						
Simple RTs	426.08 (172.45)			453.25 (211.9)		
Dot Enumeration	90.20 (6.82)	2082.05 (734.05)	23.01 (7.85)	85.38 (4.33)	2278.05 (803.4)	26.68 (9.73)
Number Comparison	92.22 (6.36)	695.67 (277.62)	7.56 (3.02)	89.67 (4.83)	1092.35 (366.5)	12.27 (4.09)
Exact Addition	88.49 (5.44)	4177.25 (2448.7)	48.28 (30.08)	58.57 (9.09)	3783.4 (2099.2)	65.25 (33.14)
Number tasks						
Approximate Comparison	68.05 (13.9)	715.19 (252.49)	45.89 (18.18)	60.0 (13.06)	703.0 (355.23)	52.13 (29.3)
Approximate Addition	69.21 (16.8)	795.39 (470.53)	55.69 (50.06)	76.67(8.38)	784.5(532.8)	45.17 (34.3)
Approximate Subtraction	60.65 (12.3)	711.08 (330.21)	53.99 (35.83)	52.92 (8.57)	824.1 (521.67)	67.39 (46.76)
Discrete Numerosity	92.94 (5.68)	1015.97 (197.97)	22.93 (5.12)	84.79 (21.3)	1175.9 (496.04)	41.39 (56.18)
Analogue Area	83.56 (12.1)	1307.30 (245.23)	33.05 (6.80)	76.25 (13.3)	1263.8 (399.26)	36.4 (15.8)

Table 5.3. Dyscalculia Screener and Number tasks' scores for normally achieving children (NA) and for children with low numeracy (LN). Cells in Bold indicate significant differences between the groups. Accuracy indicates % of correct responses.

5.7.2. Experimental design

All experimental tasks were the same as the ones described in **Chapter IV** (see **Figure 4.1**). Stimuli, task parameters and timing were also identical to those described in **Chapter IV**, Experiment 3A, B, & C (see **paragraph 4.3.2.1**).

5.8. Results 4B

5.8.1. Accuracy analyses

As in **Chapter IV**, we took deviants as our indices of accuracy. A deviant was defined as the absolute value of the subtraction between the participants' response and the target value (in pixels on the line). A 4 (notation condition) x 2 (age-group) mixed model ANOVA was performed. Pair-wise comparison analyses were adjusted for multiple comparisons (Bonferroni corrected). When the sphericity assumption was violated, Greenhouse-Geisser corrections were applied to all factors with more than two levels.

In the NP task, the analyses revealed a main effect of notation condition [$F(1.254, 47.66; \eta^2 = .582) = 52.99; p < .001$], a main effect of group [$F(1,38) = 6.568; p < .05; \eta^2 = .147$], while the interaction was not significant [$F(1.254, 47.66) = 0.472; p = .538$] (**Figure 5.2A**).

Subsequent pair-wise comparison analyses, after adjusting for multiple comparisons (Bonferroni), revealed that the accuracy on the Fractions notation condition was significantly lower than all the other notation conditions ($p < .001$). Performance on the Integers notation condition was significantly better than the Fractions and Decimals notation conditions ($p < .001, p < .05$ respectively) but not than the Money condition ($p = \text{n.s.}$). The Decimals notation condition was significantly worse than both the Integers and Money conditions ($p < .05$); and the Money notation condition was significantly better than the Fractions and Decimals conditions ($p < .001$ and $p < .05$ respectively). The main effect of group indicates that overall the LN group was more accurate than the NA group (**Figure 5.2A**).

In the PN task, all effects were significant: notation condition [$F(1.403, 53.313) = 158.747; p < .001; \eta^2 = .790$], group [$F(1,38) = 29.297; p < .001; \eta^2 = .438$], and the interaction notation condition by group [$F(1.403, 53.313) = 35.247; p < .001; \eta^2 = .387$] (**Figure 5.2B**). Subsequent pair-wise comparison analyses, after adjusting for multiple comparisons (Bonferroni corrected), revealed that the effect was only significant in the Fractions notation condition. The performance of both groups in the Fractions notation condition showed to be significantly less accurate compared with the other notation

conditions (all $p < .001$). The Integers notation condition was significantly different than the Decimals notation condition ($p < .05$); while no other differences were found between the other notation conditions (all $p > .05$).

The two groups only differed in the Fractions and Integers notation conditions where the LN group was more accurate than the NA group (both $p < .001$). No difference was found between the two groups in the Decimals ($p = .54$) and Money ($p = .3$) notations (**Figure 5.2B**).

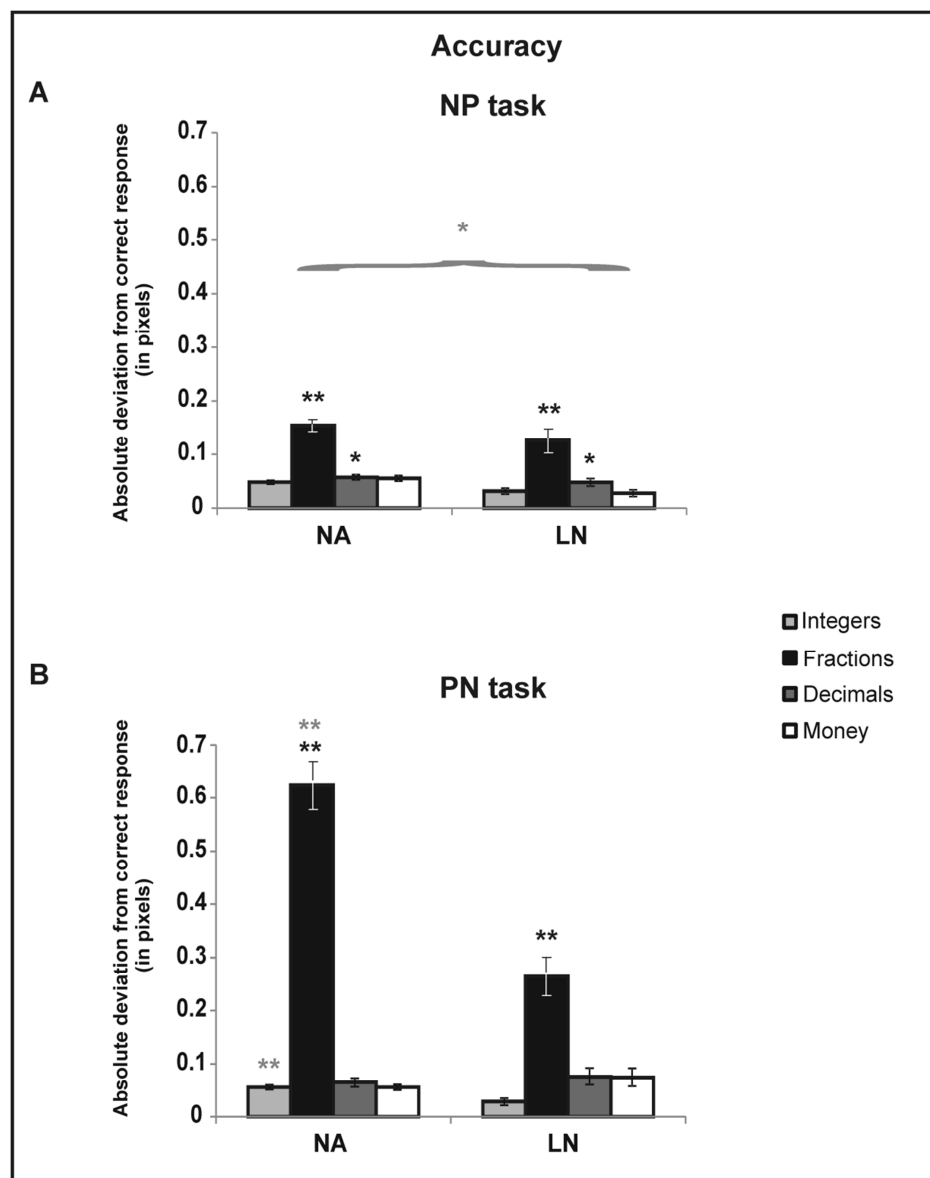


Figure 5.2. Accuracy in terms of the absolute deviation from correct responses (in pixels) plotted against age group for both tasks. A) Number to Position (NP); B) Position to Number (PN). Black asterisks indicate significant differences between notation conditions * $p < .05$, ** $p < .005$. Grey asterisks indicate significant differences between groups * $p < .05$, ** $p < .005$. Error bars indicate 1 standard error of the mean.

5.8.2. Latency analyses

These analyses were performed on median RTs. As for the analyses of accuracy, we performed a 4 x 2 mixed model ANOVA with notation condition as the within subjects factor (four levels) and group as the between subjects factor (two levels).

In the NP task, the analyses revealed a main effect of notation condition [$F(2.456, 93.344) = 12.123$; $p < .001$; $\eta^2 = .242$], a main effect of group [$F(1,38) = 18.648$; $p < .001$; $\eta^2 = .329$], while the interaction was not significant [$F(2.456, 93.344) = 0.213$; $p = .851$] (**Figure 5.3A**).

Subsequent pair-wise comparison analyses (Bonferroni corrected) revealed that the main effect of notation condition was solely driven by the Fractions notation which appeared to be significantly slower than the Integers ($p < .001$), Decimals and the Money notation conditions ($p < .005$). The LN group was significantly slower in responding compared to the NA group in all notation conditions ($p < .01$) (**Figure 5.3A**).

In the PN task, the ANOVA revealed a main effect of notation condition [$F(1.882, 71.525) = 144.04$; $p < .001$; $\eta^2 = .791$], a main effect of group [$F(1,38) = 18.434$; $p < .001$; $\eta^2 = .327$], and a significant interaction [$F(1.882, 71.525) = 20.906$; $p < .001$; $\eta^2 = .355$] (**Figure 5.3B**).

Subsequent pair-wise comparisons (Bonferroni corrected) revealed that the effect was only significant for the Fractions notation condition. The performance of both groups in the Fractions notation condition was significantly slower compared to all the other notation conditions (all $p < .001$).

The two groups only differed between each other in the Fractions and Integers notation conditions (both $p < .001$) where the LN group was significantly slower than the NA group. No significant difference between the groups was found for the Decimals and Money (both $p > 0.05$) notation conditions (**Figure 5.3B**).

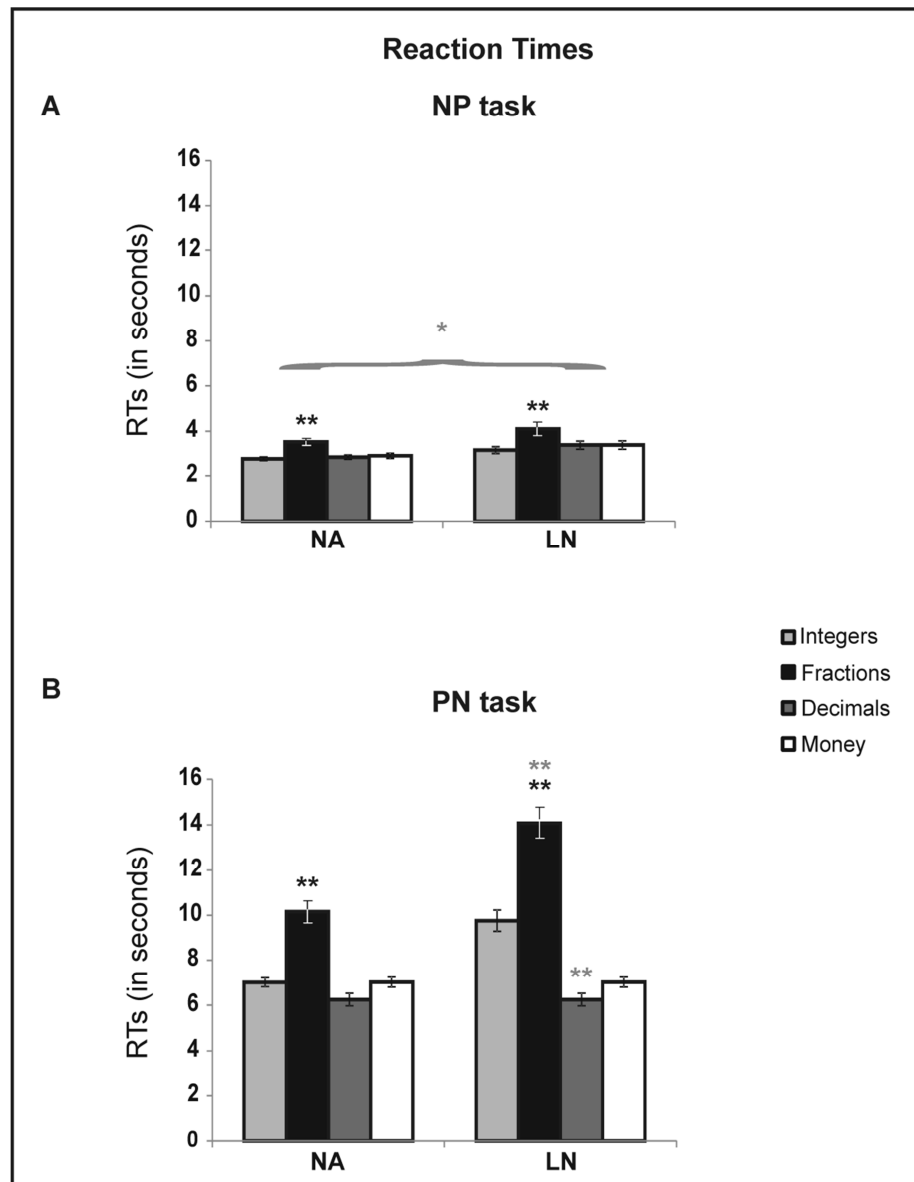


Figure 5.3. Reaction times (seconds) plotted against age group for both tasks. A) Number to Position (NP); B) Position to Number (PN). Black asterisks indicate significant differences between notation conditions * $p < .05$, ** $p < .005$. Grey asterisks indicate significant differences between groups * $p < .05$, ** $p < .005$. Error bars indicate 1 standard error of the mean.

5.9. Interim Discussion 4B

In Experiment 4B we investigated how well children with low numeracy understood the real value of four types of number notations: integers, fractions, decimals and money. In order to do so we compared performance of children with low numeracy to normal achievers.

In the NP task, our accuracy analyses revealed that fractions were less accurate to be mapped on the number line compared to all other numerical notations (integers and money, but also decimals). Moreover, in both groups there seemed to be a hierarchy in the accuracy of the estimation, with fractions and decimal stimuli at the bottom, immediately followed by money and finally integer stimuli. Furthermore, our analyses indicated that overall the LN group was more accurate than the NA group (**Figure 5.2A**).

The same pattern of results held for the PN task, where the fractional stimuli were estimated less accurately compared to the other notations. This was true in both groups. Moreover, integer stimuli seemed to be better mastered than all the other types of stimuli. Interestingly, as in the NP task, the LN group made more accurate estimates than their typically developing peers with fractional and integer stimuli, while no differences were found with decimals and money stimuli (**Figure 5.2B**).

The latency data showed the same pattern as the accuracy data. In the NP task fractions were processed slower than the other types of numerical stimuli in both groups. Yet, the hierarchical pattern found in the accuracy data did not occur in our reaction time data. Moreover, the integer notation was not processed quicker than the other notations. Finally, children with LN were consistently slower than their NA peers (**Figure 5.3A**). In the PN task, the results on latency revealed that the fractional stimuli were slower to process than the other types of stimuli in both groups. As we have seen in the previous Chapter (see **Chapter IV**), this latter result should be treated with caution as it could just be the consequence of choosing the numbers to type in or the natural demand on speed of responses required by the action of typing the numbers into two separate cells.

Interestingly, as in the accuracy data, the LN group was consistently slower than the NA group with fractional and integer stimuli, while no differences were found with decimal and money stimuli (**Figure 5.3B**).

All in all, the results of Experiment 4B suggest that even though fractions are accurately and linearly represented by typically and atypically developing children, the processing of this notation is more difficult than other types of numerical stimuli. This is less true for decimal stimuli.

Moreover, the results of Experiment 4B show an intriguing general effect of group in terms of both accuracy and reaction times. Interestingly, and quite unexpectedly children with low numeracy were overall more accurate than normal achievers when asked to map double-digit numbers on physical lines. Yet, this accuracy effect can readily be explained by the latency data which demonstrated that children with low numeracy were consistently slower in processing double-digit numbers compared to typically developing children.

These results support the notion that children with low numeracy have an intact and accurate representation of double-digit numbers (see also Experiment 4A). Moreover, their mastering of these types of notations, although more difficult with rational numbers, does not differ from their typically developing peers.

5.10. Final Discussion

In this study we investigated the nature of different double-digit numerical representations in children with low numeracy (LN). The *Dyscalculia Screener* (see **Chapter II**) determined these children to have deficits in their exact calculation abilities, but without disruptions in the core number sense (DD). It has been proposed that a deficient representation (i.e. logarithmic) of numbers (i.e. integers) could actually become beneficial when processing other types of ‘paradoxical’ notations (Opfer & DeVries, 2008). Populations who struggle with maths could therefore be ideal in order to test this hypothesis. Contrary to what has been previously suggested (Geary et al., 2007, 2008), we found that children with a selective impairment in arithmetic do not actually show a deficient (i.e. logarithmic) representation of integers in either of our number line tasks (NP and PN). Moreover, we found that at least when asked to map numerical values on a given line, children with low numeracy display an accurate (i.e. linear) representation also for other types of numbers including fractions and decimals. Indeed, in the NP task, our regression analyses revealed that the linear model was the best to account for the data in all notation conditions, including rational numbers. This demonstrates that even if this group experiences difficulties in performing exact calculation, the mapping of double-digit numbers into space (i.e. number line representation) remains intact.

It has also been suggested that difficulties in naming and ordering numbers such as fractions and decimals is predictive of general mathematical disabilities (Mazzocco & Devlin, 2008). Our correlational analyses demonstrate the existence of such link even when looking at the internal representation of these notations rather than just the ability to name and rank them. In other words, the better the mapping of rational numbers (but also whole numbers) in space (i.e. the number line), the better the accuracy on an addition verification task (or more generally arithmetic). Moreover, we demonstrate this link in a population with a specific type of maths impairment (*see above*) rather than more general mathematical disabilities as previously reported (Mazzocco & Devlin, 2008).

As previously demonstrated for well-educated adults and typically developing children (**Chapter IV**), the results of the PN task were slightly different from the NP task also in children with low numeracy. Thereby, the linear representation was not evident for fractions, and also money. On the contrary, an accurate and linear representation was in place for integer and decimals stimuli which is in further disagreement with the hypothesis of low mapping and estimation abilities in this population (Geary et al., 2007, 2008). Altogether the results of the LN group are comparable to that of adults and typically developing children (see **Chapter IV**).

Interesting, the LN group actually performed better on the NP task compared to their age and education matched controls and similarly in the PN condition for most numerical notations. This is in accordance with the notion that these individuals do not suffer from a disruption of their core numerical representation (as in the case of DD). On the contrary, they only seem to show difficulties when this information has to be manipulated, such as in exact calculations or in the mapping between numerals and numerosities (see **Chapter II**). This is reflected in the slowing of their processing speed (RTs) in both the NP and PN conditions. In accordance with the results of **Chapter II**, these children seem to exhibit intact basic numerical concepts, but their ability to utilize this information tends to be inefficient (Rousselle & Noël, 2007, see also **Chapter II**). Importantly, this stresses that future learning interventions in these children should focus on the disrupted linkages of the core numerical representation to the application and utilization of this information.

5.11. Summary

The results of Experiments 4A& B show that children with low numeracy can accurately represent (i.e. linearly) various types of number notations including double-digit integers, fractions and decimals. Moreover we show that in terms of accuracy, children with low numeracy performed actually better than their age-and education matched peers. This is in line with previous investigations which show that children with low numeracy have no difficulties in understanding and representing numerosities (see **Chapter II**). However, their reaction times were significantly slower suggesting that when the numerical information has to be manipulated in any way, this will result in a slower processing speed. It has been proposed that a deficient representation of integers is strongly linked with mathematical disabilities (Geary et al., 2007, 2008). Moreover, it has been suggested that in certain cases, a deficient representation of integers could result in a better processing of proportional notations such as fractions and decimals (Opfer & DeVries, 2008). Our data speaks against both of these hypotheses as we show that, when asked to place a numerical value on a physical line, children with low numeracy show an accurate representation for integers, and also rational numbers. Furthermore, our data favors the hypothesis of a link between the ability to represent rational numbers and arithmetical performance (see Mazzocco & Devlin, 2008). Finally, the results obtained here demonstrate the feasibility of these tasks for the quick evaluation of different aspects of number understanding in children with arithmetical learning disabilities.

VI

‘If’ and ‘how’ of exceptional maths abilities in Autism Spectrum Disorders

“Mathematicians are born, not made” --Henri Poincaré

Autism Spectrum Disorder (ASD) is a neurodevelopmental disorder characterized by a complex phenotype including social and emotional processing deficits. Yet, recent evidence has started to suggest that some individuals with ASD might have remarkable cognitive strengths, one of them being mathematics. Through a series of behavioural, neuroimaging and multivariate approaches we demonstrate that to possess special skills (i.e. mathematics) is a distinguishing mark of ASD, both at a cognitive and a neuronal level. Distinct fine-scale neural representations for arithmetic problems were evident in areas of the ventral and dorsal visual streams, the medial temporal lobe and the prefrontal cortex in children with ASD compared to typically developing children. Moreover, multivariate pattern analysis demonstrated that the anterior fusiform gyri together with two prefrontal cortical regions might play an essential role in the neural orchestration of these exceptional maths abilities. These results support the idea that ASD use distinct types of information processing. Moreover, these findings have important implications for the life chances of these individuals.

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6.1. Introduction

Nearly 70 years ago, the Viennese paediatrician Hans Asperger wrote: “it seems that for success in science or art a dash of autism is essential”. Moreover, Asperger was the first to highlight the crucial implications of the connection between the syndrome and mathematical talents when he noted that for a large proportion of people with Autism Spectrum Disorder (ASD) “their mathematical ability determined their professions” (Asperger, 1944). However, there has also been evidence of maths weaknesses in ASD (Mayes & Calhoun, 2006). Moreover, in a recent review, which included a pool of 322 individuals with ASD, Chiang and Lin, reported average maths abilities in ASD compared to the normal population. Yet, the same study also highlights the fact that some individuals with ASD show high average to very superior mathematical ability (Chiang & Lin, 2007).

Critically, up till now evidence of higher mathematical and numerosity skills in ASD has mostly been anecdotal (Kanner, 1943; Sacks, 1986; *see also* McMullen, 2000; Ward & Alar, 2000) and descriptive (Baron-Cohen, Wheelwright, Skinner, Martin, & Clubley, 2001, Baron-Cohen, Richler, Bisarya, Gurunathan, & Wheelwright, 2003; Baron-Cohen, 2007). Notably, it has been shown that scientists score higher than non-scientists on self-administered questionnaire for ‘autism associated traits’ (Baron-Cohen et al., 2001, 2003) and there is a ‘three to sevenfold increase for autism spectrum condition amongst mathematicians’ (Baron-Cohen, Wheelwright, Burtenshaw, & Hobson, 2007). Furthermore, a recent study has shown a bigger discrepancy between IQ and mathematical abilities in 14-16 years old with ASD (around 14 standard score points) in the mathematical domain (Jones, Happé, Golden, Marsden, Tregay et al., 2009; *but see* Dickerson Mayes, & Calhoun, 2003).

In a recent theoretical account Baron-Cohen and Belmonte (2005) have proposed that systematic, logical and analogical thinking is enhanced in individuals with ASD, hinting at the idea that ‘hyper-systemizable’ abilities might be an adaptive mechanism in these individuals since they represent a way to ‘reduce environmental variance to a series of regular and more effective sets of repeatable rules’ (Baron-Cohen & Belmonte, 2005). Mathematics is the most concrete instantiation of such abilities as it is built upon the principles of rules and order. Moreover, mathematical skills are essential for educational and professional success, but also crucially important in

everyday life (Geary, 1994). Thus, mathematics represents an ideal domain to experimentally measure areas of potential cognitive strengths in ASD.

Up to now, no attempt has been made to investigate the neural basis of maths abilities in this population. At present, one fMRI study has looked at the neural correlates of arithmetical and calendrical skills in two autistic savants (Cowan & Frith, 2009). An increase in parietal activation was found during calendar date questions, as well as during arithmetic problem solving. As previously mentioned (**Chapter I**) the parietal lobes have been critically implicated in arithmetical (*see* Ansari, 2008 and Zamarian, Ischebeck & Delazer, 2009 *for a review*) and numerical (Dehaene, Piazza, Pinel, & Cohen, 2003) processes. Moreover, the parietal activity in the two autistic savants was modulated by the temporal distance of calendar date questions, leading the authors to conclude that such skills, at least in the two individuals tested, are the result of intensive practice with calculation rather than memorization of dates. However, there have been no investigations of the neural correlates of numerical abilities in ASD individuals who are not savants.

In this chapter we describe three experimental studies that (i) provide the first systematic characterization of mathematical abilities in children with High Functioning Autism (HFA) compared to age and IQ matched typically developing peers; (ii) demonstrate the extent to which the neural recruitment during an arithmetic task is different in HFA; and (iii) show a causal relation between HFA’s exceptional abilities and the distinct fine-scale neural representation that characterizes this population when solving an arithmetic task.

Experiment 5A.Characterizing maths abilities in Autism Spectrum Disorder

6.2. Aim 5A

We have explored the behavioural signature of mathematical impairments in its manifestations as Developmental Dyscalculia and Low Numeracy demonstrating the need for a distinction between the two conditions. Moreover, we have investigated in depth the heterogeneous nature of the discipline of mathematics showing that different aspects of it (i.e. approximate calculation, judgments of analogue quantities, linking quantities to symbols, rational numbers understanding) could be spared or not depending on the type of maths impairment (i.e. Developmental Dyscalculia or Low Numeracy). In this experiment we sought to investigate and characterize the other side of ‘abnormal’ mathematical abilities by looking at exceptional skills. Particularly, we investigate whether maths is an islet of relative ability, and a potential strength in children with Autism Spectrum Disorder. Moreover we address the question of whether possible strengths of abilities might be associated with better or more sophisticated strategies.

6.3. Method 5A

6.3.1. Participants

We studied a sample of 7-12 year old high functioning children with autism spectrum disorder (HFA) (N = 21, mean age = 9.13; SD = 1.65) and a sample of typically developing children (TD) (N = 21, mean age = 9.12, SD = 1.65). HFA status was assessed by expert clinical evaluation and scores in the autism range on the Autism Diagnostic Interview – Revised (ADI-R) and/or the Autism Diagnostic Observation Schedule (ADOS) (**Table 6.1**)¹⁴.

The TD cohort was selected by a customized matching algorithm from a larger sample of TD individuals (N= 139) who were part of a longitudinal study at Stanford University. The TD cohort was matched on age, full scale IQ and gender ratio (4:1 boys

¹⁴ Specific ADI-R and ADOS scores were unavailable for two participants due to accidental loss of primary data following confirmation of eligibility for the study.

to girls) to the HFA group (**Table 6.1**). The optimization algorithm used here is less biased than the most common matching criteria (i.e. one to one matching) which incorrectly make the assumption that one individual in the experimental population is identical to their control subject with the only exception that the latter does not present a clinical characteristic (e.g. Autism Spectrum Disorder). Instead, the present optimization method uses the general properties of the experimental group in order to select a well matched sample of typically developing children through the principles of genetic algorithms. Once given the relevant criteria of the experimental group (in our case: gender ratio, age and full IQ’s mean and SD), the algorithm randomly creates 10,000 combinations of 21 people each (the size of our experimental group) and computes a fitness score (as intended by genetic algorithms and thereby in biological terms) for each of these combinations. The fitness score is then entered into a ruler probability distribution to find - using crossing-over on the pairs - the best matched subset of participants (i.e. how well the fitness score of each of the created combinations matches the original criteria).

Parental consent forms and children assent forms were obtained for each child. All protocols were approved by the Human Participants Institutional Review Board at the Stanford University School of Medicine.

Measure	HFA (N=21)	TD (N =21)	t-test	p-value
Male to Female ratio	16 : 5	16 : 5		
Age (years)	9.13, SD = 1.65	9.12, SD = 1.65	0.019	.985
IQ –WASI scale				
Full IQ	114.52, SD = 13.89	115.09, SD = 13.69	-0.134	.894
Verbal IQ	112.57, SD = 17.54	113.09, SD = 15.28	-0.103	.918
Performance IQ	113.81, SD = 18.13	112.09, SD = 14.35	0.340	.736
ADI-R				
Social	19.71, SD = 7.05			
Communication	15.18, SD = 6.72			
Repetitive Behaviour	5.94, SD = 3.03			
ADOS				
Social	7.75, SD = 1.69			
Communication	3.25, SD = 1.39			
Algorithm	11.00, SD = 2.73			

Table 6.1. Demographics and mean IQ scores for the HFA and TD group separately. Mean ADI-R and ADOS scores for the HFA group.

6.3.2. Experimental design

6.3.2.1. Standardized tasks

Mathematical abilities. Mathematical abilities were assessed using the WIAT-II (Wechsler, 2001). This is an achievement battery of nationally standardized measures of academic achievement and problem-solving abilities normed by grade and academic year (mean = 100, SD = 15). The scale includes two maths subtests: *Numerical Operations* and *Mathematical Reasoning*. The *Numerical Operations* subtest is a paper-and-pencil test that measures number identification and writing, rote counting, numbers production, and simple calculation with one to two-digit numbers (e.g. $4 - 2 = \dots$; $37 + 54 = \dots$).

The *Mathematical Reasoning* subtest is a verbal problem-solving test that measures counting, identification of geometrical shapes, and single to multi-step word problem-solving involving time, money and measurements with verbal and visual prompts. The child is required to solve problems with whole numbers, fractions or decimals, interpret graphs, identify mathematical patterns, and solve problems of statistics and probability. For example, a probability problem asks: ‘If you flipped a coin ten times, how many times would the coin be most likely to land on the heads?’.

Working Memory. The Working Memory Test Battery for Children (WMTB-C) (Pickering & Gathercole, 2001) was used in order to assess the three components of Working Memory (Baddeley, 1986). The Central Executive was assessed by the *Counting Recall* and the *Backwards Digit Recall* subtests. The *Digit Recall* subtest and the *Block Recall* subtest assessed the Phonological Loop and the Visuo-Spatial Sketchpad components respectively.

6.3.2.2. Experimental tasks

Strategy assessment. Fourteen out of twenty-one children in each of the two groups performed a strategy assessment test for single-digit addition problems (e.g. $5 + 6 = ?$). The problems were presented one at a time on a computer screen. There were 18 problems with random pairs of integers from 2 to 9 and sums ranging from 6 to 17. Problems with identical addends were excluded as well as problems such as $n+0$ and $n+1$ as they show less strategy variability (see Siegler, 1987). Children were instructed

to say the answer out loud ‘as soon as they had it in their head’. The experimenter then probed the child on which strategy was used during problem solving. Responses were categorized as: retrieval (e.g. ‘just knew it’), count (e.g. counted on my fingers’, ‘counted in my head’), and decomposition (e.g. $9 + 5 = 9 + (1 + 4) = (9 + 1) + 4 = 10 + 4 = 14$). Trials where the experimenter noted overt signs of counting even when the child reported retrieval were classified as counting. For each child, the proportion of trials in which retrieval, counting or decomposition strategies were used to correctly solve the problem was computed (Cho, Ryali, Geary, & Menon, 2011; *see also* Wu, Meyer, Maeda, Salimpoor, Tomiyama, et al., 2008 *for validation principles and procedures*).

6.4. Results 5A

6.4.1. Standardized measures of mathematical abilities

Children in the HFA group performed significantly better than TD children on both standardized maths measures of the WIAT-II scale: *Numerical Operations* ($t(40) = 3.605$, $p < .001$) and *Math Reasoning* ($t(40) = 2.379$, $p < .022$) (**Figure 6.1A**).

6.4.2. IQ and maths Discrepancy scores

Discrepancy scores calculated by subtracting full IQ scores from each of the two standardized maths measures (*see* Jones et al., 2009) were higher for the HFA group compared to the TD group: *Numerical Operations* ($t(40) = 3.272$, $p < .002$) and *Math Reasoning* ($t(40) = 3.862$, $p < .007$) (**Figure 6.1B**).

6.4.3. Working Memory abilities

No significant differences between the groups were found on any of the tasks from the Working Memory Test Battery for children: *Counting Recall* ($t(40) = 0.619$, $p = .54$), *Backwards Digit Recall* ($t(40) = .733$, $p = .468$), *Digit Recall* ($t(40) = 1.77$, $p = .084$) and *Block Recall* ($t(40) = -1.58$, $p = .123$).

6.4.4. Strategy assessments

This analysis revealed a significant difference between the groups in their use of the decomposition strategy ($t(26) = 2.39$, $p = .024$). Children in the HFA group adopted this sophisticated strategy significantly more often than their typically developing peers (**Figure 6.1C**). The groups did not differ on their utilization of the other two strategies of interest: retrieval ($t(26) = 0.257$, $p = .724$) and counting ($t(26) = -1.289$, $p = .209$).

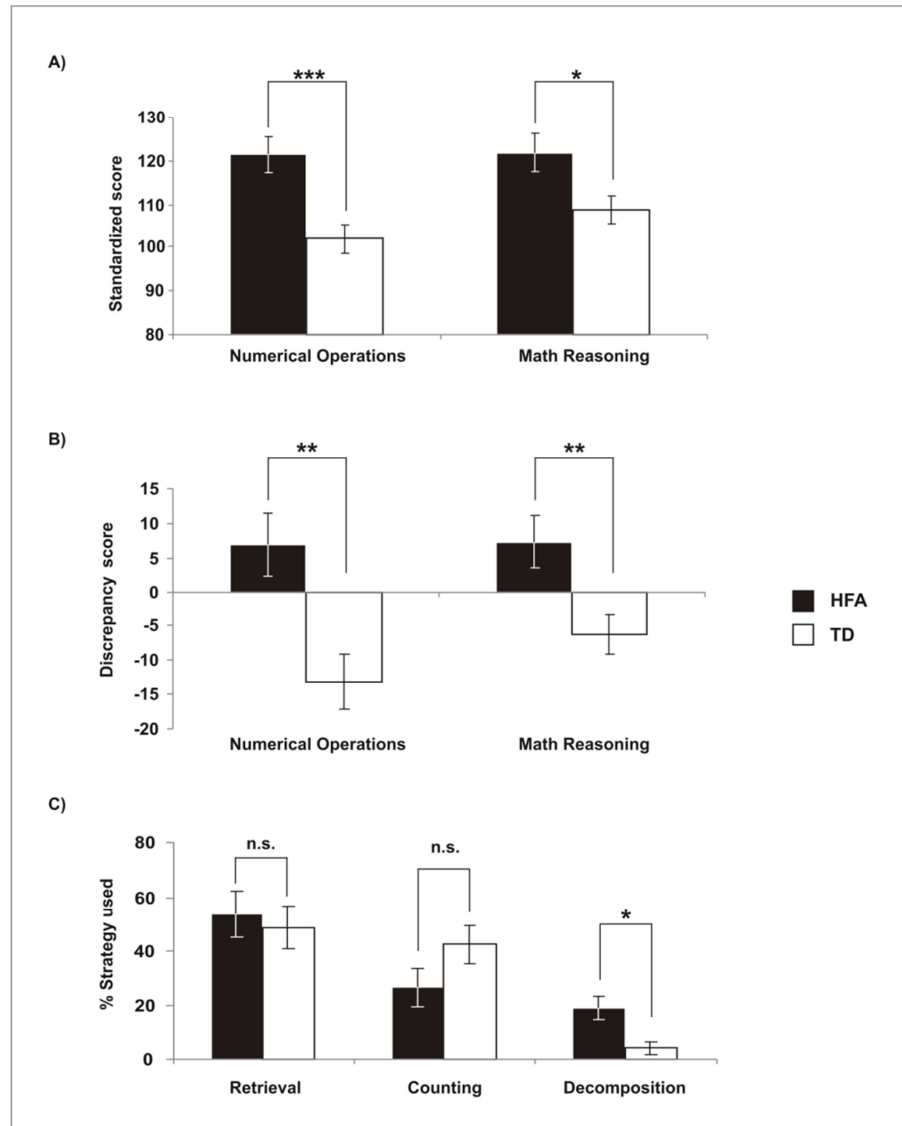


Figure 6.1. A) Standardized maths achievement scores plotted against group. The two groups differed significantly on both standardized maths measures wherein the HFA group over-performed the TD group. * $p < .05$; *** $p < .001$. B) Discrepancy scores between the two standardized maths measures and full IQ scores plotted against group. The HFA group displays a discrepancy score significantly higher than the TD group. ** $p < .01$. C) Different strategy use in HFA and TD. The two groups differed significantly on percentage of trials in which decomposition was used. * $p < .05$. Error bars indicate one standard error of the mean.

6.5. Interim Discussion 5A

In this section we aimed to investigate the relationship between the clinical condition of Autism and mathematical talent. The results show that children with HFA are significantly better at mathematical tasks compared to their age and IQ matched controls. Notably, children with HFA displayed higher scores at the maths subtests of the WIAT-II scale compared to their typically developing peers and showed a greater use of decomposition strategies when solving arithmetic tasks. The literature on strategy use of arithmetic problems mainly reports a clear distinction between two different strategy uses: counting and retrieval (Cho et al., 2011; Núñez-Peña, Gracia-Bafalluy, & Tubau, 2011; Siegler, 1996; *see also* Geary & Brown, 1991; Geary, Hoard, Byrd-Craven, & DeSoto, 2004 *for a learning disability account*) while the decomposition strategy is much less documented, discussed and most often not included in the theoretical framework of strategy refinement and development. The idea behind most of these studies is that proficiency in arithmetic problem solving is accompanied by a more consistent reliance on effective and automatized strategies such as retrieval - rather than on immature strategies such as counting - as a consequence of development and education. In our sample, no differences were found between the groups in their use of the two aforementioned strategies (counting and retrieval), yet the HFA group adopted the decomposition strategy much more often than their control group. However, it is important to note that decomposition was the least utilized strategy in both groups (see **Figure 6.1C**). Interestingly, two behavioural studies on typically developing children have reported that even if decomposition was the least common strategy in their sample, it was associated with higher performance's accuracy on arithmetic problem solving (Cowan, Donlan, Shepherd, Cole-Fletcher, Saxton, & Hurry, 2011; Siegler, 1987). We could therefore speculate that exceptional mathematical abilities in HFA might rely on different processes and mechanisms and that the use of this less common strategy by these children might reflect their unusual – yet effective – way of solving arithmetic problems.

The initial proposal by Baron-Cohen and Belmonte (2005) postulates that the condition of ASD is characterized by exceptional systemizing and analogical abilities, we therefore suggest that the greater use of the decomposition strategy could reflect the predilection of this population for manipulating the problems through the act of partitioning the equation into its distinct components. This is also in line with the ‘weak

central coherence theory’ (Frith, 1989) as these children seem to be highly focusing on the local details of the equation rather than its global properties. Moreover, this strategy would clearly reflect a more analytical and ‘personalized’ way to approach the problem compared to the automatic retrieval from memory, and to ineffective and immature processes such as counting. Furthermore, it is important to note that the use of a less popular, and commonly thought of as less effective strategy – compared to fact retrieval from Long Term Memory - is not negatively affecting their performance on any of the maths tasks. If anything, it seems to be very beneficial for them and it could represent one of the behavioural signatures of the mathematical strengths in children with ASD.

Our study extends previous findings which have investigated the relation between academic achievement (i.e. maths) and IQ (Jones et al., 2009) by looking at the discrepancy scores in a younger population of ASD (7-12 years old) and using a between group design. We report a better - and positive - discrepancy score in the HFA group compared to the TD group. This result has important implications for the educational practice of ASD in its initial years (i.e. primary and secondary school). Furthermore, it is important to note that the ASD sample tested in the study by Jones and colleagues (2009) had a mean full-scale IQ score which falls on the lower end of the distribution (mean = 84.3, SD = 18) with standardized scores ranging between 50 and 119, while the present study only included children with high functioning Autism (i.e. average or better than average full-score IQ) and still demonstrates a high positive discrepancy with their mathematical abilities.

Finally, the pattern of no differences in Working Memory (WM) abilities between the two groups is coherent with the few and rather inconsistent reports from the literature which have found both increased visual memory skills (Martos-Pérez & Ayuda-Pascual, 2003) and poor working memory abilities (Mayes & Calhoun, 2007, 2008). However, at this stage it would be hard – and beyond the scope of this study - to speculate on this finding rather than attribute it to individual differences within the sample and the heterogeneity of the disorder. Yet it is important to note that a pattern of no differences in WM skills between the groups suggests that superior maths abilities in the ASD sample could not be attributed to a better WM system.

Experiment 5B. Neuronal signature of maths abilities in Autism Spectrum Disorder

6.6. Aim 5B

Experiment 5A demonstrated that children with High Functioning Autism perform better at mathematical tasks compared to typically developing children of the same age and IQ. In Experiment 5B we therefore sought to test whether the exceptional maths abilities characterizing this population might be ‘supported’ by different brain areas or distinct fine-scale neural representations for arithmetic problem solving compared to typically developing children.

6.7. Method 5B

6.7.1. Participants

The same two groups of participants described in **6.3.1. (Table 6.1)** took part in Experiment 5B.

6.7.2. Experimental design

6.7.2.1. Experimental tasks

Children performed Addition verification tasks. Stimuli were presented in a block fMRI design in order to optimize signal detection (Friston, Zarahn, Josephs, Henson, & Dale, 1999). Block length was randomly jittered between 22.5 and 27 seconds. Each run consisted of four task conditions: (i) Complex problems, (ii) Simple problems, (iii) Number identification and (iv) Passive Fixation, with 18 trials per condition. Complex and Simple problems were contrasted while the other two conditions were modelled into the fMRI analysis but not analyzed further. Critically, the Complex and Simple conditions have the same numerical and symbolic formats as well as response selection requirements, but differ in arithmetic processing demands (Campbell & Metcalfe, 2007). Complex problems consisted of equations with unique addends different from ‘1’ (e.g. $3+4 = 7$). One operand ranged from 2 to 9, the other from 2 to 5 (tie problems such as, $5 + 5 = 10$ were excluded). Simple addition problems were identical in format,

except that one of the addends was always ‘1’ (e.g. $5 + 1 = 7$). The child indicated via a button box whether the answer was correct or incorrect. Answers were correct on 50% of the trials, the order of the accurate and inaccurate trials was randomized across participants and incorrect answers deviated by ± 2 or ± 1 from the correct answer. Accuracy and median reaction time of correctly solved problems were computed for each participant. Each equation was displayed for 5 seconds with an inter-trial interval of 500 milliseconds.

6.7.2.2. fMRI data acquisition

Functional brain images were acquired on a 3T G.E.Signa scanner (**Figure 6.2**). A total of 29 axial slices (4.0 mm thickness, 0.5 mm skip) parallel to the AC-PC and covering the whole brain were imaged using a T2* weighted gradient echo spiral in-out pulse sequence with the following parameters: TR = 2 sec, TE = 30 msec, flip angle = 80° , 1 interleave. The field of view was 20 cm, and the matrix size was 64x64, providing an in-plane spatial resolution of 3.125 mm. In order to reduce blurring and signal loss from field inhomogeneity, an automated high order shimming method based on a spiral acquisition was used prior to the acquisition of functional MRI scans (Kim, Adalsteinsson, Glover, & Spielman, 2002).



Figure 6.2. The 3 Tesla G.E.Signascanner located at the Richard M. Lucas Center for Imaging, Radiological Sciences Laboratory, Department of Radiology, Stanford University School of Medicine (image courtesy of Stanford website www.spectrum.stanford.edu).

6.7.2.3. fMRI data analysis

Data were analyzed using the general linear model implemented in SPM8 (<http://www.fil.ion.ucl.ac.uk/spm/>).

fMRI preprocessing. The first five volumes were not analyzed to allow for signal equilibration. ArtRepair software was used to correct for excessive participants’ movement (<http://sfnl.stanford.edu/tools/ArtRepair/ArtRepair.htm>). Images were realigned to correct for movement, smoothed with a 4 mm FWHM Gaussian kernel and motion adjusted. Deviant volumes resulting from sharp movement or spikes in the global signal were then interpolated using the two adjacent scans. No more than 20% of the volumes were interpolated. Finally, images were corrected for errors in slice-timing, spatially normalized to standard MNI space, resampled to 2mm isotropic voxels, and smoothed again at 4.5 mm FWHM Gaussian kernel. The double step sequence of first smoothing with a 4 mm FWHM Gaussian kernel and later with 4.5 mm FWHM Gaussian kernel approximates a total smoothing of 6 mm.

Individual Subjects analysis. Task related brain activation was identified using the General Linear Model (GLM) implemented in SPM. During individual subjects analysis, the interpolated volumes which were flagged at the preprocessing stage were de-weighted. Brain activity related to task conditions was modeled using boxcar functions with a canonical hemodynamic response function and a temporal derivative to account for voxel-wise latency differences in hemodynamic response. Low frequency drifts at each voxel were removed using a high pass filter of 0.5cycles/mm. Serial correlations were accounted for by modeling the fMRI time series as a first-degree autoregressive process. Finally, voxel-wise contrast and *t*-statistics images were generated for each participant by contrasting Complex versus Simple problems.

Univariate group analysis. Group level analyses were conducted as t-tests which compared hemodynamic signal changes between the HFA and TD groups on contrasted images that corresponded to the Complex versus Simple conditions. Significant cluster were identified at a height threshold of $p < .005$ with Family Wise Error (FWE) correction for multiple comparisons at the cluster level of $p < .01$. We used a parametric approach based on Monte Carlo simulations to determine the cluster extent that corresponded to the minimum cluster size that controls for false positive rates at $p < .005$ and $p < .01$ for heights and extent respectively. This approach avoids making any assumptions about the underlying distribution of cluster size under the null hypothesis. Monte Carlo simulations were implemented in Matlab using methods equivalent to the AlphaSim procedure in AFNI (Forman, Cohen, Fitzgeralds, Eddy, Mintun, & Noll, 1995;

Wards, 2000). Ten thousand iterations of random 3D images, with the same resolution and dimensions as the fMRI data were generated and the resulting images were smoothed with the same 6mm FWHM Gaussian kernel used to smooth the fMRI data. The probability distribution of cluster size across all iterations was then estimated. The cluster threshold – masked for grey matter – corresponding to a significance level of $p < .005$ was determined to be 87 voxels¹⁵.

Multivariate Pattern analysis. At a group level, images were analyzed using a multivariate statistical pattern recognition-based method (MVPA) at the whole brain level on the contrast Complex versus Simple. This method identified brain regions that discriminated spatial activation patterns between children with HFA and TD children. The MVPA analyses used a nonlinear classifier based on support-vector machine algorithms with radial basis function (RBF) kernels (Muller, Mika, Ratsch, Tsuda, & Scholkopf, 2001). At each voxel v_i , a $3 \times 3 \times 3$ neighborhood centered at v_i was defined. Therefore, the spatial pattern of voxels in this neighborhood was defined by a 27-dimensional vector. Support vector machine (SVM) classification was then performed using the LIBSVM (<http://www.csie.ntu.edu.tw/~cjlin/libsvm>) software. For the nonlinear SVM classifier, we specified two parameters: C (regularization) and α (parameter RBF kernel) at each searchlight position. Subsequently we estimated optimal values of C , α as well as the generalizability of the classifier at each searchlight position by using a combination of grid cell and cross-validation procedures. Contrary to previous approaches in which the free parameter C was arbitrarily set (Haynes, Katsuyuki Sakai, Rees, Gilbert, et al., 2007), we here optimized both free parameters (C and α) based on the data, thereby designing an optimal classifier.

Significant clusters of activation were identified at the whole brain level using a height threshold of $p < .005$, with corrections at $p < .01$ for multiple comparisons following the probability distribution for non-parametric tests determined by Monte Carlo simulations (*see above*). Thus, the cluster threshold - grey matter masked – corresponding to a significance level of $p < .005$ was determined to be 35 contiguous voxels.

¹⁵Please note that all stereotaxic coordinates are reported in MNI space.

6.8. Results 5B

6.8.1. Behavioural performance

Maths performance during fMRI scanning was analysed using a 2-Way ANOVA with Difficulty (Complex and Simple problems) and Group (HFA, TD) as factors. In terms of accuracy, there was a main effect of Difficulty [$F(1,40) = 12.05$, $p < .0001$, $\eta^2 = .232$] as Complex problems were harder than Simple problems. There was no main effect of Group [$F(1,40) = 0.129$, $p = .721$] and no interaction [$F(1,40) = 4.056$, $p = 0.52$]. Speed of responses also showed a main effect of Difficulty [$F(1,40) = 51.121$, $p < .001$, $\eta^2 = 0.561$] in the same direction as accuracy. No main effect of Group [$F(1,40) = 0.001$, $p = .980$] neither interaction [$F(1,40) = 0.814$, $p = .372$] were found.

6.8.2. Univariate differences in intensity of activation between HFA and TD

In this analysis we looked at task related activation using the General Linear Model (GLM) on the whole brain. Particularly, we sought to examine brain regions that showed significant differences in brain activation levels between HFA and TD children contrasting Complex and Simple arithmetic problems. There were no significant differences in activation levels between the two groups at the determined threshold of height $p < .005$, with Family Wise Error (FWE) correction for multiple comparisons at the cluster level (extent) of $p < .01$.

6.8.3. Multivariate differences in brain activation between HFA and TD

Using the searchlight method (Abrams, Bhatara, Ryali, Balaban, Levitin, & Menon, 2010, Choet al., 2011; Kriegeskorte, Goebel, & Bandettini, 2006), we identified sets of cortical regions that showed significant differences in multivariate activation patterns during arithmetic problem solving between HFA and TD (**Table 6.2**). Notably, very high classification accuracies (CA) (between 81% and 90%) were found in areas of the ventral visual stream: the right and left anterior fusiform gyri, the left posterior fusiform gyrus, the right lateral occipital cortex (LOC) and the right lingual gyrus (**Figure 6.3A**). Furthermore, a large cluster in a key area of the dorsal visual stream, the left superolateral parietal lobe - including the intraparietal sulcus (IPS) and the

neighbouring angular gyrus - displayed nearly 90% accuracy (**Figure 6.3B**). Moreover, in the medial temporal lobe (MTL), high classification accuracy values were evident in the right enthorinal cortex adjoining the parahippocampal gyrus (**Figure 6.3C**). Interestingly, high classification accuracies (near 90%) were also observed in some areas of the prefrontal cortex: the left and right middle frontal gyri (MFG), the left rostrolateral prefrontal cortex (RLPFC) and the right insula (**Figure 6.3D**).

Region	MNI coordinates			Cluster size	Max CVA (%)
	X	Y	Z		
Ventral visual stream					
R Lingual Gyrus	22	-62	0	108	90
R Inferior LOC	42	-64	-2	80	88
L Posterior Fusiform Gyrus	-22	-72	-18	85	86
R Anterior Fusiform Gyrus	42	-14	-26	72	83
L Anterior Fusiform Gyrus	-28	-30	-28	47	81
Dorsal visual stream					
L IntraparietalSulcus	-38	-68	34	96	88
R Precuneus	4	-48	54	49	81
Medial Temporal Lobe					
R Enthorinal Cortex/ParahippocampalGyrus	34	-16	-28	72	83
Prefrontal cortex					
R Middle Frontal Gyrus	30	16	38	136	90
L Middle Frontal Gyrus	-34	22	30	56	88
R Insula	40	-10	-4	56	88
L RostralateralPrefronal Cortex	-24	52	20	39	88

Table 6.2. Multivariate pattern analysis (MVPA) - Cortical areas. Brain regions showing significant differences in multivariate activation patterns of arithmetical problem solving between HFA and TD.

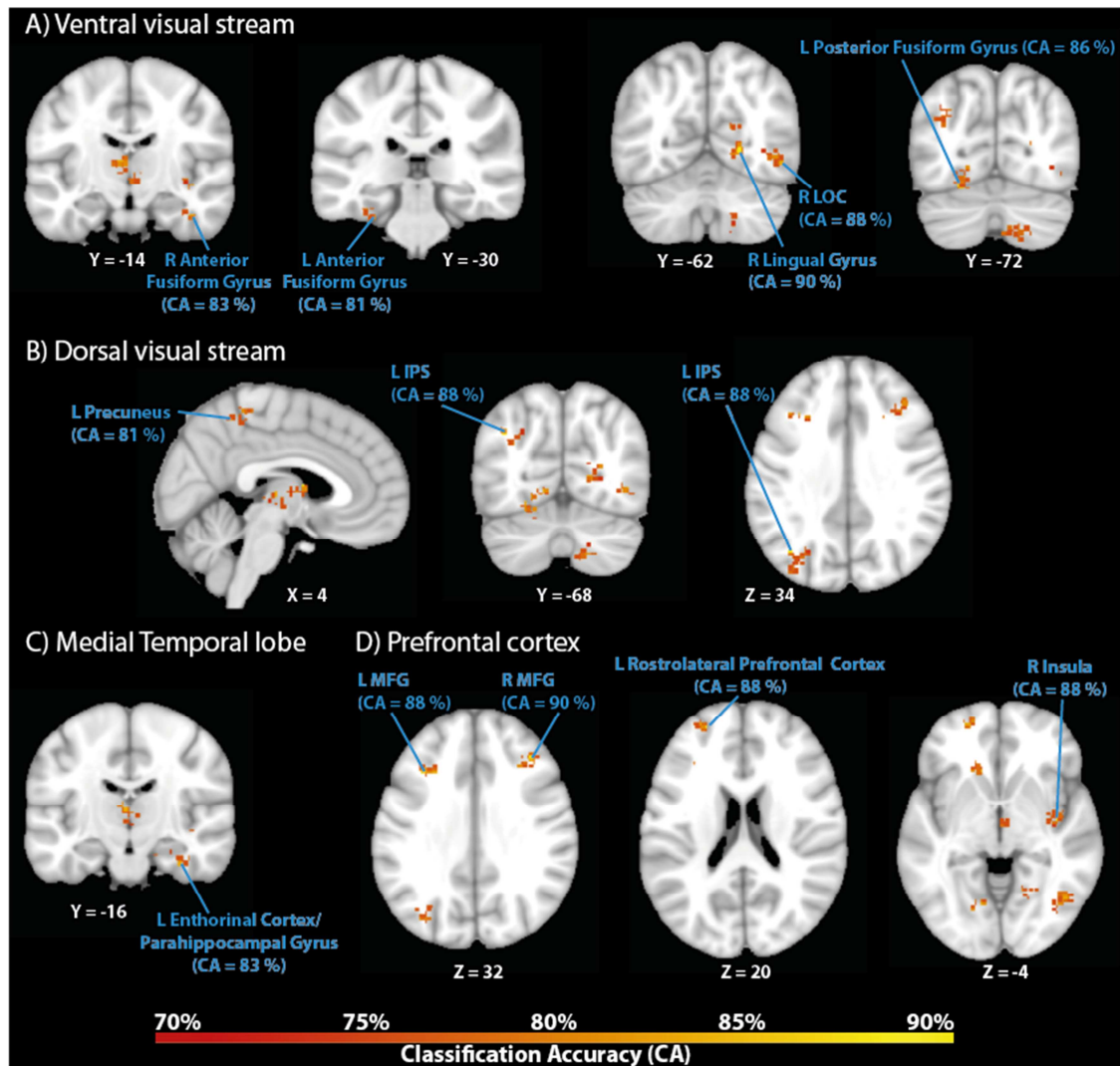


Figure 6.3. Distinct fine-scale neural representations between HFA and TD during arithmetical problem solving. Multivariate pattern analysis (MVPA) revealed significant differences in spatial activation patterns between HFA and TD in four brain complexes. **A)** Ventral visual stream: right and left anterior fusiform gyri, right lingual gyrus, right lateral occipital cortex (LOC), and left posterior fusiform gyrus; **B)** Dorsal visual stream: left intraparietal sulcus (IPS) adjoining caudal inferior parietal lobe (PGp), and right precuneus; **C)** Medial Temporal Lobe: right entorhinal cortex adjoining parahippocampal gyrus; **D)** Prefrontal areas: left and right middle frontal gyri (MFG), left rostralateralprefrontal cortex (RLPFC), and right insula. Peak classification accuracies (CA) are shown in parentheses. CA denotes Classification Accuracy (i.e. patterns of activation in these regions predicted groups' discrimination significantly above chance – e.g. with 83% accuracy).

Furthermore, few subcortical regions displayed differences in the spatial pattern of activation between the groups: the right cerebellum, the left thalamus and the left caudate (Table 6.3, Figure 6.4).

Region	MNI coordinates			Cluster size	Max CVA (%)
	X	Y	Z		
Subcortical areas					
L Caudate	-16	30	0	73	90
L Thalamus	-6	-10	8	199	90
R Cerebellum (Crus II)	18	-76	-46	155	83

Table 6.3. Multivariate pattern analysis (MVPA) - Subcortical areas.Subcortical brain regions showing significant differences in multivariate activation patterns during arithmetical problem solving between HFA and TD.

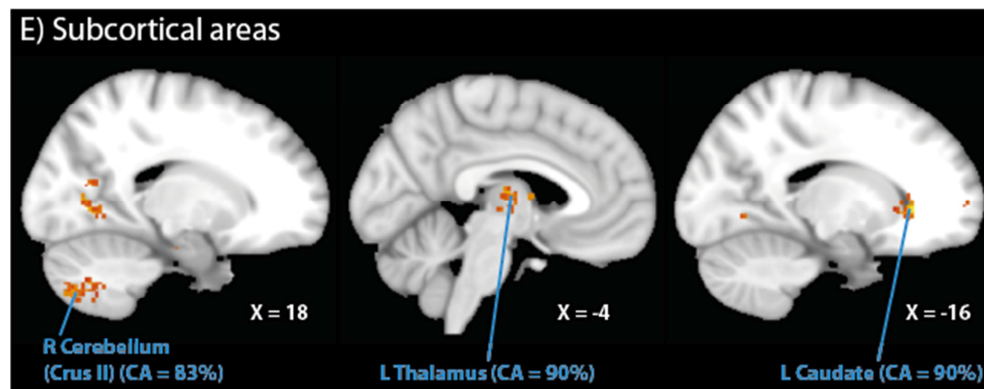


Figure 6.4. Multivariate pattern analyses – Subcortical . Multivariate pattern analysis (MVPA) revealed significant differences in spatial activation patterns between children with HFA and TD children in three subcortical areas. E) right cerebellum, left thalamus and left caudate.

6.9. Interim Discussion 5B

Behavioural analyses showed no significant differences between the two groups in their speed of responses or in their accuracy performance for complex or simple arithmetic. This is in line with previous studies that used arithmetic verification tasks as their baseline condition when investigating emotional processing deficits in Autism (Kennedy & Courchesne, 2008). Moreover, the fact that accuracy and reaction times were well matched between the two groups on the scanner task represents a critical point for our further analyses. Thus, on the basis of such tight performance matching, we can be confident that the differences seen in multivariate brain activity patterns across neuronal populations reflect a fundamentally distinct fine-scale representation associated with the condition of autism while performing these tasks.

In Experiment 5B, we compared brain responses in children with HFA and typically developing peers using both univariate and multivariate (Abrams et al., 2010;

Cho et al., 2011) approaches. Interestingly, at the individual voxel level, no brain regions displayed signal intensity differences between the two groups on the contrast of interest (Complex *versus* Simple problems). Yet, group differences were reflected in multivariate activity patterns across neuronal populations. Critically, such differences were prominent in areas of the ventral and dorsal visual streams (**Figure 6.3A,B**), in the medial temporal lobe (**Figure 6.3C**), and in areas of the prefrontal cortices (**Figure 6.3D**). These results indicate that the neural network activated by children with HFA involves regions previously associated with calculation tasks in typical populations of children and adults (Ansari, 2008; Dehaene et al., 2003; Zamarian et al., 2009). Yet, differences in neural patterns of activity were found within this network between children with HFA and their control peers suggesting a qualitative rather than a quantitative difference. This result is in line with previous studies on exceptional calculators (Cowan & Frith, 2009; Pesenti, Zago, Crivello, Mellet, Samson, et al., 2001).

In their original theoretical account, Baron-Cohen & Belmonte (2005) proposed that the ‘analytical autistic brain’ was the result of abnormally high activation in uni-modal low processing areas. According to them, support for this claim comes from studies of visual attention which reveal enhanced activity in ventral occipital cortex regions and abnormally low activity in the parietal and prefrontal cortices (*see also* Belmonte & Yurgelun-Todd, 2003). Moreover, this account could reflect the preference of ASD individuals for certain visual tasks over verbal tasks, as well as their propensity for perceptual rather than verbal reasoning (Mayes & Calhoun, 2007). In turn, this could reflect their reliance on visual rather than verbal strategies when solving given assignments (i.e. arithmetic problem solving) (see Experiment 5A). In line with this proposal, our results show neural fine-scale representational differences between children with autism and their typically developing peers in numerous regions of the ventral visual stream (**Figure 6.3A**). However, differences were also found in the parietal and pre-frontal cortices (**Figure 6.3B&D**), suggesting that the aforementioned proposal might not fully account for our data. This pattern of differences seems to be more in line with studies of calendrical savants showing that differences in neural activity would be prominent in areas of the dorsal stream, and particularly in the intraparietal sulcus (Cowan & Frith, 2009; *see also* Wallace, Happé & Giedd, 2009 *for*

an account on structural brain differences between talented savants and controls in this region).

Interestingly, the ventral and dorsal visual pathways have been reported to be crucial for the processing of category-specific information. Particularly, the anterior fusiform gyrus in the ventral visual stream, has been reported to play a crucial role within the network mediating face individuation (Haxby, Hoffman, & Gobbini, 2000; Kanwisher, 2000; Nestor, Plaut, & Behrmann, 2011); while, as mentioned earlier, the anterior segment of the intraparietal sulcus (see **Chapter I paragraph 1.4**), has been implicated in the manipulation of numerical and quantitative stimuli (Dehaene et al., 2003; Cohen-Kadosh, Cohen-Kadosh, Kaas, Henik, & Goebel, 2007; Piazza Pinel, Le Bihane, & Dehaene, 2007; *see also* Zamarian et al., 2009 *for a review*). In general, both pathways have been associated with the processing of symbolic stimuli such as words and letters (Halgren, Baudena, Heit, Clarke, Marinkovic, & Clarke, 1994; Polk, Reed, Keenan, Hogarth, & Anderson, 2002), graphemes (van Leeuwen, den Ouden, & Hagoort, 2011), characters and symbols (Kuriki, Takeuchi, & Hirata, 1998), and Arabic digits (Ansari, 2007; Rykhlevskaia, Uddin, Kondos, & Menon, 2009). Interestingly, Polk and colleagues (2002) claimed that letter identities and number identities are associated with different cortical mechanisms and that the fusiform gyrus responds more to letters than to digits, while the parietal lobe responds more to digits than letters (Polk, Reed, Keenan, Hogarth, & Anderson, 2000). However, a recent behavioural study has elegantly demonstrated, using a masked priming lexical decision paradigm, that the cognitive system that modulates the shape of the leet digits (numbers as part of words) and the one that is involved in letter symbols might be far less independent than it has been initially thought (Perea, Duñabeitia, & Carreiras, 2008), thus supporting the idea of a shared cortical substrate for symbol processing, possibly located within both visual pathways. This is similar to the proposal put forward by the ‘functional degeneracy’ theory which postulates that ‘there is more than one neuronal system for producing the same response’ (Price & Friston, 2002, 2003). Moreover, according to these authors, the cortical system devoted to a task might vary depending on the individual, with the greatest variability found between typical and atypical populations (Price & Friston, 2002, 2003). Furthermore, it has been proposed that brain areas devoted to the processing of symbols (i.e. words and numbers) and objects (i.e. faces),

might be subject to a high ‘competition for cortical space’ (Dehaene & Cohen, 2007), a notion that has been included in a theoretical framework termed the ‘cortical recycling model’ (Dehaene & Cohen, 2007). We could therefore speculate that the differences in spatial patterns of activation in the ventral and dorsal visual stream could be the consequence of intrinsic and extrinsic differences between children with HFA and their typically developing peers in shaping these visual pathways under the aforementioned neural mechanisms and constraints. Under this notion, we propose the following tentative explanation. The ventral visual stream, and particularly the fusiform gyrus, has been described as part of the neural system for social cognition, an area of reported weakness in autism (Castelli, Happè, Frith, & Frith, 2000; Semrud-Clikeman, Fine, & Zhu, 2001; van Rijn, Swaab, Baas, de Haan, Kahn, & Aleman, 2011). Moreover, hypoactivation of this area has been systematically shown in cases of ASD (Critchley, Daly, Bullmore, Williams, Van Amelsvoort, et al., 2000; Schultz, Gauthier, Klin, Fulbright, Anderson, et al., 2000; Schultz Grelotti, Klin, Kleinman, Van der Gaag, et al., 2003). Furthermore, it is well established that ventral visual areas are quite plastic and can be modelled by early experience (Fujita, Tanaka, Ito, & Cheng, 1992). Notably, individuals with autism pay much less attention to faces (Osterling & Dawson, 1994) and this might result in a ‘failure’ to acquire normal perceptual skills within this domain which will consequently affect the cortical maturation of these areas and presumably lead to their hypoactivation in certain tasks. Hence, inadequate attention to faces within the critical period for shaping the visual cortices could result in these areas becoming better tuned to other cognitive functions/representations (i.e. an excellent representation for numbers rather than a defective representation for faces). According to our reasoning, this process of ‘recycling’ (Dehaene & Cohen, 2007), coupled with the notion of ‘functional degeneracy’ (i.e. certain areas of the brain that are ‘silent’ or ‘semi-silent’ in the typical developing brain might get recruited in neurodevelopmental conditions with the consequence that the typically dedicated area serving that function will in turn remain ‘silent’), might help explain the different spatial pattern of activation in the dorsal visual stream. In this sense, a distinct recruitment (in terms of multivariate patterns of activation) of ventral visual areas by the ASD group while solving the arithmetic task should result in a different engagement of the typical ‘calculation areas’ in the parietal lobes. Under this notion, this cognitive function would utilize different

brain mechanisms functioning more effectively to suit the ‘analytical brain’. However, it is also possible that differences within the dorsal stream might reflect a higher reliance on these regions by the ASD group, as previously proposed by studies on ‘savants’ (Cowan & Frith, 2009; Wallace et al., 2009) and individuals with exceptional calculation abilities (Pesenti, et al., 2001). These hypotheses will be tested in Experiment 5C.

The MVPA analyses also showed differences between the two groups in two areas of the Medial Temporal Lobe (MTL). Given the behavioural results from the strategy assessment showing similar use of retrieval strategy in the two groups (**paragraph 6.4.4**), it is hard to predict that the differences seen in patterns of neural activity would be due to a different engagement of such areas in terms of their involvement in retrieval strategies (Cho et al., 2011; Gabrieli, Brewer, Desmond, & Glover, 1997; Kirwan & Stark, 2004). Yet, the studies that have related different involvements of MTL areas to different strategies used have specifically reported differences in the hippocampus, while our results show distinct pattern of neural representations between HFA and TD in the entorhinal cortex and the parahippocampal gyrus (**Figure 6.3C**). Interestingly, the parahippocampal gyrus has also been implicated in the recognition and processing of familiar faces (Brambati, Benoit, Monetta, Belleville & Joubert, 2010). Under this notion, the differences in multivariate activity patterns could be interpreted in line with the aforementioned proposal of higher reliance on ventral visual areas by the ASD group and could therefore reflect a different engagement of these areas by this group. This interpretation is similar to the original model proposed by Milner and Goodale (1995) which predicts ultimate connections of the ventral visual stream to MTL areas.

Interestingly, our multivariate analyses also show differences in spatial patterns of neural activity in areas of the prefrontal cortex: the bilateral middle frontal gyrus (MFG) and the left rostrolateral prefrontal cortex (RLPFC) (**Figure 6.3D**). The MFG has been largely implicated in the processing of social interactions (Semrud-Clikman et al., 2011; van Rijn et al., 2011) but also in sustained mnemonic responses and the storage of spatial information (Leung, Gore, & Goldman-Rakic, 2002). Under this notion, the decoded numerical information in lower level visual areas –by expert manipulation of maths equations through decomposition (Experiment 5A) – could subsequently be supported by higher level areas in the prefrontal cortex specifically devoted to the

storage of visuo-spatial information. The RLPFC is thought to play a critical role in the functional integration of diverse inputs of distinct mental representations. Interestingly, the RLPFC has been proposed to have privileged access to cognitive representations depending on its degree of coupling with other cortical regions (Wendelken, Chung, & Bunge, 2011). This result suggests that the RLPFC could represent the area responsible for the absolute need of ASD children to create principles and rules from concrete inputs and representations (*see* Baron-Cohen & Belmonte, 2005), which they can then easily apply when solving arithmetic problems.

Finally, the MVPA differences found in the subcortical regions (**Figure 6.4**) could be tentatively interpreted on the basis of the well documented involvement of these areas in the neurobiology of Autism. Specifically, we think differences in these areas could be reflecting intrinsic aberrancies in the neural profile of ASD – possibly within the resting state network (Pagani, Manouilenko, Stone-Elander, Odh, Salmaso, et al., 2011), rather than a specific involvement in solving the arithmetic task. Moreover, this explanation might also account for the differences in multivariate activity patterns found in the insula (**Figure 6.3D**).

In summary, our findings show that distinct fine-scale neural representations between HFA and TD children are evident during arithmetic problem solving in areas of the ventral and dorsal visual streams, in the Medial Temporal Lobe regions, and in areas of the prefrontal cortex, plus in few in subcortical areas. These results validate the idea of maths as a cognitive strength in individuals with autism. Moreover, they give support to the notion that describes ASD as individuals with a different information processing system (*see* Baron-Cohen & Belmonte, 2005; Frith, 1989). Indeed, our findings suggest that even if the same brain areas are engaged to the same extent in the two groups of children, arithmetic problem solving can evoke distinct fine-scale neural representations between HFA and TD children.

Experiment 5C. Decoding maths performance from hemodynamic signal change

6.10. Aim 5C

We demonstrated that children with High Functioning Autism (HFA) display a fundamentally different spatial pattern of brain activity compared to age and IQ matched typically developing peers when solving arithmetic problems (Experiment 5B). Notably, group differences were reflected in multivariate activity patterns across neuronal populations in areas of the ventral and dorsal visual streams, in the Medial Temporal lobe and in two areas of the prefrontal cortex. These areas have all been implicated in arithmetic problem solving in typical adults and children (Zamarian et al., 2009). However, the MVPA analysis provides no information on the ‘directionality’ of the data. In other words, from the MVPA analysis we can detect that there is a fundamentally different pattern of activation between the two groups but, unlike the GLM analysis of signal intensity, multivariate pattern analyses alone cannot reveal which brain areas within the aforementioned ‘calculation network’ would show a greater contribution to maths problem solving between our two groups. It is therefore important to determine whether differences in spatial patterns of brain activity can causally relate to maths performance on measures taken outside the scanner. In this section we will investigate the causal relation between brain areas’ activity patterns and mathematical abilities in children with HFA and their control peers using the Support Vector Regression approach (SVR) (see **paragraphs 1.9.1.1** and **6.11.2.2**).

6.11. Method 5C

6.11.1. Participants

The same two groups of participants described in paragraph **6.3.1** (**Table 6.1**) took part in Experiment 5C.

6.11.2. Experimental design

All stimuli, task parameters, and timing were identical to those described in Experiment 5B. Conditions were presented in blocks and the order of blocks was randomized within and between subjects.

6.11.2.1. fMRI data acquisition

Here we utilized the same functional brain images acquired in Experiment 3B on a 3T G.E.Signa scanner. The same acquisition parameters therefore apply.

6.11.2.2. fMRI data analysis

Support Vector Regression analysis. We used Support Vector Regression (SVR) analysis in order to predict continuous values of cognitive characteristics from the fMRI data (Cohen, Asarnow, Sabb, Bilder, Bookheimer, et al., 2010; Kahnt, Heinzle, Park, & Haynes, 2011; Valente, Martino, Esposito, Goebel, & Formisano, 2011). SVR is an extension of the Support Vector Machine (SVM) approach and is essentially a SVM regression which allows us to make real-valued predictions (Dosenbach, Nardos, Cohen, Fair, Power, et al., 2010).

SVM methods are non-probabilistic binary linear classifiers that can predict, given a set of data for each given input, which of two possible classes (i.e. ASD or healthy controls) the input is a member of. Essentially, the SVM training algorithm builds a model that assigns new examples into one category or the other (Coutanche, Thompson-Schill, & Schultz, 2011; Ecker, Marwuand, Mourao-Miranda, Johnston, Daly, et al. 2010; Uddin, Menon, Young, Ryali, Chen, et al., 2011) where each of the samples (i.e. subjects) is treated as a point in a multi-dimensional space. In the original SVM formulation, the maximally separating hyperplane is found in hyperspace, such that it is as far as possible from the closest members of both classes.

SVR retains some of the main features of SVM classification. Yet, a difference between SVM classification and regression is that in classification a loss penalty is incurred for misclassified data (points on the wrong side of the hyperplane), whereas in SVR a penalty is assessed for points too far from the regression line in hyperspace. This is achieved by defining a tube of epsilon width around the regression line in hyperspace.

Any data points within this tube carry a loss of zero, meaning that in such cases there is no penalty for being too far away from the regression line; while data points outside this tube will have a loss penalty. In essence, SVR performs linear regression in hyperspace using epsilon-insensitive loss. In SVR, the C parameter (regularization) controls the trade-off between how strongly points beyond the epsilon-insensitive tube are penalized and the flatness of the regression line (larger C allows the regression line to be less flat). In this experiment all SVR predictions used epsilon-insensitive SVRs with the Spider Machine Learning Toolbox (<http://www.kyb.tuebingen.mpg.de/de/bs/people/spider>) default setting of C (infinity) while epsilon was set to 0.00001. Leave-one-out-cross-validation was then used to estimate prediction accuracies (Di Martino, Ross, Uddin, Sklar, Castellanos, & Milham, 2008; Pereira, Mitchell, & Botvinick, 2009). During this process, each sample was designated the test sample in turns while the remaining samples were used to train the SVR predictor. Finally, the decision function derived from the training sample was then used to make a real-valued prediction about the test sample.

In Experiment 5C our predictors were defined from the MVPA analysis (Experiment 5B) by tracing a 6mm sphere around the classification peaks (**Table 6.2**). These values were subsequently regressed with the response variable of interest: performance on the *Numerical Operation* subtest of the WIAT scale for each group separately.

6.12. Results 5C

The SVR analysis revealed that mathematical performance in the HFA group was significantly predicted ($p < .05$) by the pattern of activity in two areas of the ventral visual stream: the left ($R^2 = 0.563$) and the right ($R^2 = 0.537$) anterior fusiform gyri; and in two areas of the prefrontal cortex: the right middle frontal gyrus (MFG) ($R^2 = 0.653$) and the left rostrolateralprefrontal cortex (RLPFC) ($R^2 = 0.5426$).

In the TD group, none of the brain areas investigated was significantly predictive of their mathematical performance.

6.13. Interim Discussion 5C

Our SVR analysis revealed four brain areas whose functional activity predicts maths performance on the *Numerical Operation* subtest of the Wechsler Achievement scale in the HFA group only: the left and right anterior fusiform gyri in the ventral visual stream, and two areas of the prefrontal cortex, the right middle frontal gyrus and the left rostrolateral prefrontal cortex. These results provide support for the causal role of these areas which could only be inferred from the MVPA analyses and demonstrate their crucial involvement in arithmetic problem solving in the HFA sample. Moreover, these results are in line with the predictions made in **paragraph 6.9** and could therefore be explained in terms of the cortical competition within areas of the ventral visual stream (i.e. fusiform gyrus) and its consequent specialization for numerical stimuli (*see* Dehaene & Cohen, 2007). Moreover, it seems plausible to think that areas of the ventral visual stream are differently recruited by children with HFA compared to the typical population, as proposed by the ‘functional degeneracy’ framework (Price & Friston, 2002, 2003). Furthermore, both of these interpretations are in accordance with the perceptual expertise model proposed for the fusiform gyrus (Gauthier, Tarr, Anderson, Skudlarski, & Gore, 1999, Gauthier, Skudlarski, Gore, & Anderson, 2000). Through a series of functional magnetic resonance studies, Gauthier and colleagues have established that the neural activation of this area is selectively modulated by the person’s expertise with stimulus material (i.e. bird experts engage the fusiform gyrus more strongly when viewing birds than cars, while the reverse was true for car experts) (Gauthier et al., 2000). Even more interestingly, these authors demonstrated that neural activity in the fusiform gyrus can be selectively enhanced through extensive perceptual training to a class of novel objects (Gauthier et al. 1999). In our case, we could speculate that as ‘maths experts’, individuals with autism would have undergone a cortical reorganization of this area to become specialized for numerical stimuli. Moreover, the right fusiform gyrus has been explicitly associated with the processing of the visual form of mathematical symbols (Rykhlevskia, et al., 2009). Additionally, the causal relationship between neural activity in the fusiform gyri and maths performance’s proficiency in our HFA group supports the ‘analytical brain’ theory which highlights the close relationship between high activations in ventral visual areas and hypersystemizable abilities in these individuals (Baron-Cohen & Belmonte, 2005).

The causal link between the activation of prefrontal areas and mathematical performance in the HFA group could seem less expected given the ‘analytical brain’ theoretical framework, which predicts a higher reliance on ventral visual areas, together with smaller contributions of parietal and frontal regions to the task (Baron-Cohen & Belmonte, 2005). In this sense, a more plausible explanation might therefore be found in the cortical properties of regions in the ventral visual stream, and particularly within the fusiform gyrus (Dehaene & Cohen, 2007). In fact, it has been proposed that the cortical specialization of the fusiform gyrus is facilitated by top-down projections from higher level areas (i.e. the prefrontal cortices) and this is the mechanism that ultimately promotes the development of conceptual knowledge (Mahon & Caramazza, 2011). Under this notion, we could speculate that an interplay of bottom-up and top-down projections to and from the fusiform gyrus and areas in the prefrontal cortices (i.e. in our case the middle frontal gyrus and the rostrolateral prefrontal cortex) could favour the development of maths knowledge in this population. However, it is important to note that this is only a hypothesis, as functional connectivity analyses should be performed to directly assess the functional coupling between these regions.

6.14. Final Discussion

Consistent with previous descriptive (Baron-Cohen et al., 2001, 2003; Baron-Cohen, 2007) and anecdotal evidence (Sacks, 1986) our result empirically demonstrate that children with HFA are significantly better at mathematical tasks compared to their age and IQ matched controls, as reflected by their higher scores at the WIAT maths subtests (**Figure 6.1A**). This result supports the idea of mathematics as a domain of cognitive strength in this population (Baron-Cohen et al., 2001, 2003; Cowan & Frith, 2009; Jones et al., 2009) (**Figure 6.1A & B**) and is further validated by the finding of a greater use of decomposition strategies for arithmetic problem solving in HFA (**Figure 6.1C**). In line with the proposal of better systemizing abilities in ASD (Baron-Cohen & Belmonte, 2005), the use of a decomposition strategy could be explained by the tendency of this population to ‘partition’ the problem by focusing on the intrinsic details of the equation’s properties, rather than relying on a much less analytical approach (i.e. automatically retrieving the solution from memory, or using immature processes such as counting). This supports the ‘weak central coherence theory’ (Frith, 1989) as these

children seem to be highly focusing on the local details of the equation rather than its global properties. Additionally, this result is in line with the data on calendrical savants, who have been reported to break down calendars into ‘fragments’ of dates (Heavey, Pring, & Hermelin, 1999). More generally, this result might reflect the tendency of this population for solving calculation problems through visualization processes (Butterworth, 2006) and could in turn reflect proficient visual abilities, as it has been previously proposed for calculation prodigies (*see* Butterworth, 2006; Pesenti et al., 2001).

No previous study has investigated the neural correlates of maths abilities in Autism Spectrum Disorder in adults or children. Yet, the ‘analytical brain’ theory of ASD proposes an abnormally high activation in posterior regions (Baron-Cohen & Belmonte, 2005). Interestingly, higher reliance on posterior areas has been reported in ASD. For example, Kemner and colleagues (1995) demonstrated that when responding to auditory stimuli, ASD children would show an abnormal P3 component which generalized to the occipital sites of visual processing areas. This suggests either some perceptual filtering in an ‘all-or-none’ manner (Baron-Cohen, 2005), a hyper-sensitivity to sensory stimuli (Kemner, Verbaten, Cuperus, Camferman, van Engeland, 1995) or alternatively a critical – and aberrant - involvement of visual regions ‘in support’ or ‘in replacement’ to other cortical regions devoted to the task (Price & Friston, 2002, 2003). The ventral visual pathway has been reported to be crucial for the processing of category-specific information (*see* Dehaene & Cohen, 2011 *for a review*). In this respect one of the areas that has received great attention is the fusiform gyrus, which has been associated with the processing of the visual form of symbols such as words and letters (Halgren, et al., 1994; Polk, et al., 2002), graphemes (van Leeuwen, et al., 2011), and Arabic digits (Ansari, 2008; Rykhlevskaia, et al., 2009).

Moreover, the fusiform gyrus has been consistently implicated in the perception and processing of facial stimuli to the point of deserving its own ‘functional label’ – fusiform face area (FFA) (Kanwisher, 2000). Notably, performance deficits in face perception (Klin, Sparrow, de Bildt, Cicchetti, Cohen, & Volkmar, 1999) and facial expression recognition (Hobson, Ouston, & Lee, 1988) have been consistently reported in ASD mostly associated with hypoactivation of the FFA (Critchley et al., 2000; Shultz et al., 2003). The ventral visual cortices are quite plastic so differences or aberrancies in

the patterns of activation could be the consequence of the role of experience in shaping these areas (Grelotti et al., 2001). Thus, it has been shown that FFA seems to respond preferentially to any class of objects for which a person is perceptually an ‘expert’ (Gauthier et al., 1999, 2000). Even more interestingly, it has been demonstrated that young adults can enhance their FFA activity to a class of novel objects through extensive perceptual training (Gauthier et al., 1999). Consequently, inconsistent attention to faces (Osterling & Dawson, 1994) within the critical period of plasticity could in turn promote the cortical ‘recycling’ of these areas to serve other cognitive functions (i.e. the visual representation of different semantic information – numbers rather than faces-). This hypothesis seems to be validated by our SVR analyses which demonstrate that hemodynamic signal change in the anterior fusiform gyri can predict maths performance in HFA. Furthermore, higher recruitment of ventral visual areas by the HFA group could result in lesser engagement of areas that have been systematically implicated in arithmetical problems solving in the typically developing population, such as the intraparietal sulcus in the dorsal visual stream (Deahene et al., 2003; Zamarian et al., 2009), as proposed by the ‘functional degeneracy’ theory (Price & Friston, 2002, 2003). Indeed our multivariate regression analyses showed that activity in the dorsal areas was not predictive of maths performance in ASD. In addition to ventral areas, differences in spatial patterns of neural activity were also detected in the prefrontal cortex. Interestingly, the pattern of functional activity in the middle frontal gyri (MFG) and the left rostrolateral prefrontal cortex (RLPFC) significantly predicted maths abilities in the HFA group. Thus, the HFA ability of successfully decoding and representing numerical information through posterior visual areas would subsequently be integrated with higher level areas such as the MFG, which would be responsible for storing this information in a visuospatial format (Leung et al., 2002). The involvement of the RLPFC could represent the absolute need of HFA children for creating principles and rules from concrete inputs and representation, a crucial skill for succeeding in maths. Indeed the RLPFC seems to play a critical role in the functional integration of inputs from distinct mental representations (Christoff et al., 2001; Crone et al., 2009) and has also been implicated in abstracting general principles and rules from domain-specific details (Wendelken et al., 2011). Furthermore, the functional coupling between prefrontal regions and the fusiform gyrus could favour the cortical specialization of the

latter region, and ultimately promote the development of maths concepts in HFA by the creation of principles and rules, primarily through the RLPFC (*see Mahon & Caramazza, 2011 for a general description of this functional coupling*).

In summary, our data show that children with HFA are highly proficient in maths and utilize sophisticated strategies (i.e. decomposition) when solving arithmetic problems, which suggests a high reliance on visual and analytical processes (Baron-Cohen & Belmonte, 2005; Frith, 1989) (Experiment 5A). Moreover, given our findings from Experiments 5B & C we could tentatively suggest that in children with Autism, arithmetic problem solving is supported by a network of regions that are already devoted to maths in the typical population (*see Ansari, 2008; Zamarian et al., 2009*) (Experiment 5B). Yet, neural pattern of differences were found within this network between children with HFA and their control peers (**Figure 6.3**) suggesting a qualitative rather than a quantitative difference in line with previous studies on exceptional calculators (Cowan & Frith, 2009; Pesenti et al., 2001). Moreover, the neural orchestration of this network seems to be mediated by areas that are ‘semi-silent’ in the normal population through processes of neural reorganization (Price & Friston, 2002, 2003) and cortical recycling (Dehaene & Cohen, 2007, 2011) (Experiment 5C).

Together, these findings demonstrate that to possess special skills (i.e. mathematics) may be a distinguishing mark of ASD, both at a cognitive and at a neuronal level. The identification of such extraordinary abilities could provide researchers, and also parents, educators, and clinicians with a tool to facilitate the educational, professional and social success of this population.

6.15. Summary

In this Chapter we provide the first empirical evidence that children with ASD display exceptional maths abilities and adopt sophisticated calculation strategies compared to their typically developing peers (Experiment 5A). Moreover, we show that such an outstanding behavioural signature (i.e. exceptional maths abilities) is reflected in a distinct fine-scale neural representation for arithmetic problems in areas previously associated with calculation performance in typical adults and children (Experiment 5B). However, we show that this network of regions might be differently orchestrated in

children with HFA, suggesting a different processing system in this population which might develop as a consequence of neural reorganization and cortical recycling (Experiment 5C). These results demonstrate 'if' and 'how' maths skills are a distinguishing mark of ASD, both at a cognitive and neuronal level. Moreover, these results help to characterize the heterogeneous nature and great variability of maths abilities in the general population.

VII

Numeracy intervention in Developmental Dyscalculia and Low Numeracy

*"Life is good for only two things,
discovering mathematics and teaching mathematics"*
-- Siméon Poisson

There have been many attempts to raise the performance of children with Low-Numeracy (LN), however Developmental Dyscalculia (DD) has been largely neglected possibly because it is not widely recognized by educational authorities (Butterworth et al., 2011). In this Chapter we aimed at specifically characterizing the improvement of children with DD compared to their LN peers after a curriculum-based numeracy intervention program. The intervention lasted twelve weeks and consisted of a series of one-to-one thirty minute lessons. Entry, exit and follow up assessments were measured in terms of ‘numerical age’. Performance of both groups at the time of entry was significantly worse than expected for their chronological age. As expected, DD’s numerical age was lower than LN’s. After equating for ‘numerical age’ at entry we showed an overall effect of intervention. However, while the LN group improved to the level expected for their chronological age (CA), DD’s performance still showed a significant discrepancy. Interestingly, LN’s improvement declined with time: three months after intervention a significant discrepancy with CA recurred. Altogether our results show that Developmental Dyscalculia is more resistant to intervention, while children with Low Numeracy seem to benefit from an individualized, targeted and most importantly, continuous conceptual intervention. These results are the first to demonstrate the modulation of the effects of intervention on different types of numerical disabilities. Moreover, this study is the first of its kind that directly investigates the effects of intervention in a population of DD. Finally, our findings can inform the next round of studies in the field of educational neuroscience in its ultimate goal to investigate the possible coupling of behavioural improvements with changes at the brain level.

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7.1. Introduction

A large proportion of children going through UK schools fail to learn arithmetic to the expected level. The Every Child a Chance Trust Annual report reads: ‘Each year between 30,000 and 35,000 children aged 11 years old (6% of their age group) leave primary school with numeracy skills at or below the level expected of the average seven year old’ (Every Child a Chance Trust, 2011). Moreover, poor numeracy skills have been shown to be more of a handicap in the workplace, and in everyday life than poor literacy (Bynner & Parsons, 1997; Parsons & Bynner, 2005). There have been many attempts to raise the performance of children with low numeracy, although not specifically Developmental Dyscalculia.

In **Chapter II** we have seen how Developmental Dyscalculia could be thought of as a core deficit which manifests itself in a poor understanding of the cardinality property of numbers (Butterworth, 2005; Landerl, Bevan, & Butterworth, 2004). This core impairment results in great difficulties grasping arithmetical concepts due to arithmetical operations depending on the correct manipulation of sets and their numerosities (**Chapter I**). The proposal of a core deficit comes from studies pointing in the direction of an innate capacity for understanding numerosities. For example, many infant studies have demonstrated that the basic capacity for understanding the numerosity of a given set is innate (Antell & Keating, 1983; Izard, Dehaene-Lambertz, & Dehaene, 2008; Starkey & Cooper, 1980; Xu & Spelke, 2000). Moreover, evidence from non-enumerate cultures (Butterworth, Reeve, Reynolds, & Lloyd, 2008; Pica, Lemer, Izard, & Dehaene, 2004) and animal studies (Agrillo, Dadda, & Bisazza, 2007; Matsuzawa, 1985) also seem to point to the direction that these basic numerical capacities could be innate. There is also genetic evidence for the inheritance of basic numerical abilities (Alarcon, Defries, Gills Light, & Pennington, 1997; Shalev, Manor, Kerem, Ayali, Badichi, Friedlander, & Gross-Tsur, 2001; Shalev, Gross-Tsur, 2001). Moreover, genetic regression models show that about 30% of the genetic variance in twin studies of cognitive abilities in 7 year olds is explained by factors specific to maths (Kovas, Harlaan, Petrill, & Plomin, 2006). Furthermore, various X-chromosome disorders seem to affect numerical abilities more than other cognitive functions (Mazzocco & McCloskey, 2005; Rovet, Szekely, & Hockenberry, 1994) as

demonstrated by cases of Turner syndrome (Bruandet, Molko, Cohen, & Dehaene, 2004), Fragile X (Mazzocco & McCloskey, 2005), Klinefelters Syndrome and other extra X conditions (Semenza, *personal communication*). Additionally, brain imaging studies seem to suggest that Developmental Dyscalculia (DD) has a neurological basis given that functional (Kucian, Loenneker, Dietrich, Dosch, Martin, & von Aster, 2006; Price, Holloway, Räsänen, Vesterin, & Ansari, 2007) and structural (Isaacs, Edmonds, Lucas, & Gadian, 2001; Rotzer, Kucian, Martin, von Aster, Klaver, & Loenneker, 2008) abnormalities have been found in the parietal lobes of children with DD.

Although there is little longitudinal evidence, it seems that Developmental Dyscalculia persists into adulthood (Shalev, Manor, & Gross-Tsur, 2005; Cappelletti, *personal communication*). The question that arises is whether it is possible to remediate- through extensive and targeted intervention- a condition that seems to be congenitally determined.

On the contrary, more general low numeracy (LN) skills do not seem to be congenital but more the consequence of a more general learning impairment (Donlan, Bishop, & Hitch, 1998; Donlan, Cowan, Newton, & Lloyd, 2007; Geary, 1993; Geary & Hoard, 2005; Geary, Hoard, & Hamson, 1999; Hitch & McCauley, 1991; McLean & Hitch, 1999; see also **Chapter III**), or the result of difficulties in learning and dealing with culture-based means of representing numbers (i.e. learning numerical symbols and their association with the numerosities they represent) (*see* Rousselle & Noël, 2007). In **Chapter II**, we outlined the behavioural differences between Developmental Dyscalculia and Low Numeracy providing evidence that they are two distinct maths impairments.

Unfortunately, in most of the cases, it is unlikely that such difficulties can be compensated for by the standard teaching curriculum and therefore these children are left struggling with their learning disability and often classified as unmotivated and lazy (Bevan & Butterworth, *personal communication*; *see also* Butterworth, 2003). Moreover, the importance of a properly numerate and maths trained society promotes scientific and economic progress (Beddington, Cooper, Field, Goswami, Huppert, et al., 2008; Duncan, Dowsett, Clawaawna, Manuson, Huston, et al., 2007; Parsons, & Bynner, 2005).

It follows that there is a need for targeted and individualized intervention programs aimed at raising the standard of the lowest attaining students for (1) their personal benefit, but also on a larger scale, (2) for promoting an increase in the GDP¹⁶ growth (OECD, The High Cost of Low Educational Performance: The Long-Run economic impact of Improving Educational Outcomes, 2010).

Despite an enormous literature on maths intervention, no studies have drawn a sharp distinction between the aforementioned two conditions (Developmental Dyscalculia and Low Numeracy). For example, Kucian and colleagues (2011) conducted an fMRI study on 9 year olds who were trained using a specially designed computer game that stressed number to space associations (i.e. an extensively reported characterization of numerical proficiency, see also **Chapter VIII**). The effects of intervention were effective for both dyscalculic and typical learners both behaviourally and neurologically. However, the dyscalculic group did not improve to the level of the typically developing children in terms of their behavioural nor their neuronal profile (Kucian, Grond, Rotzer, Henzi, Schönmann, et al., 2011). Particularly, the dyscalculic group displayed more frontal, rather than parietal activation, compared with their typically developing peers, which reflects the recruitment of less efficient, specialized and mature brain processes for problem solving (Rivera, Reiss, Eckert, & Menon, 2005). One of the *caveats* of this study was that the dyscalculic sample comprised pupils who were only 1.5 standard deviations below average according to standardized maths tests. These tests usually base their diagnosis on a composite score calculated by adding the scores of several subtests which comprise items on counting, number comparison, but also oral and written calculation, verbal problem solving, etc. They are therefore not ideal for diagnosing Developmental Dyscalculia in terms of its core problem. Thus, Kucian and colleagues might have been including many children with Low Numeracy rather than strictly DD in their sample.

Here we ask: are there any differences in the benefits of intervention between Developmental Dyscalculia and Low Numeracy (Experiment 6A)? Moreover, given a theoretically driven diagnosis based on the concept of the core ‘number sense’, are there

¹⁶Gross Domestic Product.

any differences in the outcome of the intervention programs between children with DD and children with LN after equating for maths performance (Experiment 6B)?

Experiment 6A. Targeted numerical intervention in Developmental Dyscalculia and Low Numeracy

7.2. Aim 6A

There have been few attempts to raise the performance of children with Developmental Dyscalculia (DD). Critically, only small scale research in specialist schools and by specialist teachers (Bird, 2007; Butterworth & Yeo, 2004; Yeo, 2003) seem to suggest that dyscalculics, like dyslexics, need specialized help to optimize their learning and even so, their performance usually does not improve to the expected level for their age. The objective of Experiment 6A was to screen learners receiving a targeted, conceptual curriculum-based intervention (*Numbers Count*) for low numeracy skills to assess how effective it is for Developmental Dyscalculia as compared with Low Numeracy (see **Chapter II**).

Developmental Dyscalculia has been described as a congenital condition that manifests as a core deficit in the ability to grasp the basic concepts of numerosity (see Butterworth, 2005) and therefore might be more resistant to intervention. On the other hand, Low Numeracy does not seem to be a condition that is neurobiologically nor genetically determined therefore it is more likely that it can be ameliorated with the appropriate type of intervention.

7.3. Method 6A

7.3.1. Participants

A total of 20 schools in the London boroughs of Hackney and Southend-On-Sea signed up for the Numeracy Intervention program (*Numbers Count*).

Participants were selected by teachers to be part of the experimental intervention program developed by Every Child a Chance Trust and funded by charities and the UK government (see **paragraph 7.3.2.1**). Specifically, teachers were asked to nominate

children who they felt were of average general ability but had serious difficulties during numeracy lessons and could benefit from targeted one-to-one numeracy intervention.

In Experiment 6A, a total of eighty children in their 2nd year of primary school (mean age = 6.645, SD = 0.36) were screened for Developmental Dyscalculia or Low Numeracy using the *Dyscalculia Screener* test (Butterworth, 2003) (see **Chapter II**).

The *Dyscalculia Screener* software (Butterworth, 2003), which diagnoses Developmental Dyscalculia on the basis of UK norms was used to assign the children to the two groups of interest: Developmental Dyscalculia and Low Numeracy.

To be classified as dyscalculic (DD), children had to obtain a standardized score below 81¹⁷ on at least one of the two tasks of the ‘capacity subscale’ (either Dot Enumeration or Number Comparison) of the *Dyscalculia Screener*. The *Dyscalculia Screener* identified thirteen DD learners (mean age = 6.705, SD = 0.334).

To be assigned to the Low Numeracy (LN) group, participants had to obtain a standardized score below 81 on the ‘achievement subscale’ of the *Dyscalculia Screener*, but a standard score above 81 on both capacity tests. The software identified sixty-seven LN children (mean age = 6.63, SD = 0.37). The studies in Experiment 6A were approved by the UCL Ethics Committee.

7.3.2. Experimental design

7.3.2.1. Standardized diagnostic tasks

The *Dyscalculia Screener* was chosen as the standardized task to measure mathematical abilities and to provide a theoretically based diagnosis for Developmental Dyscalculia or Low Numeracy (please refer to **Chapter II** for a detailed description of the test and the tasks). Subsequent analyses on the raw data indicated that the two groups differed significantly on their speed of responses in the two Capacity tasks (*Dot Enumeration* and *Number Comparison*) where the DD group performed significantly slower than the LN group (**Table 7.1**). Moreover there was also a significant difference on the same two tasks for the composite measure calculated on accuracy and reaction times (i.e. inverse efficiency): the DD group showed a higher score compared to the LN groups (**Table 7.1**). Finally there was a significant difference on the accuracy measure

¹⁷Please refer to footnote 3 in Chapter II for a reminder on what the standardized score refers to here.

of the *Addition task* where the DD group showed a lower performance compared to the LN group (**Table 7.1**).

	DD (N = 13)			LN (N = 67)		
Measure	Accuracy	RTs	Inverse Efficiency	Accuracy	RTs	Inverse Efficiency
	Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)
Dyscalculia Screener						
Simple RTs	445.61 (66.515)			440.40 (117.27)		
Dot Enumeration	91.06 (4.00)	6287.73 (763.23)	69.16 (8.88)	89.97 (6.27)	3684.69 (813.4)	40.97 (8.83)
Number Comparison	87.44 (9.73)	2943.42 (1337.98)	35.33 (20.59)	87.61 (7.93)	1616.54 (380.6)	18.55 (4.46)
Exact Addition	57.69 (9.87)	5940.50 (4074.69)	94.89 (88.52)	70.58 (17.5)	7850.5(4278.7)	110.69(56.92)

Table 7.1. Dyscalculia Screener scores for the Simple reaction time test and the three numerical and arithmetical tasks (Dot Enumeration, Number Comparison and Exact Addition) for the DD and the LN groups. Numbers in Bold indicate significant differences between the groups. Accuracy indicates % of correct responses.

7.3.2.2. Targeted Curriculum-based Intervention – *Numbers Count*

The current National Strategy in the UK, particularly in England, has started to give special attention to children with low numeracy through the launch of the *Numbers Count* program, an individualized curriculum-based intervention developed by the Every Child Counts Trust (<https://everychildcounts.edgehill.ac.uk/>). This is a partnership between Edge Hill University and the Department for Education which aims of enabling the lowest attaining children in Years 1 to 3 to make greater progress towards expected levels of attainment in mathematics. The program provides one-to-one teaching for the children who really struggle with mathematics and may not reach Level 3 at the end of Key Stage 2 of the school curricula. For 12 weeks these children receive a daily individualized lesson for 30 minutes from a specialist ‘*Numbers Count*’ teacher who receives intensive training for 2 terms from a local teacher leader, and ongoing support thereafter from the ‘*Numbers Count*’ program.

The teacher assesses each child’s conceptual gaps in maths (see **paragraph 7.3.2.2**) and gives them targeted conceptual intervention which focuses on numbers understanding and arithmetic. The ad-hoc intervention covers topics that tap into basic numerical skills and follows a set routine. Moreover, the intervention uses special (and often concrete) material together with activities designed to specifically bridge each

child's learning gap. Activities usually include counting of concrete objects, manipulation of concrete material with numbers, number lines, number bounds, stressing the link between numerosities and numerals, basic arithmetic principles and rules and fact retrieval.

Lessons take place in a dedicated teaching area where children can use a wide variety of resources (see **Figure 7.1**).



Figure 7.1. Examples of activities and material used during the Numbers Count lessons. (Image created from images courtesy of Every Child a Chance Trusts website www.everychildachancetrust.org/every-child-counts).

7.3.2.3. Assessment task

Entry, exit and follow up assessments were measured by the Sandwell Early Numeracy Test–Revised (SENT-R) (Arnold, Bowen, Tallents, & Walden 2008). This test was initially developed to identify specific number skills that require targeted teaching and to evaluate the impact of interventions (Arnold, Bowen, Tallents & Walden, 2008). Hence, for children with special educational needs (i.e. dyscalculics or children with low numeracy) it can be used to identify targets for individualized educational plans. Most importantly, it can provide a standardized baseline to establish children's levels of numeracy at the start and end of a program and to monitor their progress throughout.

This test is designed for use with children between 4 and 12 years of age and measures five strands of basic numerical skills: 1) identification; 2) oral counting; 3) place-value/computation; 4) object counting; 5) language (see **Appendix 1A**). The

SENT-R is administered on a one-to-one basis and comprises a total of 68 questions across nine different National Curriculum levels from P6 to 2A (see **Appendix 1B**).

Children take the SENT-R when they enter and exit the program. On Entry, it is normally administered by the *Numbers Count* teacher and on Exit by a trained teacher who has not taken part in the program. The score obtained by the pupil is reported in terms of ‘numerical age’. This variable indicates the average chronological age of a standardized national sample of children, across all ability ranges and not in the Numbers Count program, who achieved the same raw score when the test was standardized. Therefore a ‘numerical age’ score that is below age indicates that children’s attainment is below the average for their age. A ‘numerical age’ score that is close to age indicates that children’s attainment is on average for their age. Finally, a ‘numerical age’ score that is above age indicates that children’s attainment is above the average for their age. This logic will be followed in subsequent analyses as ‘numerical age’ represents our dependent variable (see **paragraph 7.4**).

7.3.2.4. Experimental procedure

The intervention lasted 12 weeks and consisted of skilled specialist one-to-one 30 minutes lessons. Every day, for twelve weeks, children in the lowest end of their maths class were taken from their regular maths lesson to have a targeted intervention session with a specially trained teacher. The *Dyscalculia Screener* assessment was administered either before, during, or after the intervention phase due to practical issues in liaising with the schools and the teachers. Unfortunately it was not possible to test all these children at once at the beginning of the school year. The intervention phases were based on the academic terms and therefore had three starting point: Autumn, Spring and Summer. The assessments took place between September 2009 and April 2010.

7.4. Results 6A

In Experiment 6A we looked at performance of all 80 participants. Our dependent variable of interest was ‘numerical age’ which is a composite measure of achieved abilities based on the National Curriculum (UK) and the chronological age of the pupil (see **paragraph 7.3.2.2**). ‘Numerical age’ was investigated in relation to the

abilities expected by the chronological age of the pupil at two time points: Entry (before intervention) and Exit (after intervention). Finally, we investigated the longevity of the effects of intervention by looking at the effect of intervention immediately after the training program had been completed (Time0) and three months after completion of the program (Time3). All these effects were investigated within and between our groups of interest: Developmental Dyscalculia and Low Numeracy.

7.4.1. ‘Chronological age’ versus ‘Numerical age’

In this analysis we contrasted ‘chronological age’ versus what has been defined as ‘numerical age’ (see **paragraph 7.3.2.2** and **7.4**) within and between subjects. As expected¹⁸, no difference was evident on the ‘chronological age’ variable between the groups ($t(78) = 0.651$, $p = .517$) (**Figure 7.2**).

Paired-sample t-tests showed that at time of Entry (i.e. before the intervention started) both groups (DD and LN) did not reach the level of proficiency expected for their chronological age: DD ($t(12) = 9.99$, $p < .001$) and LN ($t(66) = 10.96$, $p < .001$) (**Figure 7.2**). Moreover, independent samples t-tests revealed a significant difference between the two groups on their ‘numerical age’ where the DD group displayed a worse performance compared to the LN group ($t(78) = -2.018$, $p < .05$).

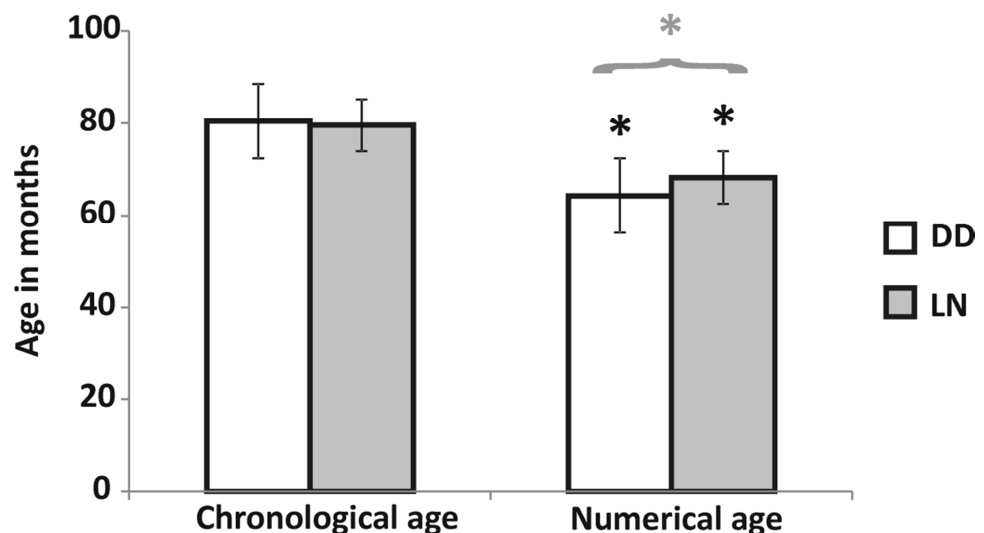


Figure 7.2. Numerical and Chronological age (in months) plotted against group. Error bars indicate 1 standard error of the mean. Black asterisks indicate significant differences between Chronological and Numerical age: neither group reached the expected performance for their chronological age (both $p < .001$). Grey asterisk represents significant difference between groups: the DD group’s performance was worse than LN ($p < .05$).

¹⁸All children were from Year 2.

7.4.2. Performance's change in DD and LN

In this analysis we were interested in looking at changes in performance before and after intervention within and between groups. Moreover, we wanted to quantify the level of improvement by comparing performance before and after intervention against chronological age. A 3 by 2 Repeated Measures ANOVA was performed with time of assessment as the within subject factor (3 levels – chronological age, performance before and after intervention) and group as the between subject factor (2 levels – DD and LN).

The results showed a main effect of performance change [$F(2,512) = 69.55$, $p < .001$, $\eta^2 = .478$], a main effect of group [$F(1,76) = 7.75$, $p < .01$, $\eta^2 = .093$], and a significant interaction [$F(2,512) = 8.799$, $p < .001$, $\eta^2 = .104$]. The DD group did not reach the proficiency expected for their chronological age, even though they did improve significantly after intervention (**Figure 7.3**). In contrast, LN group's performance improved significantly after intervention and ended up even better than expected for their chronological age (**Figure 7.3**). Performance between groups differed before and after intervention ($p < .01$) (**Figure 7.3**).

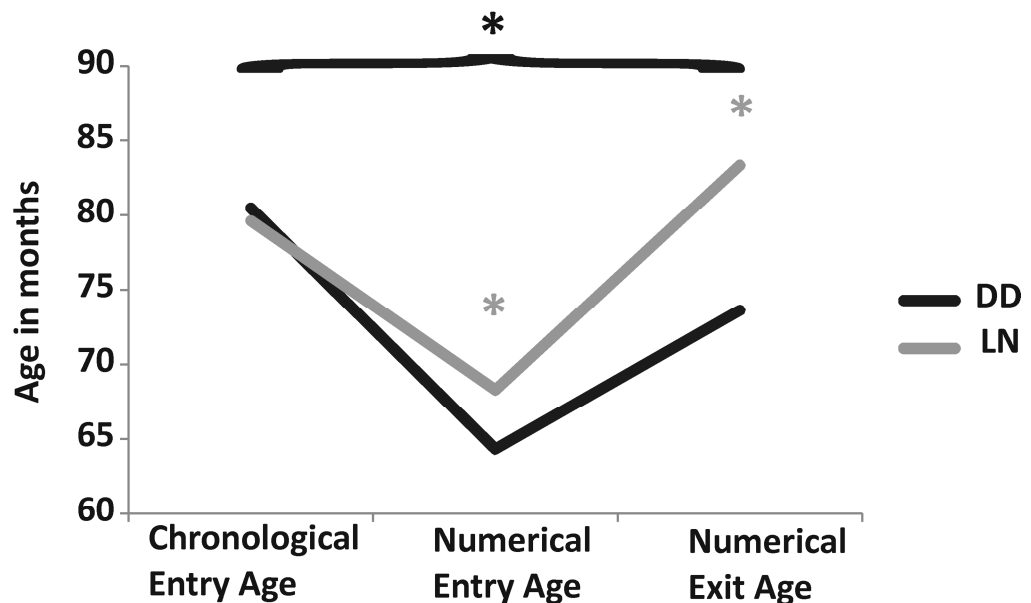


Figure 7.3. Change in performance by group. Black asterisk represents significant differences within groups: performance improved in both groups after intervention ($p < .001$). At Exit, LN's performance was significantly better than both Entry level ($p < .001$) and chronological age ($p < .005$). DD's performance at Exit was significantly better than Entry level ($p < .001$) but still significantly worse than chronological age ($p < .005$). Grey asterisk indicate significant differences between groups ($p < .01$).

7.4.3. Performance longevity in DD and LN

In this analysis we looked at performance's increase at Time0 (immediately after intervention) and at Time3 (three months after intervention). Performance's longevity was measured in terms of these two time points (immediately after intervention – Time0; and three months after intervention – Time3).

A 2 (time of measurement) by 2 (group) Repeated Measures ANOVA revealed a main effect of time of measurement [$F(1,54)^{19} = 6.915$, $p < .01$, $\eta^2 = .114$] and a main effect of group [$F(1,54) = 4.184$, $p < .05$, $\eta^2 = .072$]. The interaction was marginally significant [$F(1,54) = 2.237$, $p = .058$, $\eta^2 = .04$].

Subsequent pair-wise comparison analyses revealed that in the DD group the improvement at Time0 was small (see **paragraph 7.4.2**), but it did not decline over time (Time3) ($p = .522$). On the other hand, LN's improvement at Time0 was much greater (see **paragraph 7.4.2**), but it declined over time (Time 3) ($p < .001$) (**Figure 7.4**). The two groups differed at Time0 ($p < .01$) but not at Time 3 ($p = .976$).

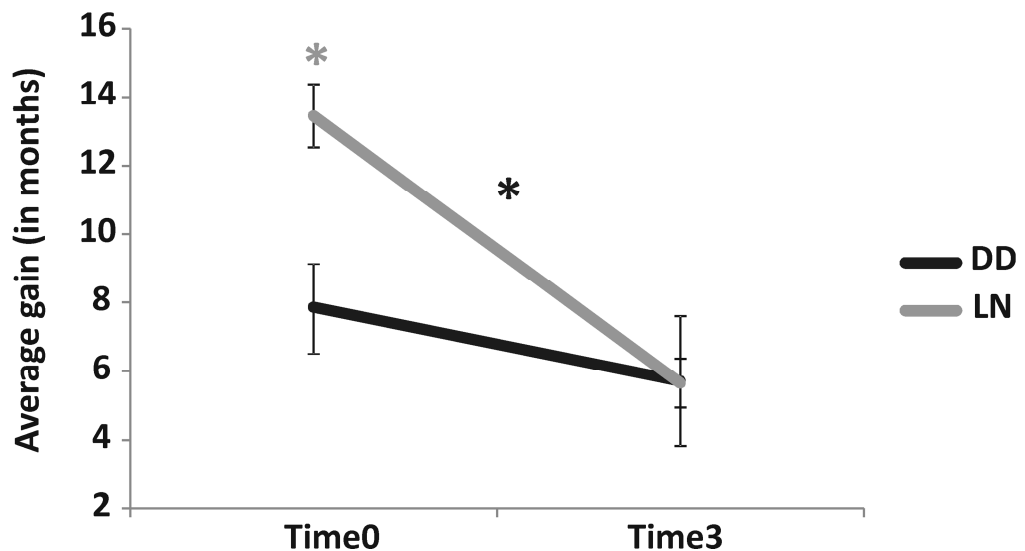


Figure 7.4. Performance's longevity measured as average increase in 'numerical age' (months) at Time0 (immediately after intervention) and at Time3 (three months after intervention) plotted against group. Error bars indicate 1 standard error of the mean. Black asterisk indicates the significant drop in LN's performance three months after intervention ($p < .001$). Grey asterisk represents significant difference between the groups at Time0 ($p < .01$).

¹⁹Please note that only a total of fifty-six children (7 in the DD group and 49 in the LN group) were tested three months after intervention. These statistical analyses therefore reflect the performance of a subset of the original data.

7.5. Interim Discussion 6A

Our first analysis aimed at quantifying the level of the maths deficits before intervention within and between groups (Developmental Dyscalculia *versus* Low Numeracy). We compared the ‘numerical age’ score obtained by our participants with their ‘chronological age’ scores. That is we contrasted their obtained score to the ideal score expected based on their age and the National curriculum goals of typical arithmetical development. All children were from the same school Year, and indeed no differences were found in terms of age (**Figure 7.2**). However, when comparing the numerical scores obtained at the Sandwell Early Numeracy Test to the scores expected for their age and the school curriculum, we show that both groups failed to reach the level of proficiency expected (**Figure 7.2**). As expected, children diagnosed with Developmental Dyscalculia had a worse performance compared to their peers in the Low Numeracy group (**Figure 7.2**).

In our second analyses we looked at the general effects of remediation (within group analysis). Moreover, we analyzed the diagnosis-specific benefits of the intervention program (between groups’ analysis). We therefore took participants’ performance before and after intervention and contrasted it with the scores expected by the chronological age of the pupil and their school curriculum. The results of this analysis revealed that the effects of intervention were modulated by group’s membership. Notably, children with Developmental Dyscalculia did not reach the numeracy level expected for their chronological age, even though they did slightly improve with intervention (**Figure 7.3**). On the contrary, children in the Low Numeracy group showed a massive improvement due to the numeracy intervention as indicated by their performance level which exceeded the one expected for their age at the time of Exit from the program (**Figure 7.3**).

Finally, our analyses tested the longevity of the improvement. Our findings indicate that the massive improvement experienced by the Low Numeracy group tended to quickly (and steeply) decline over time (three months after intervention) (**Figure 7.4**). Yet, the positive effects of intervention (although smaller) seemed to last longer for the dyscalculics (**Figure 7.4**).

Altogether, the results of Experiment 6A indicate that the *Numbers Count* intervention had a great benefit for children with low numeracy skills (LN group), yet it seemed to be only partly effective (or at least not resolving) for the dyscalculic children.

Experiment 6B. Equating performance on numerical age

7.6. Aim 6B

In Experiment 6A we have seen how the effects of targeted curriculum-based intervention can be differently beneficial depending on the type of maths difficulties. Previous investigations on the efficacy of intervention programs in low maths achievers have not drawn a clear distinction between general low numeracy skills and Developmental Dyscalculia. In fact, previous studies have been primarily concerned with the outcomes of intervention in mathematical learning disabilities (Wilson, Revkin, Cohen, Cohen, & Dehaene, 2006) or low numeracy (Räsänen, Salminen, Wilson, Aunio, & Dehaene, 2009) rather than in children with Developmental Dyscalculia as described by a theoretically driven diagnosis. In this respect, Experiment 6A represented the first attempt to specifically investigate the outcomes of numeracy remediation in pure Developmental Dyscalculia defined as a core deficit in understanding numerosities. The findings of Experiment 6A seem to point to the direction of Developmental Dyscalculia being more resistant to intervention compared to other forms of low numeracy skills. However, one could argue that these results could have been slightly biased in the sense that our two groups of interest (DD and LN) started off with a significant discrepancy in their standardized numerical age score. We therefore could not say, from Experiment 6A, whether the modulation found on the effects of intervention was solely due to the diagnosis or alternatively to the fact that the LN group started off better than the DD group aside from the given diagnosis.

In Experiment 6B we aimed to control for this potential confound by equating for scores on standardized numerical age between the two groups before intervention. This allowed us to test whether performance's change based on intervention is independently modulated by clinical diagnosis.

7.7. Method 6B

7.7.1. Participants

A total of twenty-six participants took part in Experiment 6B. They were a subsample of the eighty participants from Experiment 6A. The same criteria for groups' classification used in Experiment 6A applied here. Particularly, to be classified as dyscalculic (DD), children had to obtain a standardized score below 81²⁰ on at least one of the two tasks of the 'capacity subscale' (either Dot Enumeration or Number Comparison) of the *Dyscalculia Screener*. The *Dyscalculia Screener* identified thirteen DD learners (mean age = 6.705, SD = 0.334) (the same group from Experiment 6A). On the basis of the 'numerical age' at Entry (see **paragraph 7.3.2.3**) of the DD group, we adopted a one-to-one matching approach in order to identified a subset of 13 LN learners (mean age = 6.91, SD = 0.334) whose 'numerical age' at Entry did not differ from the one of the DD group (see **paragraph 7.8.1**). Subsequent analyses on the raw data from the *Dyscalculia Screener* indicated that the two groups differed significantly on their speed of responses in the two Capacity tasks (*Dot Enumeration* and *Number Comparison*) where the DD group performed significantly slower than the LN group (**Table 7.2**). Moreover there was a significant difference in the same two tasks on the inverse efficiency measure: the DD group showed a higher score compared to the LN groups (**Table 7.1**). There was also a significant difference on the accuracy measure of the *Addition task* where the DD group showed a lower performance compared to the LN group. Finally, there was a significant difference on speed of responses in the *Addition task* indicating that the LN group performed significantly slower than the DD group (**Table 7.2**).

The studies in Experiment 6B were approved by the UCL Ethics Committee.

²⁰Please refer to footnote 3 in Chapter II for a reminder on what the standardized score refers to here.

Numeracy intervention in Developmental Dyscalculia and Low Numeracy

	DD (N = 13)			LN (N = 13)		
Measure	Accuracy	RTs	Inverse Efficiency	Accuracy	RTs	Inverse Efficiency
	Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)	Mean (SD)
Dyscalculia Screener						
Simple RTs		445.61 (66.515)			464.23 (197.65)	
Dot Enumeration	91.06 (4.00)	6287.73 (763.23)	69.16 (8.88)	87.22 (6.61)	3487.8 (1017.9)	39.89 (10.97)
Number Comparison	87.44 (9.73)	2943.42 (1337.98)	35.33 (20.59)	87.44 (5.64)	1611.96 (458.7)	18.50 (5.30)
Exact Addition	57.69 (9.87)	5940.50 (4074.69)	94.89 (88.52)	71.16 18.8)	8128.5(4609.9)	109.02(54.82)

Table 7.2. Dyscalculia Screener scores for the Simple reaction time test and the three numerical and arithmetical tasks (Dot Enumeration, Number Comparison and Exact Addition) for the DD and the LN groups. Numbers in Bold indicate significant differences between the groups. Accuracy indicates % of correct responses.

7.7.2. Experimental design

All standardized tasks were the same as described in Experiment 6A. Moreover, the intervention (*Numbers Count*) as well as the intervention parameters were the same as described in Experiment 6A for both groups (Developmental Dyscalculia and Low Numeracy). The assessment task (Sandwell Early Numeracy Test) was also the same as in Experiment 6A.

7.8. Results 6B

7.8.1. ‘Chronological age’ versus ‘Numerical age’

As in Experiment 6A, this analysis contrasted ‘chronological age’ versus ‘numerical age’ (see **paragraph 7.4**) within and between subjects. As expected no difference between the groups was evident on ‘chronological age’ ($t(24) = 0.226$, $p = .823$). Moreover, ‘numerical age’ was equated at the time of Entry ($t(24) = -1.142$, $p = .889$) (**Figure 7.5**).

Paired-sample t-tests showed that at time of Entry (i.e. before intervention) both groups (DD and LN) did not reach the level of proficiency expected for their chronological age: DD ($t(12) = 9.99$, $p < .001$) and LN ($t(12) = 9.738$, $p < .001$) (**Figure 7.5**).

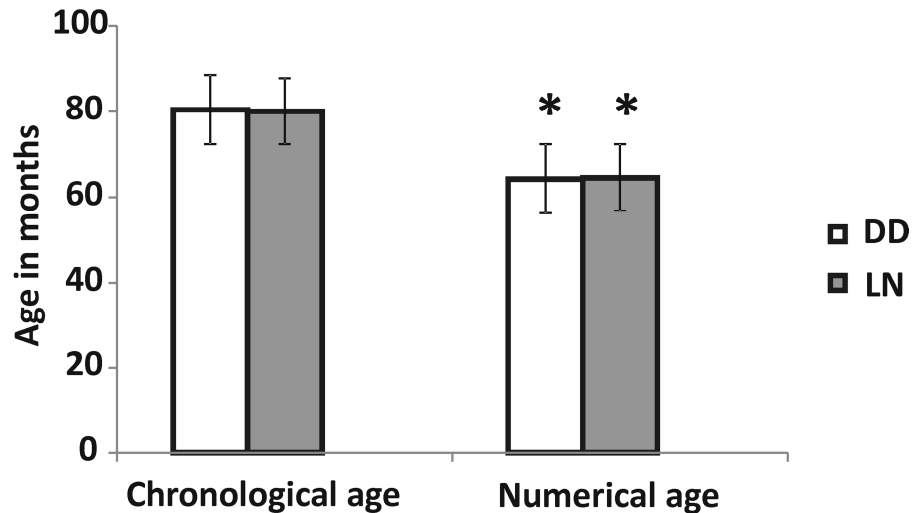


Figure 7.5. Numerical and Chronological age (in months) plotted against group. Error bars indicate 1 standard error of the mean. Black asterisks indicate significant differences between Chronological and Numerical age in both groups ($p < .001$).

7.8.2. Performance's change in DD and LN

In this analysis we were interested in looking at changes in performance before and after intervention within and between groups. Moreover, we wanted to quantify the level of improvement by comparing performance before and after intervention against chronological age. A 3 by 2 Repeated Measures ANOVA was performed with time of assessment as the within subject factor (3 levels – chronological age, performance before and after intervention) and group as the between subject factor (2 levels – DD and LN).

The results revealed a main effect of performance change [$F(2,48) = 88.39$, $p < .001$, $\eta^2 = .786$]. There was no main effect of group [$F(1,24) = .62$, $p = .439$, $\eta^2 = .025$]. The interaction was marginally significant [$F(2,48) = 2.75$, $p = .052$, $\eta^2 = .095$]. The DD group did not reach the expected level of performance for their chronological age ($p < .004$) (**Figure 7.6**). Yet, they did improve significantly after intervention ($p < .001$). In contrast, even after equating for 'numerical age' at Entry, the LN group improved to the level expected for their chronological age (or to a level that was not significantly different to the one expected for their chronological age) ($p = .416$). Performance between the two groups differed after intervention ($p < .05$) (**Figure 7.6**).

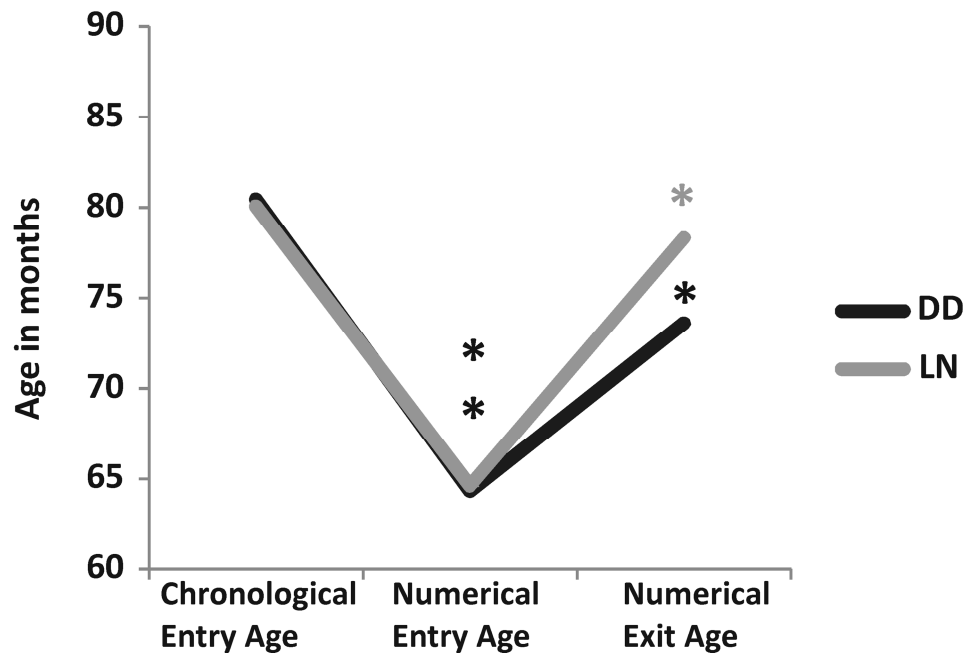


Figure 7.6. Change in performance by group. Black asterisk represents significant differences within groups. At Exit, LN's performance was significantly better than Entry level ($p < .001$) and it was not significantly different than chronological age ($p = .416$). DD's performance at Exit was significantly better than Entry level ($p < .001$) but still significantly worse than chronological age ($p < .001$). Grey asterisk represents significant difference between the groups at Exit ($p < .05$).

7.8.3. Performance longevity in DD and LN

In this analysis we looked at performance's increase at Time0 (immediately after intervention) and at Time3 (three months after intervention). Performance's longevity was measured in terms of these two time points (immediately after intervention – Time0; and three months after intervention – Time3).

A 2 (time of measurement) by 2 (group) Repeated Measures ANOVA revealed a main effect of group [$F(1,13)^{21} = 5.57, p < .05, \eta^2 = .03$]. Generally, the improvement of DD seems to be smaller than LN. There was no main effect of time of measurement [$F(1,13) = 2.975, p = .108, \eta^2 = .186$] and no significant interaction [$F(1,13) = .621, p = .445, \eta^2 = .046$]. The improvement did not decline significantly in either group, even if the slope was steeper for the LN group (**Figure 7.7**). No difference between the groups was found three months after intervention (**Figure 7.7**) ($p = .59$).

²¹Please note that only a total of fifteen children (7 in the DD group and 8 in the LN group) were tested three months after intervention. These statistical analyses therefore reflect the performance of a subset of the original data.

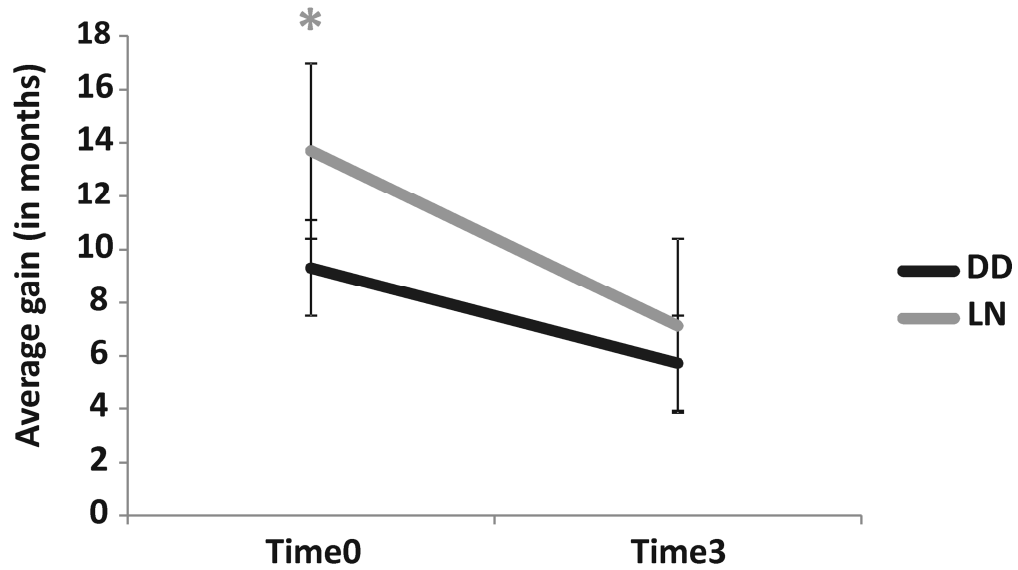


Figure 7.7. Performance's longevity measured as average increase in 'numerical age' (months) at Time0 (immediately after intervention) and at Time3 (three months after intervention) plotted against group. Error bars indicate 1 standard error of the mean. Grey asterisk represents significant difference between the groups at Time0 ($p < .05$).

7.8.4. Correlational analyses with standardized measures of numerical and arithmetical abilities

In this analysis we tested whether performance change correlated with participants' maths abilities. We therefore correlated 'numerical age' at Entry and 'numerical age' at Exit with performance on the tasks of the *Dyscalculia Screener* test - Dot Enumeration, Number Comparison and Exact Addition (see **Chapter II**).

Numerical Age at Entry significantly correlated with all measures of the *Number Comparison task* of the *Dyscalculia Screener*: Accuracy ($r = .393$, $p < .05$), Reaction Times ($r = -.502$, $p < .01$) and Inverse Efficiency ($r = -.528$, $p < .01$). None of the other correlations were significant.

Interestingly Numerical Age at Exit significantly correlated with two measures of the *Number Comparison task* of the *Dyscalculia Screener*: Reaction Times ($r = -.509$, $p < .01$) and Inverse Efficiency ($r = -.530$, $p < .01$), but not with Accuracy ($r = .28$, $p = .166$). Additionally, it correlated with the Accuracy measure of the *Addition task* ($r = .483$, $p < .05$). None of the other correlations were significant.

7.9. Interim Discussion 6B

Experiment 6B aimed at investigating the effects of intervention on Low Numeracy and Developmental Dyscalculia after controlling for numerical performance before intervention (Entry time point). Our first analysis ensured that there were no significant differences between the two groups in terms of standardized numeracy scores ('numerical age') and chronological age (see **Figure 7.5**). In other words, this analysis made us confident that our one-to-one matching procedure (see **paragraph 7.7.1**) worked successfully. Moreover, both groups' standardized scores before intervention were significantly below their expected age and curriculum criteria (**Figure 7.5**).

The second analysis directly tested the effects of numeracy intervention and its interaction with diagnosis (DD or LN). The results of this analysis showed that both groups improved with intervention (**Figure 7.6**). Yet, the improvement of the DD group was still not satisfactory given their age and curriculum expectancies. On the other hand, the LN group showed an adequate amelioration given the a priori criteria (**Figure 7.6**).

When looking at the sustained effect of intervention, we found that generally, the improvement seen in the DD group was smaller than the one in LN (**Figure 7.7**). Yet, contrary to Experiment 6A, in Experiment 6B we did not find a strong effect on the longevity of the remediation: the level of improvement did not seem to decline over time in either group (**Figure 7.7**). This suggests that the longevity of improvement might be highly related to the size of the improvement.

Finally we were interested in measuring the level of correlation between our main dependent variable ('numerical age') at its different time points and the measures that served to produce the relevant diagnoses (DD and LN). Interestingly, 'numerical age' measured before intervention significantly correlated with all measures of the Number Comparison task of the *Dyscalculia Screener* (accuracy, speed of responses and their cumulative measure – inverse efficiency) suggesting that the better the child performed on the Number Comparison task, the better their standardized numeracy score.

Even more interestingly, after intervention, ‘numerical age’ significantly correlated with the latency and inverse efficiency measures of the Number Comparison task but also with the accuracy measure of the Addition task.

7.10. Final Discussion

This Chapter investigated the potential benefits of a targeted, conceptual and curriculum-based intervention in two groups of children experiencing difficulties in mathematics: dyscalculics and children with low numeracy. The few studies which investigated the behavioural effects of various types of intervention programs have not properly differentiated Developmental Dyscalculia from other causes of Low Numeracy (Räsänen et al., 2009; Wilson et al., 2006). Moreover, the one study that has investigated the neural changes in brain activation after intervention has included children with mathematical learning disabilities irrespectively of the type of pathology (Kucian et al., 2011).

On the contrary, in the field of numerical cognition more and more evidence suggests the need for ad-hoc and theoretically based diagnoses mainly because of the heterogeneity of the discipline of mathematics and the different phenotypes of numerical disabilities encountered (Butterworth & Laurillard, 2010; Landerl, Fussenegger, Moll, & Willburger, 2009; but see also **Chapter II**). Unfortunately this notion has been somehow neglected by the one aspect of investigation that instead would find it most beneficial – intervention studies (*see above*). This is unfortunate as effective early intervention may help reduce the later impact on poor numeracy skills as it does in dyslexia (Goswami, 2006). Unlike dyslexia though, where the behavioural manifestation of the disorder is somehow ‘universal’ (i.e. slow or more error prone when reading a text), difficulties in mathematics (or more generally during numeracy lessons) can manifest in many different ways: an inability to deal with basic numerosities concepts, a difficulty in deciding which of two numbers is bigger, a difficulty in assigning the proper symbolic mean to a given numerosity, or problems in retrieving arithmetical facts from memory, and so on. Given the evidence provided above, it is clear that the most beneficial approach would lie in an *a priori* differentiation to assess the different benefits for the different learners’ type. Such approach will help to find the best way to help them during numeracy classes.

Recent neuroscientific studies suggest that numerical abilities depend on a specialized brain network (Castelli, Glaser, & Butterworth, 2006; Dehaene, Piazza, Pinel, & Cohen, 2003; Piazza, Izard, Pinel, Le Bihan, & Dehaene, 2004; Piazza, Mechelli, Butterworth, & Price, 2002; Piazza, Mechelli, Price, & Butterworth, 2006) whose basic architecture is innately specified. This raises the possibility of Developmental Dyscalculia being a congenital abnormality in this neural architecture (Isaacs, et al., 2001; Kucian, et al., 2006; Price, et al., 2007; Rotzer, et al., 2008) and consequently more resistant to intervention (or at least to behavioural intervention).

Indeed in Experiment 6A we see that when comparing pupils' numerical scores obtained at the Sandwell Early Numeracy Test to the scores expected for their age and the school curriculum, both the DD and the LN learners failed to reach the expected level of proficiency (**Figure 7.2**). Yet, the intervention seemed to be fully successful only for the LN children who showed a substantial improvement in performance. In contrast, the children in the DD group, although showing a slight amelioration, did not reach the level expected for their chronological age (**Figure 7.3**). Interestingly, the children with low numeracy seemed to strongly benefit from the intervention initially, yet with a declining longevity of this effect (**Figure 7.4**) suggesting that for these children intervention should be continuous.

In Experiment 6B we directly investigated the effects of intervention and their interaction with the type of maths impairment by equating numerical performance of the two groups before intervention. The result was in line with the findings from Experiment 6A as both groups showed an improvement (**Figure 7.6**). However, dyscalculics' performance after intervention was still not adequate for their age and curriculum expectancies. On the other hand, LN's improvement was much more substantial as it reached the given criteria (**Figure 7.6**), although the effect was not as pronounced as in Experiment 6A (**Figure 7.3**).

Altogether, Experiment 6A & B demonstrate that Developmental Dyscalculia seems to be more resistant to intervention compared to more general low numeracy skills. This suggests that different types of behavioural (or perhaps even neurobiological) interventions should be developed for this population. A promising behavioural approach could consist in devising adaptive software informed by the

neuroscience findings on the core deficit in Developmental Dyscalculia (Butterworth & Laurillard, 2010; Yeo, 2003).

7.11. Summary

The results of Experiments 6A&B confirm the existence of different types of numerical impairments that can be differently helped through intervention. We demonstrate that targeted, individualized, conceptual curriculum-based intervention was very beneficial for children with low numeracy, particularly if delivered continuously and constantly. Yet, our data show that the condition of Developmental Dyscalculia is much more resistant to intervention and that children in this group never reach the level expected for their chronological age if remediated through this type of intervention. This result lends further support to the proposal of Developmental Dyscalculia as a congenital condition characterized by a core-based deficit (Butterworth, 2005). All in all Experiments 6A and B demand for a clear distinction between different types of numerical impairments and demonstrate the resistance of Developmental Dyscalculia to the most common forms of behavioural interventions. This suggests that other types of interventions could be considered for the rehabilitation of this disorder (see **Chapter VIII**).

VIII

Neural stimulation for enhancing numerical performance

"With me everything turns into mathematics"
--Descartes

As previously mentioned, numbers are an essential aspects of our everyday life. Indeed, almost every walk of life requires the use of numerical information, from telling the time, through the use of money to complex computational procedures. It goes without saying that proficiency in maths is essential to succeed in life. In this Chapter we wished to investigate the potential benefits of utilizing non-invasive stimulation to the parietal lobes during an implicit learning paradigm to selectively enhance numerical abilities. We used transcranial direct current stimulation (tDCS) to change the polarity of the transmembrane neural potential in the parietal lobes. tDCS was applied during a 6 day training period where participants were learning an artificial number system. By changing the electrical polarity we influenced the level of excitability and modulated the firing rate of individual neurons in response to given inputs to either enhance (Anodal stimulation) or impair (Cathodal stimulation) numerical performance. Our results show that brain stimulation modified the acquisition of automatic number processing and the mapping of numbers to space. Importantly, the improvement was still present 6 months after the training. These results are important for the rehabilitation of developmental disorders in numerical cognition such as Developmental Dyscalculia.

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8.1. Introduction

As we have seen in previous Chapters (see **Chapter II**), up to 6.5% of the population struggle with basic numerical understanding, a disability termed Developmental Dyscalculia (Shalev, 2007; von Aster, & Shalev, 2007). A far higher number of the population (at least 15 to 20%) has difficulties that are less extreme or less specific, but which still cause significant practical, educational and later employment issues (Bynner & Parsons, 1997; Parsons & Bynner, 2005). Having numerical difficulties no doubt contributes to difficulties progressing in education, or leads to increased unemployment, reduced salary and job opportunities with tremendous costs for governments (Beddington, Cooper, Field, Goswami, Huppert, et al., 2008; Duncan, Dowsett, Claessens, Magnuson, Huston, et al., 2007; Parsons & Bynner, 2005). This makes numerical cognition and numerical disabilities an important field of study with valuable and relevant potential applications for society and education.

At a neuronal level we have seen that Developmental Dyscalculia is characterized by functional (Price, Holloway, Rasanen, Vesterinen, & Ansari, 2007) and structural (Rotzer, Kucian, Martin, von Aster, Klaver, & Loenneker, 2008; Rykhkevskaya, Uddin, Kondos, & Menon, 2009) abnormalities of the right parietal lobe. These correlative measurements are further supported by the indication of dyscalculic-like performance in healthy subjects following transcranial magnetic stimulation to the right parietal lobe (Cohen-Kadosh, Cohen-Kadosh, Schuhmann, Kaas, Goebel, et al., 2007). Moreover, the right parietal lobe seems to be important for the development of numerical skills already during infancy (Hyde, Boas, Blair, & Carey, 2010) and early childhood (Ansari, Garcia, Lucas, Hamon, & Dhital, 2005; Cantlon, Brannon, Carter, & Pelphey, 2006). Recent studies have shown activity modulation of the parietal lobes as a function of numerical information over development (Ansari, et al., 2005; Cohen-Kadosh, Lammertyn, & Izard, 2008; Rivera, Reiss, Eckert, & Menon, 2005), suggesting that later in development the left parietal lobe starts being recruited for these processes.

At a behavioural level, cognitive and developmental studies show automatic quantity processing as reflected by a Stroop-like effect in a numerical Stroop paradigm (Girelli, Lucangeli, & Butterworth, 2000; Rubinsten, Henik, Berger, Shahar-Shalev, 2002; Schwarz, & Ischebeck, 2003; Tzelgov, Meyer, & Henik, 1992). In these types of paradigms subjects are presented with two stimuli expressed as numerical digits and are

required to compare the stimuli according to their physical size (Numerical Stroop task) (see **Chapter I, paragraph 1.5**). A common finding, as reflected by reaction times is that incongruent trials are slower to be processed than congruent trials (congruity effect). This effect has been interpreted as an indicator that the subject processes numbers automatically even when the task does not require so. Therefore the congruity effect has been described as an index of proficiency with numbers. This idea has been corroborated by the findings that adults with Developmental Dyscalculia (Rubinsten & Henik, 2006) and typically developing children in their first year of school (Girelli, Lucangeli, & Butterworth, 2000) do not seem to show this effect.

Another signature of numerical proficiency seems to be an accurate mapping of numbers to space (see **Chapter IV and V**). Particularly, it seems to be a consensus in the field of numerical cognition that a good mastering of numerical information is characterized by a linear mapping of numbers to space (i.e. or less metaphorically to numbers onto a physical line – see **Chapter IV**). Moreover, numerate adults show a systematic spatial bias towards the larger number, while children show a bias towards the small number (de Hevia & Spelke, 2009).

In Experiment 7 we used transcranial direct current stimulation (tDCS) to affect numerical competence in healthy adults. In this brain stimulation technique, low-amplitude direct currents are applied via scalp electrodes and penetrate the skull to enter the brain. They modify the transmembrane neuronal potential (depolarize or hyperpolarize) and thus influence the level of excitability and modulate the firing rate of individual neurons in response to given inputs (Wagner, Valero-Cabre, & Pascual-Leone, 2007; but see also **Chapter I, paragraph 1.9.2**). Thus affecting behavioural performance depending on the type of stimulation (i.e. Anodal stimulation enhances performance while Cathodal stimulation impairs it).

Moreover, we coupled tDCS with a learning paradigm which used artificial digits (the Gibson figures) (Gibson, Gibson, Pick, & Osser, 1962) to investigate the development of numerical automaticity and the interaction between numbers and space in typically developing adults. Numerical automaticity was assessed by measuring the congruity effect in a Stroop-like task, while the interaction between numbers and space was tested with the *Number to Position* task described in details in **Chapter IV**. As mentioned above, numerical automaticity and the accurate mapping of numbers onto

physical lines are two well-documented behavioural signatures of numerical proficiency (*see above*). Our aim was to see whether the application of tDCS to the ‘right’ area of the brain can boost these basic numerical abilities, which in turn seem to be correlated with good maths skills.

Experiment 7. Enhancing numerical proficiency through direct current stimulation to the parietal lobes

8.2. Aim 7

In Experiment 7 we aimed at investigating whether brain stimulation (tDCS) can enhance numerical performances if applied during learning of new stimulus material (i.e. an artificial numerical system constructed using the Gibson figures in place of Arabic digits). Particularly, we were interested in delivering the direct current to two areas of the brain that have reportedly been implicated in number processing: the left and the right parietal lobes (Cohen-Kadosh, Cohen-Kadosh, Kaas, Henik, & Goebel, 2007; Piazza, Pinel, Le Bihane, & Dehaene, 2007).

Using the Numerical Stroop task with artificial digits we wished to investigate the emergence of a congruity effect and its modulation depending on the brain area stimulated. The Numerical Stroop task has long been used in the literature as a measure of number automaticity or automatic quantity retrieval (Girelli et al., 2000; Rubinsten et al., 2002; Tzelgov, Yehene, Kotler, & Alon, 2000). We therefore predict that if numerical performance was to be enhanced by stimulation to either the right or the left parietal lobes, this would be reflected in an earlier appearance of the congruity effect. On the contrary, if numerical performance was impaired by the stimulation, the congruity effect would be more likely to show the features that usually characterize populations with a less proficient numerical system (i.e. individuals with Developmental Dyscalculia, or very young children).

Using the mental number line task (*Number to Position*, see **Chapter IV and V**) with artificial digits we wished to examine whether the mapping of numbers into space followed a linear or logarithmic scale. Previous studies have suggested that a log-to-linear shift might occur due to exposure to critical educational material or culture-specific devices such as rulers or graphs (Dehaene, Izard, Spelke, & Pica, 2008; Siegler

& Opfer, 2003; see also **Chapter IV**). However, all studies that documented the log-to-linear shift involved populations that showed linear mapping due to extensively learned material (i.e., the digits 1-9 that are familiar from schooling) and/or symbolic knowledge of quantity (Dehaene, et al., 2008; Siegler & Opfer, 2003). The current paradigm allowed us to reveal whether brain stimulation can induce a performance that is characterised by linear fit independent of explicit exposure to critical educational material or culture-specific devices.

8.3. Method 7

8.3.1. Participants

Fifteen participants, right handed university students (mean age: 21.0 years, between 20-22 year-old) were randomly assigned to three groups of equal size ($N = 5$), and were stimulated simultaneously on the left and right parietal lobes in one of the following ways: 1) Right Anodal (RA) group (3 males; mean age 21.0) – who received excitatory (anodal) stimulation to the right parietal lobe of the brain, and inhibitory (cathodal) stimulation to the left parietal lobe of the brain for 20 minutes per day. 2) Right Cathodal (RC) group (3 males; mean age 21.0) – who received inhibitory (cathodal) stimulation to the right parietal lobe of the brain, and excitatory (anodal) stimulation to the left parietal lobe of the brain for 20 minutes per day. 3) Sham (control) group (2 males; mean age 21.0) – who received stimulation that lasted for 30 seconds per day. In this group, the electrodes were positioned in the same way as for the RA and RC groups. Subjects were informed that the experiment was designed to investigate effects of tDCS on cognition but were kept blind as to the specific relevance to numerical cognition and to the type of stimulation they were receiving. None of the participants reported significant neurological or psychiatric disorders. The study was approved by the local ethics committee at University College London and informed written consent was obtained for every subject before the start of each session.

8.3.2. Experimental design

8.3.2.1. Learning task

For this task we used nine artificial digits - Gibson figures (Gibson et al., 1962) which were arbitrarily assigned to the numbers 1-9 (**Figure 8.1**) and used as stimuli.









1	2	3	4	5	6	7	8	9
								

Figure 8.1. The artificial digits used as stimuli and their equivalent in everyday digits.

Subjects were unaware that these symbols represented numbers as was confirmed by a debriefing at the end of the experiment. The first session consisted only of the learning task as this session also included additional participant briefing regarding the experiment, the stimulation method, and health screening. Subjects were instructed to refer to the meaningless symbols (i.e., the artificial digits) as representing various magnitudes. Each trial began with a fixation point (in white ink) for 300 ms at the center of a black computer screen. Three hundred ms after the fixation disappeared two symbols (vertical visual angle of 2.63°) appeared on the computer screen, one symbol in the left visual field, and another in the right visual field. The center-to-center distance between the two digits subtended a horizontal visual angle of 9.7° . The symbol pair appeared and remained in view until the participant pressed a key (but not for more than 5 sec). Visual feedback (“Correct Answer”/“Mistake”/“No Response”) was provided for every trial for 500 ms. A new trial began 200 ms after the feedback. Each learning session was divided into 11 blocks of trials, each block consisting of 144 symbol pair comparisons (trials) that included 18 comparisons for each adjacent pair (e.g 1 2, 2 3, 3 4, etc.). The presentation in each block appeared in a random order. A training block with 48 trials was performed at the beginning of the task. Participants were instructed to choose the symbol they thought had a larger magnitude in each pair. They were asked to respond as quickly as possible but to avoid mistakes and to indicate their choices by pressing one of two keys (i.e., P or Q on the keyboard) corresponding to the side of the display with the selected member of the digit pair. The right answer appeared equal times on the right and left sides and all pairs appeared equally often. Participants were

provided with the average reaction time of the correct answers and percentage of errors after one third, two thirds and end of each block. The learning task was the first task to be done in all six sessions.

8.3.2.2. Experimental tasks

In addition to the learning task, sessions two through six included a numerical Stroop task (**Figure 8.2**) and the number line task (*Number to Position*) (**Figure 8.3**).

Numerical Stroop task. In the numerical Stroop task pairs of symbols (the Gibson figures – the same used in the learning task) appeared on the screen in the same manner as in the learning task, but the symbols were different in physical size (vertical visual angle of 2.2° or 2.75°). Subjects were instructed to choose the *physically* larger symbol by pressing either P or Q buttons as quickly and accurately as possible. While all the possible adjacent pairs were used (e.g., 1-2, 2-3, 3-4) in the learning phase, only non-adjacent pairs were used here (e.g., 1-3, 2-4, etc.) and congruent, incongruent, and neutral conditions were included in order to examine the possible generation of mental numerical representations (Tzelgov, et al., 2000). In a congruent pair, the numerically larger artificial digit was also physically larger. In a neutral pair, the digits differed only in the relevant dimension. In an incongruent pair, the numerically larger digit was physically smaller. The artificial digits that were the equivalent to the numbers 1 and 9 received the same classification during the learning phase (small, and large, respectively) and were not included in the analysis (Tzelgov, et al., 2000). The three conditions appeared the same number of times, with the right answer appearing equal times on the right and left sides and all pairs appearing equally often. No feedback was given on the performance in this task.

Number line task. In the *Number to Position* task, subjects mapped symbols onto a horizontal line displayed on the computer screen. The symbol corresponding to number one was placed at the left and the symbol representing number nine at the right end of the line (**Figure 8.3**). Subjects were instructed to place each of the remaining seven symbols on this line according to their magnitude. Symbols to be mapped appeared above the right and left end of the line in a randomised order to avoid any bias in responses that might arise due to stimulus location (Nichelli, Rinaldi, & Cubelli,

1989). Each symbol appeared 3 times at each location, making 42 line bisection trials in total for each session. No feedback was given on the performance in this task.

Control task: Every-day digits. On the last day, after the completion of the aforementioned tasks, the same numerical Stroop task (**Figure 8.2**) and the number line task (**Figure 8.3**) were administered to all three groups of participants. In this case, the stimuli were everyday digits instead.




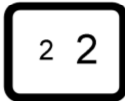

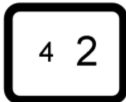
	Artificial Digits	Everyday Digits
Congruent		
Neutral		
Incongruent		

Figure 8.2. An example of the conditions in the numerical Stroop task with Artificial Digits (left column) and Everyday Digits (right column). The subject was required to determine which of the two stimuli was physically larger. The stimuli presented may be congruent, neutral or incongruent in relation to their physical size and semantic value.

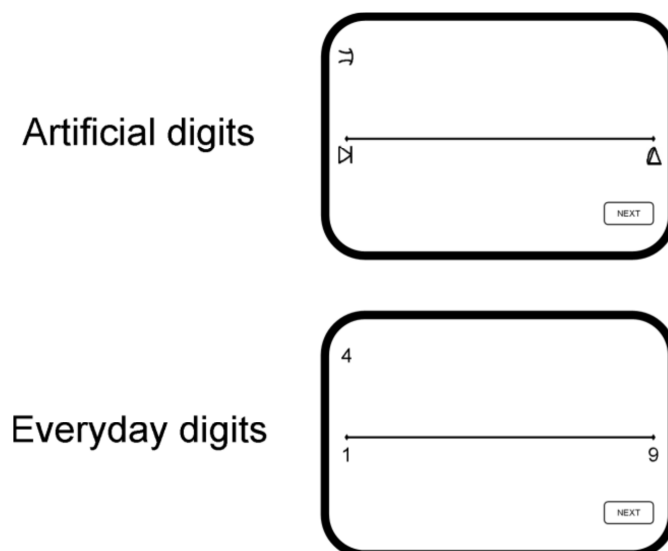


Figure 8.3. Number line task (*Number to Position*) with Artificial Digits (Top) and Everyday Digits (Bottom). Subjects were asked to map the given symbol, which appeared randomly at the left upper corner, as in the current example, or at the right upper corner, on the physical line. Subjects were instructed to place each symbol on this line according to its magnitude in relation to the presented symbols at the edges of the line.

8.3.2.3. Experimental procedure

The study consisted of six sessions for each subject. The sessions lasted approximately 120 minutes each (including electrode placement, the learning phase, and the testing phase) and were distributed over a 7 day period. Each subject attended one session per day apart from a break after the 4th day. The experiment always started on Tuesday and finished the following Monday (Saturday was the day of break for all subjects). All subjects were tested between 9am and 6pm. The first session only consisted of the learning task. During session 2-3-4 and 5 we administered the learning task plus the two experimental tasks with artificial digits (numerical Stroop and number line task). Finally on the last session (day 6), two additional tasks were added to the aforementioned protocol: the numerical Stroop and the number line task with every-day digits (see **Figure 8.4** for a schematic outline of the experimental design).

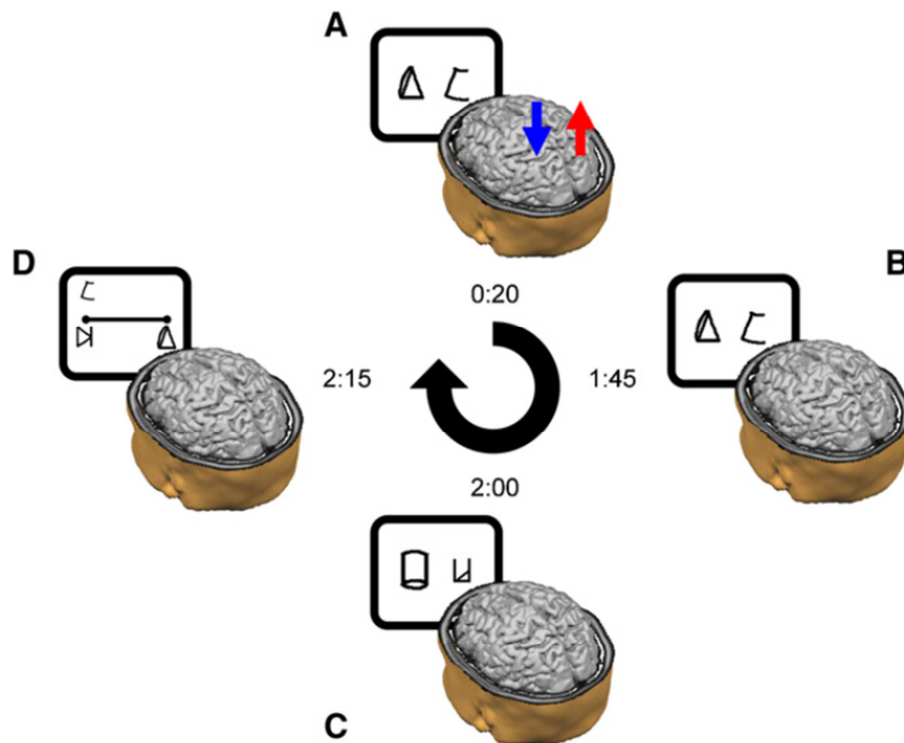


Figure 8.4. Schematic outline of the experimental design in a typical daily session (A) tDCS was delivered for 20 min from the start of the training. In this example, anodal stimulation was applied to the right parietal lobe (red arrow), whereas cathodal stimulation was delivered to the left parietal lobe (blue arrow). (B) The training continued after the termination of the stimulation. (C and D) Once the training ended, the subjects performed the numerical Stroop task (C) and the number line task (Number to Position) (D). The time next to each image reflects the elapsed time from the beginning of the daily session until its termination in a cumulative fashion.

8.3.2.4. tDCS stimulation parameters

Direct current was generated by a Magstim stimulator (The Magstim Company Ltd, UK) and delivered via a pair of identical, rectangular scalp electrodes (3x3 cm) covered with conductive rubber and saline soaked synthetic sponges.

For all groups at the beginning of the stimulation the current was increased slowly during the first 15 sec to the stimulation threshold (1mA) (ramp-up), and at the end of the stimulation the current was decreased slowly to 0mA during last 15 sec (ramp-down). Between the ramp-up and ramp-down constant direct current (1mA) was delivered for 20 minutes at the beginning of each session (i.e. at the beginning of the learning task). The latter type of stimulation was not applied to the sham group. This group therefore only received the slow current increase up to 15 sec to give the sensation of the stimulation. The stimulation was then automatically stopped.

The current was delivered through a pair of saline soaked sponge electrodes. Electrodes were positioned over the left and right parietal lobes according to the 10-20 EEG procedure on the sites corresponding to P3 and P4 respectively. We chose to place the cathodal electrode on the parietal lobe, and not on the prefrontal cortex, in order to not affect the mechanisms that might relate to learning, and feedback/reward which was provided during the learning phase (Albert, Robertson, & Miall, 2009; Duncan, 2001). In addition, the placement of the electrodes over both parietal lobes increases the specificity of the type of stimulation to each lobe, and increased its effect by increasing the current density (Nathan, Sinha, Gordon, Lesser, & Thakor, 1993).

Although stimulation ended during the learning task, electrodes were kept in place until task completion in order to avoid participant bias. The same set up applied for all groups and the subjects were unaware of not receiving full stimulation. All subjects reported a slight tingling sensation during the stimulation, which diminished rapidly due to habituation. No other discomforts or adverse effects were reported.

8.4. Results 7

8.4.1. Learning task

The learning of each participant was assessed by fitting the performance using the following power law function (Cohen-Kadosh, Sagiv, Linden, Robertson, Elinger, Henik, 2005; Newell & Rosenbloom, 1981):

$$RT = B*(N)^{-C}$$

In this equation, RT represents the mean Reaction Time in a given block, B is the performance in time on the first block ($N = 1$), N the number of the block and C is the slope of the line (i.e., the learning rate). Non-linear regression showed an equivalent fit for all three groups (RC-LA, $R=.92$; RA-LC, $R=.88$; sham, $R=.85$; $p=.46$). In addition, the speed of learning and the reaction time for the first block were not significantly different among the three groups (all $ps > .33$) (**Figure 8.5**).

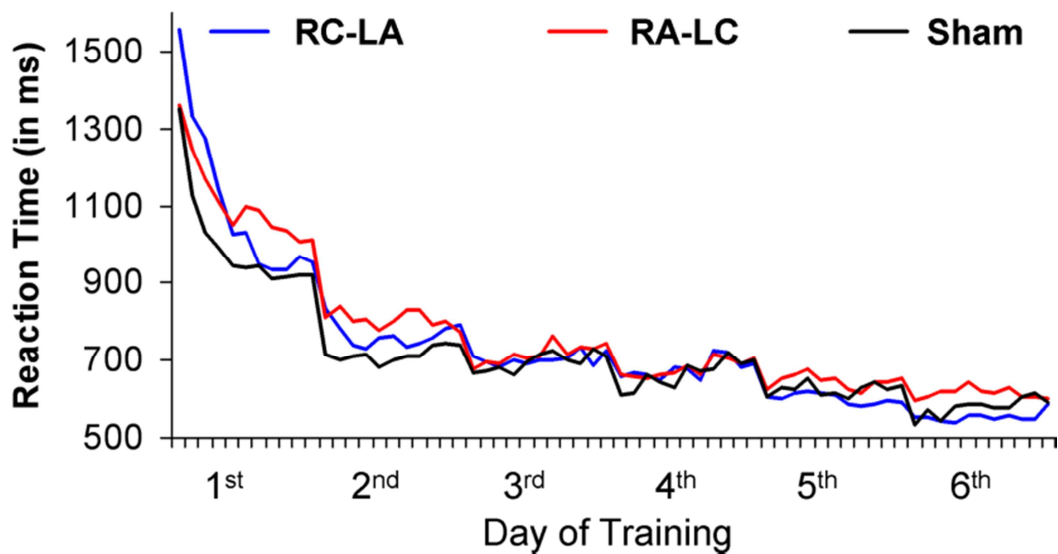


Figure 8.5. Learning function for Right Cathodal, Right Anodal, and Sham group. The improvement in the learning task over days was modelled using a power law function.

8.4.2. Numerical Stroop task

During the numerical Stroop task, the development of automaticity over time differed among the groups, as indicated by a significant three-way interaction between group, session, and congruity ($F(16,96) = 1.85$, $p = 0.035$). Further analyses revealed that the RA-LC group showed an interaction between congruity and training. This

interaction was due to a consistent congruity effect (43–50 ms) that was already present from the fourth training day ($F(2,8) = 10.81, p = 0.005$), indicating automatic numerical processing (**Figure 8.6, Table 8.1**). In contrast, the RC-LA group showed an abnormal effect ($F(2,8) = 5.67, p = 0.03$). A quadratic trend analysis (incongruent > neutral < congruent) explained 87% of the variance ($F(1,4) = 11.36, p = 0.03$), indicating that this effect was due to faster reaction times (RTs) for the neutral condition in comparison to the congruent and incongruent conditions (congruent *versus* incongruent, $p = 0.03$). However, it seems that, in contrast to the RC-LA group, which did not show a typical congruity effect, and the RA-LC group, which showed a consistent congruity effect already from the fourth day, the sham group started to display a typical congruity effect but only from day 5 ($F(2,8) = 4.52, p = 0.049$) (**Figure 8.6, Table 8.1**).

Six months after the end of the training, we contacted the participants from the RA-LC group to examine whether their performance on the tasks with artificial digits persisted. All but one of the participants was available. In the numerical Stroop task, participants showed a significant congruity effect, as indicated by slower RTs for the incongruent versus neutral and congruent conditions ($p = 0.04$). This performance was very similar to the performance on the last day of training 6 months earlier (interaction between congruity and time, $p = 0.53$; congruity effect of 44 ms at the end of training versus 36 ms after 6 months).

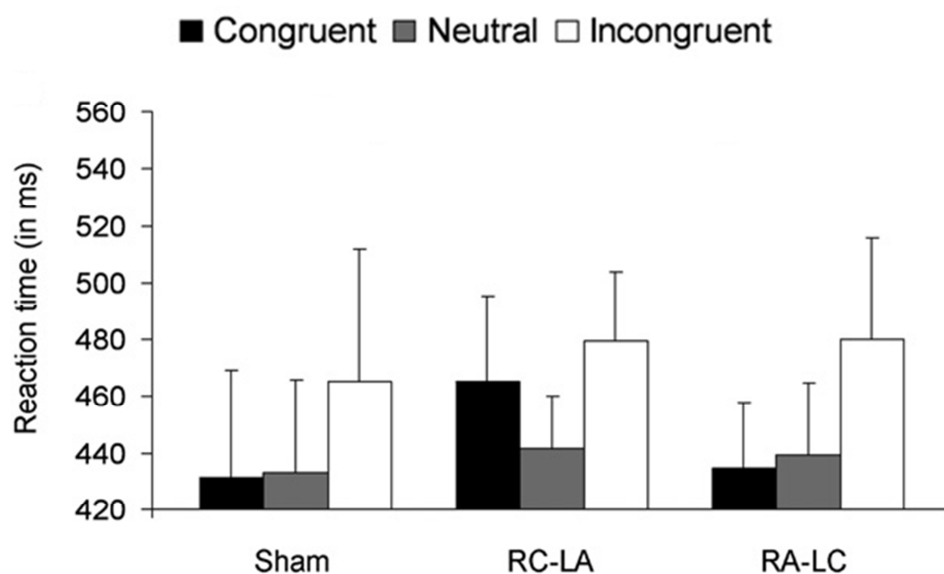


Figure 8.6. Numerical Stroop task with Artificial Digits. Congruency effect (measured in RTs) across the three stimulation groups. Data are mean \pm standard error (SE) of the mean.

			2 nd session	3 rd session	4 th session	5 th session	6 th session
RC-LA	Congruent	RT	519	473	483	428	424
		SEM	52	24	40	29	20
	Neutral	RT	487	441	465	412	401
		SEM	33	29	33	15	13
	Incongruent	RT	561	464	499	437	436
		SEM	45	25	39	22	12
RA-LC	Congruent	RT	513	447	430	433	441
		SEM	41	27	25	37	13
	Neutral	RT	523	447	443	435	440
		SEM	44	20	22	37	20
	Incongruent	RT	524	448	480	476	485
		SEM	52	22	34	47	35
sham	Congruent	RT	593	461	456	447	415
		SEM	51	20	24	51	26
	Neutral	RT	510	466	438	442	424
		SEM	33	12	19	36	30
	Incongruent	RT	581	470	466	495	436
		SEM	44	17	18	65	31

Table 8.1. Reaction Times in the numerical Stroop task for each group in each session. Bold numbers represent days in which normal congruity effect was observed. RT=reaction time; SEM= standard error of mean.

8.4.3. Number line tasks

Here we examined whether the mapping of numbers into space followed a linear or logarithmic scale. At the end of the learning phase, a logarithmic function was selected as the best predictor in the regression analysis for the Sham group and the RC group, while linear function best characterised the RA group (**Figure 8.8**). Moreover, whereas the performance with artificial digits was affected by the type of brain stimulation and showed a linear fit only for the RA-LC group, the performance with everyday digits was independent of the type of brain stimulation and showed a linear fit for all the groups (**Figure 8.8**). In addition, as indicated by a main effect for group, a rightward shift toward the large number was observed for the RA-LC group (mean = 0.59) and to a lesser degree for the sham group (mean = 0.25). In contrast, a leftward shift was observed for the RC-LA group (mean = 20.27; $F(2,12) = 5.2$, $p = 0.023$).

Finally, 6 months after the experiment participants from the RA-LC group were brought back for a follow up (see **paragraph 8.4.2**). Results of the number line task with artificial digits revealed a positive correlation between their current mapping and

their performance 6 months before ($r = 0.83$, $p = 0.02$). Moreover their performance was still best characterized by a linear function ($\beta = 0.71 \pm 0.13$, $p < 0.001$).

8.4.4. Control tasks

To examine whether tDCS affected more general perceptual or cognitive abilities, on the last day of testing, we asked the subjects to perform the same tasks with everyday digits. The performance in these tasks was not modulated by the type of brain stimulation (all $p > 0.2$). Specifically, in the numerical Stroop task all subjects showed a normal congruity effect [$F(2,24) = 14.1$, $p < .0001$]. Moreover, this effect did not vary between groups ($p = 0.46$) (**Figure 8.7**).

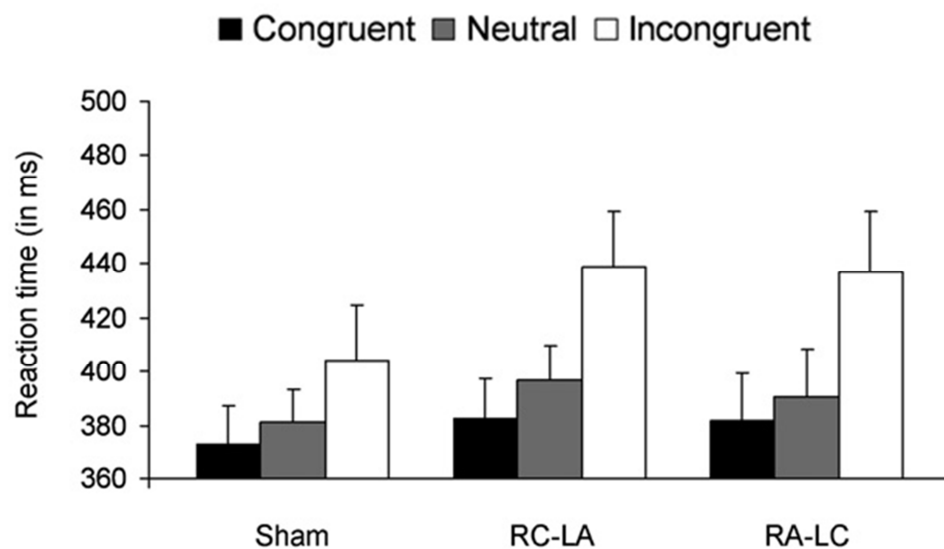


Figure 8.7. Numerical Stroop task with Everyday Digits. Congruency effect (measured in RTs) across the three stimulation groups. Data are mean \pm standard error of the mean.

In the number line task with everyday digits the linear scale showed the best fit to participants' performance in all three groups (**Figure 8.8**).

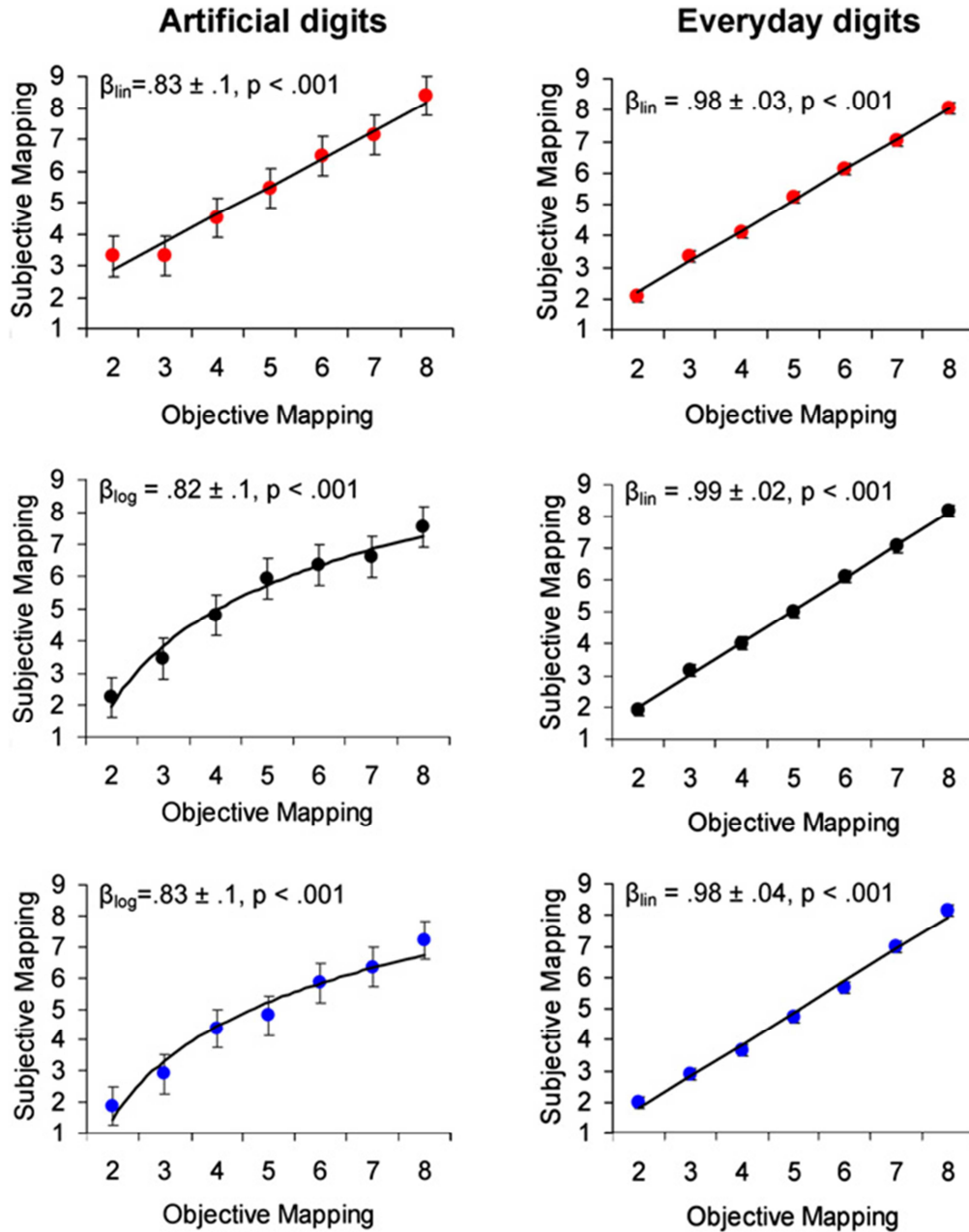


Figure 8.8. Average location of subjective responses on the number line for Artificial Digits (left column) and Everyday Digits (right column) plotted against type of stimulation. β represents the selection of the best weight, whether it was logarithmic (β_{log}) or linear (β_{lin}) in stepwise regression analysis with linear and logarithmic predictors. The first row reflects the performance of the RA-LC group (red circles), the middle row reflects the performance of the sham group (black circles), and the bottom row presents the performance of the RC-LA group (blue circles).

8.5. Final Discussion

The results of Experiment 7 show that non-invasive brain stimulation, a tool that can be used to induce plasticity in the brain in healthy subjects and special populations (Miniussi, Cappa, Cohen, Floel, Fregni, et al., 2008; Nitsche, Cohen, Wassermann, Priori, Lang, et al., 2008) during numerical learning can enhance or impair the development of automatic numerical processing and the interaction between number and space, which are critical indices of numerical proficiency (Booth & Siegler, 2008; Rubinsten & Henik, 2009).

We found that during numerical learning, anodal stimulation to the right parietal lobe and cathodal stimulation to the left parietal lobe (which enhances and reduces the excitation of neuronal populations, respectively) caused better and more consistent performance in numerical tasks (**Figure 8.6, Table 8.1**). In contrast, the opposite configuration (i.e. anodal stimulation to the left parietal lobe and cathodal stimulation to the right parietal lobe) led to underperformance that was similar to that observed in young children (Booth & Siegler, 2008; Girelli et al., 2000; Siegler & Opfer, 2003), or indigenous tribes with rudimentary numerical abilities (Dehaene et al., 2008). Sham stimulation led to a performance that fell between both stimulation groups: subjects in the sham group did process newly learnt numerical information automatically but at a later time point than the group that received right anodal stimulation to the parietal lobe (**Figure 8.6, Table 8.1**).

Moreover, the mapping of numbers into space followed a logarithmic function, similar to the outcome of the right cathodal group, rather than a linear function which was only evident in the right anodal stimulation group (**Figure 8.8**). In addition, as indicated by the main effect of group, a rightward shift toward the large number was observed for the RA-LC group and to a lesser degree for the sham group. This pattern of performance is equivalent to data on adults with everyday digits. In contrast, a leftward shift, which is associated with children's performance (de Hevia & Spelke, 2009), was observed for the RC-LA group.

Altogether, the results from the Number to Position task and the numerical Stroop task suggest that automatic number processing and accurate mapping of numbers onto space can be dissociated, and that more training might be required to obtain an

‘accurate interaction’ between numbers and space. Moreover, our results suggest that similar to the hemispheric asymmetry found in children, the acquisition of numerical competence in the adult brain depends on the intact function of the right parietal lobe. Therefore, enhanced excitation of the right parietal lobe leads to improved numerical abilities, while reduced activity of the right parietal lobe can diminish numerical abilities. In contrast, reduced excitation or enhanced excitation of the left parietal lobe does not seem to impair or improve numerical abilities, respectively. These results are in line with previous evidence that indicated the contribution of the right parietal lobes for good mastering of numerical skills. Indeed studies of Developmental Dyscalculia (Price et al., 2007; Rotzer et al., 2008) have shown that the right parietal lobe is strongly linked with maths performance. Our study provides further evidence for a causal link between numerical competence and right parietal lobe function (*see also* Cohen-Kadosh et al., 2007).

Moreover, it is important to note how tDCS did not affect the learning process itself (**Figure 8.5**), nor the automaticity and mapping of numbers into space with everyday digits. This led us to conclude that the current findings are specific to the representation and processing of the newly learnt numerical system and the artificial digits. Furthermore, one of the remarkable findings of Experiment 7 is the longevity of the effect reached by tDCS. In fact, our results demonstrate that a congruity effect in the numerical Stroop task, as well as an accurate (i.e. linear) mapping of numbers to space was still evident in the RA-LC group even 6 months after the stimulation.

To summarize, results of Experiment 7 showed that enhancement occurred when the anodal electrode was placed over the right parietal lobe, while impairment occurred after cathodal stimulation to the same area. This finding is in line with previous studies that observed improvement after anodal stimulation and impairment after cathodal stimulation (Miniussi, et al., 2008; Nitsche, et al., 2008; Utz, Dimova, Oppenlander, & Kerkhoff, 2010). However, it is important to note that it is entirely possible that the stimulation of the contralateral area contributed to the observed effect by modulating inter-hemispheric interactions/inhibitions (Cappelletti, Barth, Fregni, Spelke, & Pascual-Leone, 2007).

To conclude, the findings from Experiment 7 establish a causal link between the right parietal lobe and numerical skills. Moreover, they are important because they

establish tDCS as a potential tool for intervention in cases of deficient numerical development such as Developmental Dyscalculia. To date, purely behavioural interventions still seem not to be particularly beneficial for cases of Developmental Dyscalculia (see **Chapter VII**). On the other hand, the specificity and the remarkable longevity of the stimulation, makes the usage of tDCS attractive for future use in the field of rehabilitation and more importantly it gives potential hope for achieving effective results in terms of the remediation of core maths deficits such as Developmental Dyscalculia.

8.6. Summary

In Experiment 7 we trained subjects with artificial numerical symbols (the Gibson figures) during 6 days and tested whether brain stimulation to the parietal lobes within this time frame can modulate numerical competence. We found that the polarity of the brain stimulation could enhance or impair the acquisition of automatic number processing, and also the mapping of numbers into space, which serve as important indices of numerical proficiency. Specifically, anodal stimulation and concurrent cathodal stimulation to the right and left parietal lobes respectively boosted numerical performance, while the opposite configuration led to a less proficient performance comparable to the one of young children or individuals with low numerical skills.

Control tasks revealed that the effect of brain stimulation was specific to the representation of the learnt material (i.e. the artificial digits).

These results provide a first step to using brain stimulation as a potential intervention tool for improving numerical learning, especially in populations with developmental numerical difficulties such as Developmental Dyscalculia, a condition that as we have seen, seems to be quite resistant to other types of interventions (see **Chapter VII**).

IX

General Discussion

In a series of experiments we investigated the development of numerical and arithmetical processes in populations with different levels of maths proficiency. First, through a series of behavioural studies in typical and atypical 8-9 year olds we were able to better delineate the nature of what has been proposed as the core system for numbers. Our results support the idea of a clear distinction between two systems of knowledge by showing that the ability to represent and manipulate approximate numerosities is different from the ability to do so exactly. Moreover, our findings highlight the importance of the exact, but not the approximate system for the development of arithmetic. Additionally, we aimed at better characterizing the behavioural profile of atypical development by differentiating between core number deficits and more general maths difficulties. At least for one of our single cases, our results seem to support the idea of Developmental Dyscalculia as a domain-specific deficit in the core representation of numbers. On the contrary, Low Numeracy was characterized by a domain-general difficulty in the inhibition processes of working memory and by an inability to associate an intact representation of numbers with the symbols that denote them (i.e. Arabic digits). Secondly, we assessed the understanding of double-digit numbers, including notations such as fractions and decimals. Using two types of number line tasks we demonstrated that well-educated adults, but also typically developing children as young as 10, and even children with Low Numeracy, showed a normal understanding of these types of numbers as they did not incur in the ‘whole number bias’ (i.e. “0.65 is greater than 0.8 because 65 is greater than 8”). Thirdly, we assessed arithmetical abilities in a population of children with Autism Spectrum Disorder (ASD) to test whether this neurodevelopmental condition could provide a good model of exceptional maths skills. Through a series of behavioural, neuroimaging and multivariate approaches we provided converging evidence to suggest that maths is an area of cognitive strength in ASD. Moreover, we demonstrated that ASD’s exceptional maths skills seem to have their specific neural correlates, which suggests a different type of information processing system in this population. Finally, we looked at different intervention approaches for improving mathematical skills. We showed that targeted behavioural intervention was beneficial for children with Low Numeracy, but not as much for children with Developmental Dyscalculia, indicating that other methods should be considered for helping this population. Interestingly, we showed that maths skills can be enhanced through neuromodulation techniques (tDCS) to areas of the brain that have been implicated in number processing. This suggests that such methodology could be used for the remediation of cases of Developmental Dyscalculia. In summary, we provided behavioural and cognitive evidence for a better characterization of typical, atypical and exceptional maths abilities challenging the classical models of maths development.

9.1. Closing Discussion

There is a large consensus in the literature that the cardinal properties of numbers represent the precursors of arithmetic. Moreover, it has been proposed that humans (and non-human species) are endowed with a basic capacity for understanding such properties of numerical stimuli (Butterworth, 1999; Dehaene, 1997). However, there is some disagreement on the nature of this capacity, which ultimately leads to confusion on which are the cognitive systems that support the learning of arithmetic. This has also important implications in defining the behavioural manifestations of typical, atypical and exceptional development of maths skills. Specifically, what are the cognitive mechanisms that lead to different levels of maths proficiency? Two theoretical frameworks have so far been proposed to explain atypical development and individual differences in the population: the ‘modular view’ (Butterworth, 2005, 2010; *see also* Wilson & Dehaene, 2007) and the ‘continuum view’ (Dowker, 2005). However, none of these models seem to fully account for the coexistence of different degrees of maths proficiency in the population, including maths impairments and excellence.

In the next paragraphs I will discuss the empirical findings of this thesis and I will explain them using the two classical models that have been proposed. I will conclude by suggesting that neither of them seem to fully encompass the data thereby outlining a different framework to better account for the heterogeneous nature of maths skills in the population.

9.1.1. Typical and atypical development

It has been proposed that human and non-human species are born with a capacity for representing numerical quantity that provides the basis for arithmetic (Dehaene, Molko, & Cohen, 2004; Feigenson, Dehaene, & Spelke, 2004; *but see* Butterworth, 1999, 2005, 2010; Butterworth & Reigosa-Crespo, 2007; Gelman & Gallistel, 1987). First, we can represent the cardinality of large sets in an approximate way (i.e. ‘the approximate system’ also referred to as ‘the analogue magnitude system’) (Dehaene, et al., 2004; Feigenson, et al., 2004), and second, we possess attentional resources to track individual elements in small sets of objects (‘parallel individuation system’) (Carey, 2004; Le Corre & Carey, 2007). An alternate view is that the innate

‘system for numbers’ is exact in its nature, and uses an internal ‘numerosity code’ that represents numerosities exactly (Butterworth, 1999, 2005, 2010; Butterworth, Reigosa-Crespo, 2007; Gelman & Gallistel, 1987). Both proposals have been metaphorically explained with the construct of the ‘mental number line’. The proponents of the ‘approximate/analogue system’ have suggested that numerical quantities are cognitively represented as approximate ‘activations’ on an analogue mental number line and are best accounted for by a logarithmic function (Dehaene, 1997). On the other hand, it has been proposed that the ‘numerosity code’ activates discrete representations, so for example the activation of ‘fiveness’ does not overlap with neighboring numbers (Gallistel & Gelman, 2000; Zorzi & Butterworth 1999). The first construct presumes that the approximate system is the precursor of arithmetic (Barth, La Mont, Lipton, & Spelke, 2005; Barth, La Mont, Lipton, Dehaene, Kanwisher, & Spelke, 2006; Feigenson et al., 2004). On the other hand, the proponents of the ‘exact system’ have suggested that children are endowed with a system to represent the numerosity of sets in an exact way that supports arithmetical operations (Butterworth, 1999, 2005; Gelman & Gallistel, 1978). More recently, it has been proposed that understanding the symbol system (e.g. Arabic digits) is a key for learning about numbers and arithmetic (Ansari, 2008; Holloway & Ansari, 2009; Rousselle & Noël, 2007; *see also* Whitehead, 1948).

In Experiment 1A we aimed to adjudicate between these different theories on the development of arithmetic by testing a population of typically developing children on exact and approximate tasks utilizing symbolic and non-symbolic stimuli. Moreover, we used a task of analogue magnitude in order to test whether the system of approximate numerosity is reducible to the system for analogue magnitude. Our results indicated that the ability to compare approximate sets of numerosity was highly correlated with the ability to perform approximate computations on them (i.e. addition and subtraction). However, this ability was not related to performance on exact arithmetic tasks. These results are in line with other findings that did not show a significant association between the ability to deal with approximate numerosities and arithmetic (Mazzocco, Feigenson, & Halberda, 2011; Piazza, Facoetti, Trussardi, Berteletti, Conte, et al., 2010). Our findings suggest that the system for representing and processing approximate numerosities on non-symbolic material does not seem to be sufficient for the development of arithmetic as it has been previously proposed (Barth, et al., 2005, 2006;

Gilmore, McCarthy, & Spelke, 2007, 2010; Halberda, Mazocco, & Feigenson, 2008). Furthermore, performance on the analogue task was not correlated with the ability to compare or manipulate numerosities approximately, suggesting that the system for approximate numerosities is not reducible to the system for analogue quantities. Instead, exact arithmetic was significantly predicted by symbolic and non-symbolic tasks of exact numerosity. This suggests that at least by the age of 8 to 9 years old, children solve exact arithmetic through the support of the exact system alone, with no reliance on the approximate system. Moreover, the lack of correlation between the approximate tasks and the exact arithmetic task makes it difficult to think of the approximate system as the direct precursor of arithmetic as intended in school settings. On the other hand, it is possible that a comprehensive understanding of the symbolic system for numbers helps the development of arithmetic as recently suggested (Ansari, 2008; Holloway & Ansari, 2009; Rousselle & Noël, 2007).

In Experiment 1B we assessed two distinct populations struggling with learning arithmetic (i.e. Developmental Dyscalculia and Low Numeracy) to investigate the causes that could lead to their difficulties. Specifically, we asked whether impairments in arithmetic are the result of a problem in dealing with exact numerosities, approximate numerosities, or more generally with analogue magnitudes. Moreover, we sought to test the hypothesis put forward by Rousselle and Noël (2007) who describe Developmental Dyscalculia as a ‘mapping deficit’, that is these children might show an impairment in mapping an intact representation of numerosities with the symbols that denote them. Our results suggest a distinction between a domain-specific deficit (Developmental Dyscalculia) and a more domain-general one (Low Numeracy). At least in one of our single cases of Developmental Dyscalculia, such a distinction was evident from both our standardized as well as experimental measures. Specifically, using a standardized task (Butterworth, 2003) we were able to identify, from a pool of children who were struggling in their maths classes, a subset of children that showed core impairments in dealing with basic concepts of numerosities (i.e. dyscalculics), from children who did not show a core deficit but were still struggling in learning arithmetic (i.e. children with low numeracy). As expected the former condition was much less prevalent than the latter (Reigosa-Crespo, Valdés-Sosa, Butterworth, Estévez, Rodríguez, et al., 2012). Children with Low Numeracy (LN) performed as well as their control peers on tasks of

approximate numerosities and analogue magnitude and on tasks of exact non-symbolic numerosities. On the contrary, their ability to quickly compare numerosities presented in a symbolic format was impaired compared to controls. Altogether these findings fit well within the ‘deficient mapping’ hypothesis (Rousselle & Noël, 2007) and they validate our previous findings on normally achieving children by showing that the approximate system does not predict arithmetic performance in typical or atypical development. Moreover they support the idea that the understanding of the symbol system (e.g. Arabic digits) seems to be the key for learning about numbers and arithmetic both in typical (Ansari, 2008; Holloway & Ansari, 2009) and atypical development (Mazzocco et al., 2011; Piazza et al., 2010).

Developmental Dyscalculia (DD) was defined from a theoretically driven diagnosis based on the ‘*defective number module hypothesis*’, which assumes that the cognitive deficit in these individuals is an inability to deal with exact numerosities (Butterworth, 2003, 2005; *but see* Dehaene, 1997 *for a different account on the ‘number module’*). Interestingly, the results of our standardized measures identified two distinct cases of Developmental Dyscalculia: in one case (DD1), a deficient performance was evident in the *Number Comparison* task. In this task the child is asked to compare two numerosities presented in a symbolic format (i.e. as Arabic digits). The second DD case (DD2) showed a significant impairment in the *Dot Enumeration* task, which asks to compare a numerosity presented non-symbolically to a numerosity presented as an Arabic digit. Interestingly, both cases were normal on tests of approximate arithmetic and analogue magnitude, however, their profiles on our experimental tasks were not exactly identical. DD2 displayed defective performance in the approximate and exact non-symbolic comparison tasks, which was not the case in DD1. This suggests that DD2 might have suffered from a more deep-rooted deficit in the capacity to represent exact (and also approximate) numerosities. However, his performance on the approximate calculation tasks (i.e. addition and subtraction) was to the same level as controls, providing further evidence that the ability to perform approximate arithmetic is not necessary as a building block for the proper development of exact arithmetical skills. As mentioned, DD1 seemed to have a less pronounced deficit as his problems were confined to the symbolic tasks for both comparison and addition. Hence, his case seems to be better explained within the ‘deficient mapping’ hypothesis (Rousselle &

Noël, 2007). Nonetheless, both DD cases were diagnosed on the basis of tasks that tapped on the basic understanding of the cardinal properties of numbers and their numerosities, which is thought to be one of the key features of the ‘number module’ as originally proposed (Butterworth, 1999). Together, the results of Chapter II emphasize the critical role of the exact system for numbers for the development of arithmetic (Butterworth, 1999, 2005, 2010; Gallistel & Gelman, 1987, 2000). Additionally, our findings highlight the possibility that mathematical difficulties can have multiple origins (Dowker, 2005; Mazzocco & Myers, 2003; Temple, 1994). Moreover, our data seems to suggest that also Developmental Dyscalculia can have multiple origins (*as suggested by* Kosc, 1974).

In Experiment 1B, we have seen that children who struggle learning arithmetic, but are not dyscalculic, showed a rather pronounced difficulty in comparing number stimuli presented symbolically as Arabic digits, at least when directly compared to their control population. However, a large body of literature has suggested that these children have problems in more general cognitive domains, such as Working Memory, Long Term Memory (Geary, 1993; Geary & Hoard, 2005), language (Donlan, Bishop, & Hitch, 1998) or low IQ (Kovas, Harlaar, Petrill, & Plomin, 2000; O’Connor, Cowie, & Samella, 2000). The children in our LN group had an average IQ and no sign of language impairments, suggesting that if we were to look for a more general cognitive deficit, it would not be sensible to do so within these domains. Moreover, one of the cognitive domains that has been closely linked to the development of arithmetic skills is Working Memory (WM) (Gathercole & Pickering, 2000; Geary, 1993; Jordan & Montani, 1997; Russell & Ginsburg, 1984; Shalev & Gross-Tsur, 2001). Interestingly, dissociations have been found in tasks of Working Memory, suggesting that the stimulus-material to be remembered is what drives differences between poor and average attainers. Hence, Siegel & Ryan (1989) found that children with maths difficulties performed worse than controls on a working memory task that involved counting and remembering digits, but not on a non-numerical WM task. Furthermore, it has been speculated that there could be a WM system specialized for number stimuli (Butterworth, Cipolotti, & Warrington, 1996; Semenza, Miceli, & Girelli, 1997; Siegel & Ryan, 1989). Moreover, it has been proposed that different components of WM are distinctly implicated in arithmetical problem solving (De Rammelaere, Stuyven, &

Vandierendonck, 1999, 2001; Hecht, 2002; Logie, Gilhooly, & Wynn, 1994; Noël, Seron, & Trovarelli, 2004). Experiment 2 tested these possibilities on a group of children with LN identified in Experiment 1B. Our results support a complex relationship between the distinct components of WM and arithmetical abilities. Particularly, the LN group performed at the same level as their peers in tasks tapping the Phonological component of WM. On the other hand, we showed that tasks measuring the Central Executive (CE) component of WM discriminated between the groups. We also found an effect of the type of stimulus material used: the LN group performed worse on numerical compared to non-numerical stimuli. This seems to suggest that poor arithmetical performance could be the result of a subtle deficit in the CE component of WM, which seems to be material-specific (Knops, Nuerk, Fimm, Vohn, & Willmes, 2006). Moreover, we could speculate that what leads to low numeracy, at least in our population, seems to be a subtle deficit in updating and manipulating numerical information in WM (Experiment 2), coupled with difficulties in accessing the symbolic information of numbers (Experiment 1B).

As children progress in their school curriculum, they do not only have to deal with comparing numbers expressed as integer-units, and perform arithmetical operations on them, but they have to start dealing with other types of number notations such as fractions and decimals (DfEE, 1999, Key Objectives, p.3). In doing so, they have to make a large conceptual transition from thinking of numbers as integer-units - which can be counted - to numbers that are not part of the counting list such as rational numbers (Smith, Solomon, & Carey, 2005). Such a conceptual difficulty, which has been described as the ‘whole number bias’, has been identified in children (Ni & Zhou, 2005) and sometimes even persists into adulthood (Bonato, Fabbri, Umiltà, & Zorzi, 2007).

In Chapter IV and V we sought to investigate the understanding of double-digit numbers expressed as fractions and decimals in typical children and adults, and in children with Low Numeracy. In order to do so, we used a computerized version of number line tasks (Siegler & Opfer, 2003) which asked to either (i) map the values of numbers expressed as different notations on a given line or to (ii) estimate the value of a hatch mark on the given line expressed as a fraction, integer, decimal or money amount.

Experiment 3A tested well-educated adults with a variety of academic backgrounds and demonstrated that, contrary to what has been predicted by previous accounts (Bonato et al., 2007), subjects showed an accurate representations of these types of stimuli, especially when asked to map the numerical values on the number line. Moreover, our data suggests that they were accurately representing the real value of fractional and decimals stimuli as revealed by their linear responses in all notations. This was not the case when asked to estimate the fractional value of a hatch mark on the line, as in this case the linear model was not the best predictive model given the data. On both tasks, these results held true irrespective of academic background that is people who were taking maths at a university level did not differ in their mapping or estimation from people who did not even take maths as A-levels. Experiments 3B & 4A assessed performance of typically and atypically developing children of 10-11 years of age. Quite surprisingly, the results were very similar to that of Experiment 3A in the sense that children as young as 10 were not affected by the ‘whole number bias’ as revealed by their accurate (and linear) mapping of all numerical notations including fractions. This was true even for children with low numeracy (Experiment 4A). Furthermore children populations and adults performed similarly in the estimation task, whereby responses were successfully fit by the linear model in all notations except the fractions one. Taken together, Experiments 3A&B and Experiment 4A suggest that when the task does not encourage a ‘whole number bias’, both adults and children with and without maths difficulties can accurately represent double-digit integers and rational numbers in a linear manner.

Interestingly, the ability to linearly represent numbers has long been described as an index of maths proficiency (Booth & Siegler, 2006, 2008; Dehaene, Izard, Spelke, & Pica, 2008; Siegler & Opfer, 2003). With Experiments 3A & B we were the first to demonstrate that the linear representation which has previously been shown for integers is also evident for rational numbers in both adults and children as young as 10, and does not seem to be ‘disrupted’ in cases of low numeracy (Experiment 4A). Furthermore, by showing that children with low numeracy represent integers but also rational numbers linearly and correctly, we challenge the idea that the lack of a linear representation is what causes arithmetic problems in this population (Geary, Hoard, Nugent, Byrd-Craven, 2008). Additionally, the correlational data between indices of representational

accuracy for fractions and decimals and accuracy on the addition task (Experiment 4A) speaks in favor of the hypothesis put forward by Mazzocco and Devlin (2008), which suggests a link between the ability to represent rational numbers and arithmetical performance.

Furthermore, it has been suggested that what drives the development of arithmetic is the acquisition of number vocabulary used in counting. Moreover, a linear (and accurate) representation for numbers could only be evident after language has been acquired and fully developed (Carey, 2004; Le Corre & Carey, 2007). However, even if this concept were true, it could only be applied to integer-units, as they are part of the counting sequence. The same does not hold true for rational numbers as they are not part of the counting sequence. Hence, Experiments 3A,B & 4A do not support Carey's account as they show that fractions and decimals, which cannot be counted, are linearly represented by adults, typically developing children and even by children with arithmetic difficulties.

In Experiment 3C we assessed the mastering and processing of different notation conditions, including fractions and decimals in typical adults and children. Moreover, we sought to investigate whether longer experience with rational numbers results in a more efficient processing of these types of stimuli. In contrast to Experiments 3A&B, these results seem to favour the hypothesis of the 'whole number bias', as fractional stimuli were processed slower than other stimuli by both adults and children. This suggests that the accurate representation of fractions might take longer to access, and it is also harder to master as inhibition processes might take place to prevent the 'whole number bias'. The same was not true for decimal stimuli. Altogether, the results of Experiments 3A,B&C suggest a complex relationship between the stimulus type, the particular task used, and the participants' group. First, fractional stimuli do not seem to be reducible to decimal stimuli, at least in our tasks: a linear representation was always evident for decimal stimuli in both groups, independent of task demands; while the same did not occur for fractional stimuli where task demands seemed to play an important role in the type of representation expressed by the subjects. Moreover, fractional stimuli were harder to process compared to other types of stimuli in both tasks. An effect of group was found on accuracy but not on latencies, suggesting that the role of experience interacts with the ability to properly master fractions, yet it seems to

have little or nothing to do with its processing speed. Interestingly, decimal stimuli were both harder and slower to map on the line for children compared to adults mostly suggesting that the role of experience has a bigger effect on decimal stimuli. Finally, it is important to note that the estimation task (i.e. estimating the numerical values of hatch marks on the line) was considerably more demanding for both groups, as reflected by accuracy and reaction times. This result could be interpreted using a more general theoretical framework which describes two generic modes of cognitive functions: (i) an 'intuitive mode' in which judgments and decisions are made automatically and rapidly; and (ii) the 'control mode', which is deliberate and slower (Kahneman, 2003).

Experiment 4B tested the mastering and processing of double-digit numbers in children with low numeracy. Surprisingly, the results show that these children were overall more accurate than their typically developing peers when asked to map double-digit numbers onto a physical line. Moreover, this was true regardless of stimulus type, thereby including fractions and decimals. Yet, children with low numeracy were consistently slower than their peers with all stimuli. In line with Experiment 1B, this finding seems to suggest that children with low numeracy have an intact and accurate representation of double-digit numbers. However, their ability to utilize this information tends to be inefficient, particularly when dealing with symbolic stimuli (Rousselle & Noël, 2007). This suggests that when the numerical representation has to be manipulated in any way, they show slower processing which is ultimately reflected in their calculation performance (Experiment 1B). Altogether, the results of Experiment 1B and 4A&B (see also Experiment 2) provide evidence for a difficulty in manipulating an otherwise intact representation of numbers by this population. Moreover, Experiment 1B and 4A&B support the 'deficient mapping' hypothesis for low numeracy learners (Rousselle & Noël, 2007). Importantly this stresses that future learning interventions in these children should focus on the disrupted linkages of core numerical representations with the application and utilization of this information.

9.1.2. Exceptional abilities

Even if an innate specialized capacity for representing numerosities has been widely documented, it is unclear if and how this alone can result in exceptional maths abilities (Butterworth, 2006). Hence, according to Smith (1983), motivation represents a key aspect in the acquisition of excellent skills. For the sake of our argument it is important to note that motivation is usually triggered by a great interest for certain disciplines or stimuli and this is how it is intended here.

Another idea is that exceptional skills in maths are the result of very well developed general skills such as working memory, visuo-spatial abilities or visual attention, which are successfully applied to maths (the calculating prodigy Bidder, *quoted by* Smith, 1983). Finally, it has also been proposed that maths experts are endowed with a ‘domain-specific gift’ for maths (*see* Gardner, 1983 on maths as part of logical mathematical intelligence; *also* Mitchell, 1907). This is similar to the proposal put forward to explain cases of atypical development such as Developmental Dyscalculia (see **paragraph 9.1.1**). Up to now, only few studies have investigated exceptional maths abilities and their neural correlates. Moreover, these are usually anecdotal evidence presented as single case studies.

In the case study conducted by Pesenti and colleagues, it was evident that their outstanding calculator had exceptional domain-general skills such as working memory (Pesenti, Seron, Samson, & Duroux, 1999) and tended to solve calculation problems through visualization processes as demonstrated by a great recruitment of posterior visual areas when solving the task (Pesenti, Zago, Crivello, Mellet, Samson, et al., 2001). This suggests that at least in this case, exceptional maths abilities were in turn supported by exceptional domain-general abilities (such as visual and working memory abilities), rather than endowed as an ‘exceptional starting kit’ for numbers (i.e. an exceptional ‘number sense’). However, Pesenti’s exceptional calculator also activated regions that have been systematically associated with calculation abilities in the normal population, namely the superior parietal lobes. Single cases studies conducted on autistic savants also found enhanced activity (Cowan & Frith, 2009) and increased cortical thickness (Wallace, Happè, & Giedd, 2009) in areas of the superior parietal lobe, particularly, in the horizontal segment of the intraparietal sulcus. These data have been interpreted as the consequence of intense practice with numbers by these

individuals (Cowan & Frith, 2009; Wallace et al., 2009; *but see also* Pesenti et al., 2001). Taken together these studies seem to point in the direction that exceptional maths skills could be the result of a fortuitous interaction between (i) motivational factors - intended here in the sense of a great interest for a certain discipline and category of stimuli (i.e. numbers) - and the intensive practice that might follow from that (*see* Cowan & Frith, 2009; Wallace et al., 2009) and (ii) excellent domain-general abilities for numbers (i.e. working memory, visual abilities) (Pesenti, et al., 1999; 2001). In Chapter VI we tested this possibility in a group of children with High Functioning Autism (HFA). Autism Spectrum Disorder (ASD) has been often associated with exceptional maths abilities (Asperger, 1944; Baron-Cohen, Wheelwright, Skinner, Martin, & Clubley, 2001, Baron-Cohen, Richler, Bisarya, Gurunathan, & Wheelwright, 2003; Kanner, 1943; Sacks, 1986; McMullen, 2000; Ward & Alar, 2000; *but see* Chiang & Lin, 2007; Mayes & Calhoun, 2006). However, only few systematic studies have so far explicitly investigated the relation between mathematical talent and the condition of autism. For example, Jones and colleagues have found high discrepancies between IQ and achievement scores in the ASD population. Moreover, such discrepancies were mostly evident in the maths domain (Jones, Happè, Golden, Marsden, Tregay, et al., 2009).

In Experiment 5A we tested a population of 7-12 year old children with HFA on a series of maths tasks and compared their performance to typically developing children. Particularly, we sought to test the possibility that maths is an islet of relative ability, and a potential strength in this population, as previously suggested (Baron-Cohen et al., 2001, 2003; Baron-Cohen & Belmonte, 2005; Happè & Frith, 2010; Jones et al., 2009). After carefully matching for IQ, our results revealed that children with HFA scored significantly better than their typically developing peers on standardized tests of maths abilities. Moreover, they showed a higher discrepancy between their IQ and their maths scores (*see also* Jones et al., 2009). Finally, we assessed their strategy use and found that children with HFA adopted the so called ‘decomposition strategy’ significantly more often than controls. This might suggest that they were relying on good visualization abilities, as expert calculators often report (i.e. their strategies often involve manipulables that are usually visual - *see* Butterworth, 2006). Interestingly, the literature on autistic savants has proposed that these individuals tend to break down

calendars into ‘fragments’ of dates (Heavey, Pring, & Hermelin, 1999). This is the same concept of ‘decomposition’ of equations as intended here. Moreover, this result could be interpreted within the theoretical account proposed by Baron-Cohen and Belmonte (2005) who suggested that children with ASD are characterized by highly proficient analogical and systematic skills, an idea that has been ultimately summarized in the so called ‘analytical brain theory’. Moreover, this finding is also in line with the ‘weak central coherence’ theory proposed by Frith (1989), which states that individuals with autism tend to process information at a local (analytical) rather than a global level. All in all, the results of Experiment 5A demonstrate for the first time, using a cross-sectional study, that children with HFA display exceptional maths abilities compared to age, gender and IQ matched controls.

To better clarify the roots of exceptional maths abilities, in Experiment 5B we used fMRI to investigate the brain processes underlying arithmetic problem solving in autism. Our multivariate pattern analyses showed that numerous brain areas displayed a different pattern of activity between children with HFA and their typically developing peers. Particularly, distinct fine-scale neural representations were detected in areas of the ventral and dorsal visual streams, in the medial temporal lobe and the prefrontal cortex. These brain regions have been systematically implicated in solving arithmetical tasks in typical adults and children. Specifically, the intraparietal sulcus in the dorsal visual stream has been showed to support numerical (Dehaene, Piazza, Pinel, & Cohen, 2003) and arithmetical processes (*see* Ansari, 2008 and Zamarian, Ischebeck & Delazer, 2009 *for a review*). The fusiform gyrus in the ventral visual stream has been implicated in the processing of symbolic stimuli (i.e. Arabic digits) (Ansari, 2008; Cantlon, Brannon, Carter, & Pelphrey, 2006; Rykhevskaya, Uddin, Kondos, & Menon, 2009); and the Medial Temporal lobe has been reported to activate during arithmetic fact retrieval (Cho, Ryali, Geary, & Menon, 2011). Finally, the prefrontal cortex has been implicated in number comparison tasks (Ansari, Garcia, Lucas, Hamon, & Dhital, 2005), arithmetic problems solving, particularly in children (Rivera, Reiss, Eckert, & Menon, 2005) and in more abstract mathematical thinking (Shallice & Evans, 1978). These results seem to be in line with previous studies showing that exceptional calculators activate the same brain areas as average individuals, but to a different extent and with additional areas sometimes recruited (Cowan & Frith, 2009; Fehr, Weber,

Willmes, & Herrmann, 2010; Pesenti et al., 2001). Particularly, from the results of Experiment 5B it appears that differences are qualitative rather than quantitative (*as proposed by* Pesenti et al., 2001; *see also* Fehr et al., 2010). Support for this idea comes from the fact that in our samples we found no overall differences in signal amplitude (i.e. univariate analyses) and distinct patterns of activation in these brain areas were only evident on fine-scale neural representations as measured by multivariate analyses.

However, in order to better understand whether this network is differently orchestrated in children with HFA, thereby indicating a different processing system, in Experiment 5C we used another Support Vector Machine method (Support Vector Regression - SVR) and showed that hemodynamic signal change in the anterior fusiform gyrus, but not in the dorsal visual stream (i.e. the intraparietal sulcus), predicted performance on a standardized arithmetic task in the HFA group only. This finding might suggest that exceptional maths abilities are tightly coupled with the recruitment of a more posterior visual system, as initially suggested by the ‘analytical brain theory’ (Baron-Cohen & Belmonte, 2005). However, this account also proposes that dorsal and frontal areas would be less involved in the task. This does not seem to be the case in our data as activity in two areas of the prefrontal cortex were also highly predictive of maths abilities in the HFA group. Instead, an explanation might be sought within the functional ‘properties’ of the fusiform gyrus per se. This brain area has been largely linked with the ability to discriminate and make judgments about faces (Kanwisher, 2000). Yet, as mentioned earlier, this area has also been implicated in symbol processing (Ansari, 2008; Cantlon, Pinel, Dehaene, & Pelphrey, 2011; Dehaene & Cohen, 2011). Moreover, it has been shown that faces and written words (Mei, Xue, Chen, Xue, Zhang, & Dong, 2010), but also numerical stimuli (Cantlon et al., 2011) activate very close or even overlapping regions within the fusiform gyrus, possibly because these types of stimuli place high demands on high resolution foveal processing (Yoncheva, Blau, Maurer, & McCandliss, 2010). It follows that that fusiform gyrus might be subject to a high ‘competition for cortical space’ (Dehaene & Cohen, 2011), as suggested by the theoretical framework of ‘cortical recycling’ (Dehaene & Cohen, 2007). Although we have not tested our population on a facial discrimination task, we could speculate that this area could have been ‘recycled’ to account for stimuli that are interesting, rather than hard to process (i.e. numbers rather than faces). Given that

ventral visual areas are quite plastic (Fujita, Tanaka, Ito, & Cheng, 1992) limited attention to facial stimuli within the critical period of visual cortices specializations, would promote their ‘cortical recycling’ to serve other cognitive processes that are associated with great expertise in a stimulus-domain (Gauthier et al., 1999, 2000; *see also* Dehaene & Cohen, 2011). Furthermore, it has been proposed that the emergence of specialization in certain areas of the brain is favored by top-down projections in a process that ultimately shapes the organization of conceptual knowledge (Mahon, & Caramazza, 2011). Interestingly, one of the regions that significantly predicted performance on the standardized maths task in the HFA group was the rostrolateral prefrontal cortex (RLPFC), a region that has been recently implicated in the integration of domain-specific representations through sets of principles and rules (Wendelken, Chung, & Bunge, 2011). In this sense, we could speculate that projective connectivity from this prefrontal region could support the cortical specialization of the fusiform gyrus for number stimuli and concurrently use the concrete inputs and representations sent from this ventral region in order to create principles and rules and ultimately promote the development of (maths) conceptual knowledge in this population. Yet, this is only a speculation as functional connectivity analyses are essential in order to experimentally test the functional coupling between these regions.

Finally, what has been proposed as the ‘functional degeneracy’ hypothesis implies that our brain has more than one neuronal system for producing the same response (Price & Frith, 2002, 2003). According to this framework, in the typical population a given brain function (i.e. arithmetic) is preferentially processed by certain brain regions (i.e. the intraparietal sulcus) yet, that is not the only region that could potentially perform the task. This theory proposes that are other brain regions that are kept either ‘silent’ or ‘semi-silent’ within normal development, but could still potentially get recruited to the task when necessary, mostly due to neural reorganization. This is the case of acquired clinical conditions, but also neurodevelopmental disorders in which neural reorganization is due to intrinsic and extrinsic maturational factors. Hence, we could tentatively suggest that, through processes of neural reorganization (i.e. the recruitment of ‘semi-silent’ areas) and cortical recycling, arithmetic problem solving in children with Autism Spectrum Disorder is orchestrated by different brain areas compared to typical populations.

In summary, the results of Chapter VI demonstrated superior maths abilities in a population of children with high functioning autism which are reflected in a different neural processing system compared to controls. Moreover, in line with previous studies (Cowan & Frith, 2009; Pesenti et al., 2001) our results seem to suggest that highly developed domain-general abilities, possibly coupled with a great interest in numerical stimuli, is how these exceptional maths abilities originate. Indeed, it has been recently documented that a variety of psychiatric disorders strongly covary with intellectual interests. Particularly, it has been reported that students aspiring to majors such as science and mathematics were more likely to report siblings with autism spectrum disorder (Campbell, & Wang, 2012).

9.1.3. Intervention approaches

The importance of maths training has received increased interest in our society as it has been closely linked with scientific and economic progress (OECD, *The High Cost of Low Educational Performance: The Long-Run economic impact of Improving Educational Outcomes*, 2010). It follows that aiming to find affective remediation approaches to raise performance of low maths attainers has become more and more crucial. Interestingly, learning studies on adults have demonstrated that intensive practice (i.e. ‘learning by drill’) with multiplication and subtraction problems is reflected in more proficient mastering of these problems both at a behavioural and neuronal level (Delazer, Domahs, Barthä, Brenneis, Lochy, et al., 2003; Delazer, Ischebeck, Domahs, Zamarian, Koppelstaetter, et al., 2005). On the other hand, highly targeted and more conceptual interventions (i.e. ‘learning by concept’, not ‘by drill’) could be very beneficial for learners, especially in cases of atypical development. There have been many attempts by neuroscientists and psychologists to raise performance of children with maths difficulties (e.g. Dowker, 2004, 2009; Räsänen, Salminen, Wilson, Aunio, & Dehaene, 2009; Wilson, Revkin, Cohen, D., Cohen, & Dehaene, 2006), although not specifically Developmental Dyscalculia (Butterworth, 2011). Moreover, no studies have drawn a sharp distinction between Developmental Dyscalculia and more general low numeracy skills, to adjudicate what are the most effective intervention approaches for different types of maths impairment. In Chapter VII we aimed at specifically characterizing the improvement of children with Developmental

Dyscalculia compared to children with low numeracy after a conceptual intervention program. Performance was assessed after an intervention program that lasted 12 weeks and consisted of targeted one-to-one 30 minute lessons with specially trained teachers. The findings in Experiment 6A indicate that children with Developmental Dyscalculia were significantly below their peers with low numeracy on a standardized maths measure even before intervention. Interestingly, the 12-weeks intervention improved performance in both groups. However, children with low numeracy showed a much higher improvement compared to dyscalculics who still fell below the curriculum standards expected for their age, even after intervention. Even when we equated for performance before entering the program (Experiment 6B), the results still showed a significant difference in the improvement between the two groups. Altogether these findings suggest that while children with low numeracy show a great benefit from this targeted intervention, dyscalculics seem to suffer from a deep-rooted deficit in numerosity understanding (*see also* Experiment 1B) that is more resistant to this type of intervention alone. Furthermore, our results seem to support the idea that Developmental Dyscalculia is a core neurobiological deficit that could derive from structural abnormalities of the parietal lobes (Isaacs, Edmonds, Lucas, & Gadian, 2001; Kucian, Loenneker, Dietrich, Dosch, Martin, & von Aster, 2006; Price, Holloway, Räsänen, Vesterinen, & Ansari, 2007). Thus, these results show that different forms of atypical development can be differently affected by intervention. Moreover, it highlights the importance of differentiating between these two conditions by showing that intervention needs to be specifically targeted to the type of maths impairment, as children with a core deficit for numerosity cannot fully benefit from a curriculum-based approach.

Interestingly, the field of neuroscience has started to utilize neuromodulation techniques such as transcranial Direct Current Stimulation (tDCS) in order to rehabilitate acquired neurological conditions, such as aphasia (Baker, Rorden, & Fridriksson, 2010; Holland, Leff, Josephs, Galea, Desikan, et al., 2011).

In Experiment 7 we wished to see whether this technique can be successfully applied to the field of maths cognition and whether it could be thought of as a tool for remediating conditions such as Developmental Dyscalculia that is more resistant to certain types of behavioural intervention, as Experiments 6A&B have shown.

Particularly, we looked at the effects of tDCS on the learning of novel symbols (Gibson figures) in a population of young adults. The symbols were deliberately associated with the Arabic digits 1 to 9, yet the subjects were unaware of this (i.e. implicit learning). The training lasted one week and involved a daily 20 minutes stimulation of the parietal lobes which was delivered during the learning phase. Stimulation was modulated by hemisphere and by group. More importantly, neuromodulation could have been affected either in a positive way (anodal stimulation) or in a negative way (cathodal stimulation) so that behavioural performance could either be enhanced or impaired depending on the polarity of the stimulation. Results showed that anodal stimulation to the right parietal lobe improved learning of the novel symbols as it transferred to more general maths proficiency skills as measured by tasks of number representation and automaticity. Moreover, this effect was specific for the learnt material and did not affect performance on tasks that used every-day digits. On the contrary, cathodal stimulation to the right parietal lobe impaired performance in all tasks. These results give support to the evidence suggesting that the right parietal lobe is a key area for number processing and further highlights the importance of this region in the representation of numerosity (Price, et al., 2007) and in creating number to symbols associations (Cohen-Kadosh, Cohen-Kadosh, Kaas, Henik, & Goebel, 2007, *but see* Piazza, Pinel, Le Bihane, & Dehaene, 2007; Rykhlevskaia, Uddin, Kondos, & Menon, 2009). Even more importantly, the results of Experiment 7 suggest that direct intervention on neural processes could help learning, particularly in cases of Developmental Dyscalculia. In the future it would be important to investigate children and adults with this condition using tDCS to assess its potential benefits in the remediation of this deficit.

9.1.4. Theoretical perspectives

Up to now, two distinct models of maths development have been proposed: the first one has been introduced in the context of atypical development and specifically Developmental Dyscalculia (Butterworth, 1999, 2005, 2010) and has been termed '*the defective number module hypothesis*'. Here we will refer to it as the '*modular view*' as it seems to be a high specification of Fodor's theory on the modularity of the mind (Fodor, 1983). The second one has been introduced to explain individual differences in maths across the range of typical and atypical populations (Dowker, 2005) and will here be

termed '*the continuum view*', as it postulates a performance continuum, which is very reasonable. However, this theoretical framework also assumes that there is a 'cognitive system continuum', which is more unlikely, unless it is believed that there is only one cognitive system. Hence, as postulated, this view seems to be a reminiscent of the *g* factor and the general theory of intelligence (Spearman, 1904), and this is how it will be intended here.

The '*modular view*' states that Developmental Dyscalculia (DD) is a core deficit in the representation of numerosities. Our data in Experiment 1B seem to support this claim, at least in one of our cases of DD (see also Experiments 6A&B). However, our findings on children with low numeracy from the same experiment seem to be harder to fit within this theoretical framework. Of course there are many possible causes for low numerical skills, including low intelligence, poor teaching, low general cognitive abilities, etc. (see also **paragraph 1.6.2**) whose understanding is beyond the scope of this thesis. However, our data from Experiment 2 show that, at least in some cases, the condition of low numeracy could be the result of a complex interaction between inhibition processes of Working Memory, stimulus-material and ultimately the processing of symbolic number stimuli (see Experiment 1B). Moreover, data from Experiment 1B seem to suggest that certain numerical abilities could be spared in either case of atypical development (Dyscalculia and Low Numeracy). Moreover, we found that heterogeneity could occur even within the condition of Developmental Dyscalculia itself, as originally proposed by Kosc (1974) (*see also* Rubinsten & Henik, 2009). Specifically in Experiment 1B, we showed that even after basing our diagnosis on the assessment of the most intuitive property of numbers (i.e. cardinality) (*see* Starkey & Cooper, 1980; Starkey, Spelke, & Gelman, 1990; van Loosbroek & Smitsman, 1990), a dissociation was found between the ability to match Arabic numbers to a given set of dots and the ability to compare two Arabic digits. This pattern of results could not be accounted by the '*continuum view*' (as it shows a fundamental deficit), but neither entirely by the '*modular view*' (as it shows that within the core deficit there could be multiple behavioural manifestations).

Furthermore, in cases of exceptional maths abilities our data does not seem to suggest that they could be reducible to a general intelligence factor, as we found that our HFA population showed significantly higher maths abilities compared to their control

population, who was IQ matched. Moreover, proficient maths skills were significantly better than IQ in children with autism (*see also* Jones et al., 2009). Whether these abilities would be the consequence of a highly proficient system for basic number processing, it is more difficult to tell. In fact, we did not assess this population on tests of basic numerical capacities. However, we could raise the possibility that if these children were endowed with an exceptional ‘starting kit’ for numbers and arithmetic, its best neural candidate would be located in the intraparietal sulcus. Yet, our data showed that functional activation in the bilateral fusiform gyrus, together with two regions in the prefrontal cortex, but not the intraparietal sulcus, was significantly correlated with maths performance in this population.

Altogether the results of this thesis are highly suggestive of the need of a less radical, and more precise theoretical framework to account for all the phenotypical manifestations of maths abilities in the population.

Recently, Westermann and colleagues have proposed the ‘neuroconstructivism view’ which has tried to explain brain specialization from a developmental perspective. In their model, neural specializations are a combination of experience and biological factors, described as ‘intrinsic biases in neural receptivity’ (Westermann, Thomas, & Karmiloff-Smith, 2010). Some aspects of this proposal have recently been revisited to account for maths development (Butterworth, Varma, & Laurillard, 2011). In their ‘causal model of possible inter-relations between biological, cognitive and simple behavioural level’ these authors suggest that Developmental Dyscalculia, a congenital deficit, would be the result of brain areas for basic numerosity processes (i.e. the parietal lobes) failing to develop normally, which will most likely be expressed as an inability to deal with numerosities exactly (Experiment 1B).

In the case of Low Numeracy, at least in the population tested here, one might suggest that if neural connections between the key areas for processing symbols and processing numerosities (Rykhlevskaia et al., 2009; *but see also* Ansari, 2007, 2008; Piazza et al., 2007; Cohen-Kadosh et al., 2007) fail to develop normally, a ‘mapping deficit’ will occur, which seems to be one of the main behavioural characteristics of our population (Experiment 1B). However, children with low numeracy, at least in our sample (*but see also* Noël et al., 2004; Siegel & Ryan, 1989) also present a selective deficit in inhibition processes of Working Memory (Experiment 2). In this case, a

‘connection failure’ might be coupled with less functioning areas devoted to working memory processes, possibly in the prefrontal cortices (Soltész, Szucs, Dékány, Márkus, & Csépe, 2007). In this sense, multiple brain ‘perturbations’ could be responsible for low numeracy (*see* Rubinsten & Henik, 2009). Moreover, low numeracy does not seem to be a congenital disorder, and therefore might manifest only later in development, when biological and cognitive factors would be highly coupled with experience (e.g. including educational and motivational factors), as also predicted by Westermann and colleagues (2010).

Finally, as originally proposed by Galton (Galton, 1979, *originally published in* 1869), exceptional maths skills might be the result of “zeal and the ability to do a very great deal of hard work”, coupled with highly developed cognitive abilities (*see* Pesenti et al., 2001).

In the case of autism, the neurobiology of the disorder could affect multiple brain areas (Carper, Moses, Tigue, & Courchesne, 2002; Muller, 2008) which could in turn result in a different neuronal organization of weaknesses and strengths. In the case of maths, such neural reorganization coupled with the right amount of motivation and interest, which could be intrinsic to autism (*see* Campbell, & Wang, 2012), might make these children ahead of their peers (Experiment 5A).

9.2. Closing Summary

First we delineated the distinct properties of the two systems for numbers (i.e. the exact and the approximate system) and demonstrated their different contributions in the representation and processing of numerosities. Furthermore, we showed how the exact, but not the approximate system seems to be crucial for the development of arithmetical abilities (Chapter II). Second, we were able to better characterize the behavioural profile of Developmental Dyscalculia from the one of Low Numeracy and identify what are the underlying mechanisms that lead to arithmetical impairments in both populations (Chapter II). Developmental Dyscalculia was identifiable, at least in one of our cases, as a deep-rooted deficit in numerosity understanding (Chapter II), while Low Numeracy showed impaired inhibition processing deficits of Working Memory specific for numerical material (Chapter III) and a subtle impairment in linking symbols to numerosities (Chapter II). Third, we assessed the understanding of double-digit

numbers, including notations such as fractions and decimals in adults (Chapter IV) and typical and atypical developing children (Chapter IV and V). We showed that adults and even children as young as ten represent these ‘paradoxical notations’ in a linear manner, which is an index of representational acuity. Moreover, children with low numeracy were also proficient in these tasks (Chapter V). Fourth, we established that the condition of Autism Spectrum Disorder is characterized by exceptional arithmetical abilities, sophisticated calculation strategies, and a fundamentally different neural processing system for solving arithmetic (Chapter VI). Finally, we showed that different forms of atypical development are differently susceptible to intervention (Chapter VII) and that maths proficiency can be ameliorated through brain stimulation techniques (Chapter VIII).

Altogether this evidence points to a combination of intrinsic (biological and cognitive) and extrinsic (experience driven) factors to account for various types of maths skills. The challenge for the future is to understand the possible cognitive causes of all the phenotypical manifestations of maths abilities, such as Developmental Dyscalculia, Low Numeracy and exceptional skills, and to develop appropriate intervention for children with low numeracy and Developmental Dyscalculia.

References

- Abrams, D.A., Bhatara, A., Ryali, S., Balaban, E., Levitin, D.J., & Menon, V. (2010). Decoding temporal structure in music and speech relies on shared brain resources but elicits different fine-scale spatial patterns. *Cerebral Cortex*, 21(7), 1507-1518
- Agrillo C., Dadda M., & Bisazza A. (2007). Quantity discrimination in female mosquitofish. *Animal Cognition*, 10(10), 63-70
- Agrillo C., Dadda M., Serena G., & Bisazza A. (2008). Do fish count? Spontaneous discrimination of quantity in female mosquitofish. *Animal Cognition*, 11(3), 495-503
- Alarcón, M., Defries, J., Gills-Light, J., & Pennington, B. (1997). A twin study of mathematics disability. *Journal of Learning Disabilities*, 30, 617-623
- Albert, N.B., Robertson, E.M., & Miall, R.C. (2009). The resting human brain and motor learning. *Current Biology*, 19(12), 1023-1027
- Almeida, R.A., Dickinson, J.E., Maybery, M.T., Badcock, J.C, & Badcock, D.R. (2010). A new step towards understanding Embedded Figures Test performance in the autism spectrum: The radial frequency search task. *Neuropsychologia*, 48, 374-381
- American Psychiatry Association (APA) (1994). *Diagnostic and Statistical Manual of Mental Disorders* (DSM-IV). American Psychiatric Publishing Inc.
- Ansari, D. (2007). Does the parietal cortex distinguish between “10”, “ten”, and ten dots? *Neuron*, 53(2), 165-167
- Ansari, D. (2008). Effects of development and enculturation on number representation in the brain. *Nature Reviews Neuroscience*, 9(4), 278-291
- Ansari, D. & Dhital, B. (2006). Age-related changes in the activation of the intraparietal sulcus during nonsymbolic magnitude processing: an event-related functional magnetic resonance imaging study. *Journal of Cognitive Neuroscience*. 18, 1820-1828
- Ansari, D., Dhital, B., & Siong, S.C. (2006). Parametric effects of numerical distance on the intraparietal sulcus during passive viewing of rapid numerosity changes. *Brain Research*, 1067(1), 181-188
- Ansari, D., Garcia, N., Lucas, E., Hamon, K. & Dhital, B. (2005). Neural correlates of symbolic number processing in children and adults. *Neuroreport*, 16, 1769-1773
- Antell, S.E., & Keating, D.P. (1983). Perception of numerical invariance in neonates. *Child Development*, 54(3), 695-701
- Arnold, C., Bowen, P., Tallents, M., & Walden, B. (2008). *Sandwell Early Numeracy Test*. GL assessment: The measure of potential
- Ashcraft, M.H. (1992). Cognitive arithmetic: a review of data and theory. *Cognition*, 44, 75-106

- Ashcraft, M.H. (1995). Cognitive psychology and simple arithmetic: a review and summary of new directions. *Mathematical Cognition*, 1, 3-34
- Ashcraft, M.H., & Kirk, E.P. (2001). The relationships among working memory, math anxiety, and performance. *Journal of Experimental Psychology: General*, 130(2), 224-237
- Asperger H. (1944); translated and annotated by Frith U. (1991) 'Autistic psychopathy' in childhood. *Autism and Asperger syndrome*. (pp. 37-92). Cambridge University Press
- Baddeley, A. D. (1986). *Working Memory*. Oxford: Clarendon Press
- Baddeley, A. D. (2000). The episodic buffer: a new component of working memory? *Trends in Cognitive Sciences*, 4, 417-423
- Baddeley, A.D., & Hitch, G.J. (1974). Working Memory. In: G.A. Bower (Ed.). *Recent advances in learning and motivation*. (Vol. 8, pp. 47-90). New York: Academic Press
- Baddeley, A. D. & Hitch, G. J. (1977). Commentary on "Working Memory". In: G. Bower (Ed.), *Human memory: basic processes* (pp. 191-197). New York: Academic Press
- Baker, J., Rorden, C., & Fridriksson, J. (2010). Using transcranial direct current stimulation (tDCS) to treat stroke patients with aphasia. *Stroke*, 41(6), 1229-1236
- Baron-Cohen, S., Ashwin, E., Ashwin, C., Tavassoli, T., & Chakrabarti, B. (2009). Talent in autism: hyper-systemizing, hyper-attention to detail and sensory hypersensitivity. *Philosophical Transactions of the Royal Society B: Biological Sciences*, 364(1522), 1377-1383
- Baron-Cohen, S. & Belmonte, M.K. (2005). Autism: a window onto the development of the social and the analytical brain. *Annual Review of Neuroscience*, 28, 109-126
- Baron-Cohen, S., Richler, J., Bisarya, D., Gurunathan, N., & Wheelwright, S. (2003). The systemizing quotient: an investigation of adults with Asperger syndrome or high-functioning autism, and normal sex differences. *Philosophical Transactions of the Royal Society, Series B (special issue on autism mind and brain)*, 358, 361-740
- Baron-Cohen, S., Scott, F.J., Allison, C., Williams, J. Bolton, P., Matthews, F.E., & Brayne, C. (2009). Prevalence of autism-spectrum conditions: UK school-based population study. *British Journal of Psychiatry*, 194, 500-509
- Baron-Cohen, S., Wheelwright, S., Burtenshaw, A., & Hobson, E. (2007). Mathematical talent is linked to autism. *Human Nature*, 18, 125-131
- Baron-Cohen, S. Wheelwright, S., Skinner, R., Martin, J., & Clubley, E. (2001). The autism-spectrum quotient (AQ): evidence from Asperger syndrome/high-functioning autism, males and females, scientists and mathematicians. *Journal of Autism and Developmental Disorders*, 31, 5-17

- Barth, H., La Mont, K., Lipton, J., Dehaene, S., Kanwisher, N., & Spelke, E. S. (2006). Non-symbolic arithmetic in adults and young children. *Cognition*, 98(3), 199-222
- Barth, H., La Mont, K., Lipton, J., & Spelke, E. S. (2005). Abstract number and arithmetic in preschool children. *Proceedings of the National Academy of Sciences of the United States of America*, 102(39), 14116-14121
- Barth, H., Starr, A., & Sullivan, J. (2009). *Children's mappings of large number words to between the shape of the mental number line and familiarity with numbers in 5- to 9-year*. Cambridge, MA: Harvard University Press.
- Beddington, J., Cooper, C.L., Field, J., Goswami, U., Huppert, F.A., Jenkins, R, ... & Thomas, S.M. (2008). The mental wealth of nations. *Nature*, 455(7216), 1057-1060
- Belmonte, M.K. & Yurgelun-Todd, D.A. (2003). Functional anatomy of impaired selective attention and compensatory processing in autism. *Cognitive Brain Research*, 17, 651-664
- Berteletti, I., Lucangeli, D., Piazza, M., Dehaene, S., & Zorzi, M. (2010). Numerical estimation in preschoolers. *Developmental Psychology*, 46(2), 545-551
- Bird, R. (2007). *The Dyscalculia Toolkit*. London: Paul Chapman Publishing
- Boddaert, N., Chabane, N., Gervais, H., Good, C.D., Bourgeois, M., Plumet, M.H., ..., & Zilbovicius, M. (2004). Superior temporal sulcus anatomical abnormalities in childhood autism: a voxel-based morphometry MRI study. *NeuroImage*, 23, 364-369
- Boggio, P.S., Bermanpohl, F., Vergara, A.O., Muniz, A.L., Nahas, F.H., ... & Fregni, F. (2006). Go-no-go task performance improvement after anodal transcranial DC stimulation of the left dorsolateral prefrontal cortex in major depression. *Journal of Affective Disorders*, 101, 91-98
- Boggio, P.S., Ferrucci, R., Rigonatti, S.P., Cobre, P., Nitsche, M., Pascual-Leone, A., & Fregni, F. (2006). Effects of transcranial direct current stimulation on working memory in patients with Parkinson's disease. *Journal of Neurological Sciences*, 249(1), 31-38
- Bonato, M., Fabbri, S., Umiltà, C., & Zorzi, M. (2007). The mental representation of numerical fractions: Real or integer? *Journal of Experimental Psychology: Human Perception and Performance* 33(6), 1410-1419
- Bonilha, L., Cendes, F., Rorden, C., Eckert, M., Dalgalarondo, P., Li, L.M., & Steiner, C.E. (2008). Gray and white matter imbalance - typical structural abnormality underlying classic autism? *Brain Development*, 30, 396-401
- Booth, J.L., & Siegler, R.S. (2006). Developmental and individual differences in pure numerical estimation. *Developmental Psychology*, 42(1), 189-201
- Booth, J.L., & Siegler, R.S. (2008). Numerical magnitude representations influence arithmetic learning. *Child Development*, 70(4), 1016-1031

- Brambati, S.M., Benoit, S., Monetta, L., Belleville, S., & Joubert, S. (2010). The role of the left anterior temporal lobe in the semantic processing of famous faces. *NeuroImage*, 53(2), 674-681
- Brambilla, P., Hardan, A., di Nemi, S.U., Perez, J., Soares, J.C., & Barale, F. (2003). Brain anatomy and development in autism: review of structural MRI studies. *Brain Research Bulletin*, 61, 557-569
- Brannon, E.M. (2002). The development of ordinal numerical knowledge in infancy. *Cognition*, 83(3), 223-240
- Brannon, E.M., & Terrace, H.S. (1998). Ordering of the numerosities 1 to 9 by monkeys. *Science*, 282(5389), 746-749
- Brewer, J.B., Zhao, Z., Desmond, J.E., Glover, G.H., & Gabrieli, J.D. (1998). Making memories: brain activity that predicts how well visual experience will be remembered. *Science*, 281, 1185-1187
- Bright, G. W., Behr, M. J., Post, T. R., & Wachsmuth, I. (1988). Identifying fractions on number lines. *Journal for Research in Mathematics Education*, 19(3), 215-232
- Broadbent, D. E. (1958). *Perception and Communication*. London: Pergamon Press
- Bruandet, M., Molko, N., Cohen, L., & Dehaene, S. (2004). A cognitive characterization of dyscalculia in Turner syndrome. *Neuropsychologia*, 42, 288-298
- Brun, C.C., Nicolson, R., Lepore, N., Chou, Y.Y., Vidal, C.N., DeVito, T.J., ... & Thompson, P.M. (2009). Mapping brain abnormalities in boys with autism. *Human Brain Mapping*, 30, 3887-3900
- Bull, R. & Johnston, R. S. (1997). Children's arithmetical difficulties: Contributions from processing speed, item identification, and short-term memory. *Experimental Child Psychology*, 65, 1-24
- Bull, R., Johnston, R. S., & Roy, R. S. (1999). Exploring the roles of the visual-spatial sketch pad and central executive in children's arithmetical skills: views from cognition and developmental neuropsychology. *Developmental Neuropsychology*, 15, 421-442
- Bunge, S.A., Wendelken, C., Badre, D., & Wagner, A.D. (2005). Analogical reasoning and prefrontal cortex: Evidence for separable retrieval and integration mechanisms. *Cerebral Cortex*, 15, 239-249
- Burani, C., Barca, L., & Arduino, S.L. (2001). Una base di dati sui valori di eta' di acquisizione, frequenza, familiarita', immaginabilita', concretezza, e altre variabili lessicali e sub-lessicali per 626 nomi dell'italiano. *Giornale Italiano di Psicologia*, 4, 839-854
- Burr D., & Ross J. (2008). A visual sense of number. *Current Biology*, 18(6), 425-428
- Butterworth, B. (1999). *The Mathematical Brain*. London: Macmillan.

- Butterworth, B. (2003). *Dyscalculia Screener*. London: nferNelson Publishing Company Ltd.
- Butterworth, B. (2005). Developmental Dyscalculia. In: J.I.D. Campbell (Ed.). *Handbook of mathematical cognition* (pp. 455-467). Hove: Psychology Press
- Butterworth, B. (2006). Mathematical expertise. In: K. A. Ericsson, N. Charness, P. J. Feltovich & R. R. Hoffmann (Eds.), *Cambridge Handbook of Expertise and Expert Performance*. (pp. 553-568). Cambridge: Cambridge University Press
- Butterworth, B. (2010). Foundational numerical capacities and the origins of dyscalculia. *Trends in Cognitive Sciences*, 14(12), 534-541. Review
- Butterworth, B., Cipolotti, L., & Warrington, E. K. (1996). Short-term memory impairment and arithmetical ability. *Quarterly Journal of Experimental Psychology Section A - Human Experimental Psychology*, 49, 251-262
- Butterworth, B., Granà, A., Piazza, M., Girelli, L., Price, C., & Skuse, D. (1999). Language and the origins of number skills: karyotypic differences in Turner's syndrome. *Brain & Language*, 69, 486-488
- Butterworth, B., & Laurillard, D. (2010). Low numeracy and dyscalculia: identification and intervention. *ZDM Mathematics Education*, 42, 527-539
- Butterworth, B., Reeve, R., Reynolds, F., & Lloyd D. (2008). Numerical thought with and without words: Evidence from indigenous Australian children. *Proceedings of the National Academy of Sciences of the United States of America*, 105(35), 13179-13184
- Butterworth, B., & Reigosa-Crespo, V. (2007). Information processing deficits in dyscalculia. In: D. B. Berch & M. M. M. Mazzocco (Eds.), *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (pp. 65-81). Baltimore, MD: Paul H Brookes Publishing Co
- Butterworth, B., Varma, S., & Laurillard, D. (2011). Dyscalculia: from brain to education. *Science*, 332(6033), 1049-1053
- Butterworth, B., & Yeo, D. (2004). *Dyscalculia Guidance*. London: nferNelson
- Bynner, J., & Parsons, S. (1997). *Does numeracy matter?* London: Basic Skills Agency
- Campbell, B.C., & Wang, S.S.-H, (2012). Familial linkage between neuropsychiatric disorders and intellectual interests. *PLoS ONE* 7(1): e30405. doi:10.1371/journal.pone.0030405
- Campbell, J.I.D., & Metcalfe, A.W.S. (2007). Arithmetic rules and numeral format. *European Journal of Cognitive Psychology*, 19(3), 335-355
- Cantlon, J.F., Brannon, E.M., Carter, E.J., & Pelphrey, K.A. (2006). Functional imaging of numerical processing in adults and 4-y-old children. *PLoS Biology*, 4(5): e125
- Cantlon, J.F., Pinel, P., Dehaene, S., & Pelphrey, K.A. (2011). Cortical representations of symbols, objects, and faces are pruned back during early childhood. *Cerebral Cortex*, 21(1), 191-199

- Cappelletti, M., Barth, H., Fregni, F., Spelke, E.S., Pascual-Leone, A. (2007). rTMS over the intraparietal sulcus disrupts numerosity processing. *Experimental Brain Research*, 179(4), 631-642
- Carey, S. (2004). *On the origin of concepts*. New York: Oxford University Press
- Carper, R.A., Moses, P., Tigue, Z.D., & Courchesne, E. (2002). Cerebral lobes in autism: early hyperplasia and abnormal age effects. *NeuroImage*, 16, 1038-1051
- Case, R. & Sowder, J.T. (1990). The development of computational estimation: a neo-Piagetian analysis. *Cognition and Instruction*, 7, 79-104
- Castelli, F., Glaser, D. E., & Butterworth, B. (2006). Discrete and analogue quantity processing in the parietal lobe: A functional MRI study. *Proceedings of the National Academy of Sciences of the United States of America*, 103(12), 4693-4698
- Castelli, F., Happè, F., Frith, U. & Frith, C. (2000). Movement and mind: a functional imaging study of perception and interpretation of complex intentional movement patterns. *NeuroImage*, 12, 314-325
- Chiang, H-M., & Lin, Y-H. (2007). Mathematical ability of students with Asperger syndrome and high-functioning autism: a review of literature. *Autism*, 11, 547-557
- Cho, S., Ryali, S., Geary, D.C., & Menon V. (2011). How does a child solve $7 + 8$? Decoding brain activity patterns associated with counting and retrieval strategies. *Developmental Science*, 14(5), 989-1001
- Christoff, K., Prabhakaran, V., Dorfman, J., Zhao, Z., Kroger, J.K., Holyoak, K.J., & Gabrieli J.D. (2001). Rostrolateral prefrontal cortex involvement in relational integration during reasoning. *NeuroImage*, 14, 1136-1149
- Church, R.M., & Meck, W.H. (1984). The numerical attribute of stimuli. In: H.L., Roitblat, T.G., Bever, & H.S., Terrace (Eds.), *Animal Cognition*. Hillsdale: NJ: Erlbaum
- Cipolotti, L., & van Harskamp, N. (2001). Disturbances of number processing and calculation. In: R. S. Berndt (Ed.), *Handbook of Neuropsychology* (2nd. ed., Vol. 3, pp. 305-334). Amsterdam: Elsevier Science
- Clearfield, M.W. & Mix, K.S. (1999). Number versus contour length in infants' discrimination of small visual sets. *Infant Behavior and Development*, 23, 441-460
- Cohen, D.J. (2009). Integers do not automatically activate their quantity representation. *Psychonomic Bulletin and Review*, 16, 332-336
- Cohen, D.J. (2010). Evidence for direct retrieval of relative quantity information in a quantity judgement task: decimals, integers, and the role of physical similarity. *Journal of Experimental Psychology: Learning Memory and Cognition*, 36(6), 1389-1398

- Cohen, J. R., Asarnow, R. F., Sabb, F. W., Bilder, R. M., Bookheimer, S. Y., Knowlton, B. J., & Poldrack, R. A. (2010). Decoding developmental differences and individual variability in response inhibition through predictive analyses across individuals. *Frontiers in Human Neuroscience*, doi: 10.3389/fnhum.2010.00047
- Cohen-Kadosh, R., Cohen-Kadosh, K., Kaas, K., Henik, A., & Goebel, R. (2007). Notation-dependent and -independent representations of numbers in the parietal lobes. *Neuron*, 53(2), 307-314
- Cohen-Kadosh, R., Cohen-Kadosh, K., Schuhmann, T., Kaas, A., Goebel, R., Henik, A., & Sack A.T. (2007). Virtual dyscalculia induced by parietal-lobe TMS impairs automatic magnitude processing. *Current Biology*, 17(8), 689-693
- Cohen-Kadosh, R., Lammertyn, J., & Izard, V. (2008). Are numbers special? An overview of chronometric, neuroimaging, developmental and comparative studies of magnitude representation. *Progress in Neurobiology*, 84(2), 132-147
- Cohen-Kadosh, R., Sagiv, N., Linden, D.E., Robertson, L.C., Elinger, G., & Henik, A. (2005). When blue is larger than red: colors influence numerical cognition in synesthesia. *Journal of Cognitive Neuroscience*, 17, 1766-1773
- Cohen-Kadosh, R., & Walsh, V. (2007). Dyscalculia, *Current Biology*, 17, R946-R947
- Cohen-Kadosh, R., & Walsh, V. (2009). Numerical representation in the parietal lobes: abstract or not abstract? *Behavioral and Brain Sciences*, 32, 313-328; discussion 328-373
- Courchesne, E., Press, G.A., & Yeung-Courchesne, R. (1993). Parietal lobe abnormalities detected with MR in patients with infantile autism. *American Journal of Roentgenology*, 160(2), 387-393
- Courchesne, E., Redcay, E., Morgan, J.T., & Kennedy, D.P. (2005). Autism at the beginning: microstructural and growth abnormalities underlying the cognitive and behavioural phenotype of autism. *Development and Psychopathology*, 17(3), 577-597
- Coutanche, M.N., Thompson-Schill, S.L., & Schultz, R.T. (2011). Multi-voxel pattern analysis of fMRI data predicts clinical symptom severity. *NeuroImage*, 57, 113-123
- Cowan, N. (1995). *Attention and memory: An integrated framework*. New York: Oxford University Press
- Cowan, R., Donlan, C., Shepherd, D.-L., Cole-Fletcher, R., Saxton, M., & Hurry, J. (2011). Basic calculation proficiency and mathematics achievement in elementary school children. *Journal of Educational Psychology*, 103(4), 786-803
- Cowan, R., & Frith, C. (2009). Do calendrical savants use calculation to answer date questions? A functional magnetic resonance imaging study. *Philosophical Transactions of the Royal Society B: Biological Sciences*, 364(1522), 1417-1424

- Crawford, J. R., Garthwaite, P. H., Azzalini, A., Howell, D. C., & Laws, K. R. (2006). Testing for a deficit in single-case studies: effects of departures from normality. *Neuropsychologia*, 44(4), 666-677
- Crawford, J. R., & Howell, D. C. (1998). Comparing an individual's test score against norms derived from small samples. *Clinical Neuropsychologist*, 12(4), 482-486
- Critchley, H. D., Daly, E.M., Bullmore, E.T., Williams, S.C., Van Amelsvoort, T., Robertson, D.M., ... & Murphy, D.G. (2000). The functional neuroanatomy of social behaviour: changes in cerebral blood flow when people with autistic disorder process facial expressions. *Brain*, 123, 2203-2212
- Crone, E.A., Wendelken, C., van Leijenhorst, L., Honomichl, R.D., Christoff, K., & Bunge, S.A. (2009). Neurocognitive development of relational reasoning. *Developmental Science*, 12, 55-66
- Cuingnet, R., Gerardin, E., Tessieras, J., Auzias, G., Lehericy, S., Habert, M.O., ... & Colliot, O. (2011). Automatic classification of patients with Alzheimer's disease from structural MRI: a comparison of ten methods using the ADNI database. *NeuroImage*, 56(2), 766-781
- Dacke, M., & Srinivasan, M.V. (2008). Evidence for counting in insects. *Animal Cognition*, 11(4), 683-689
- de Hevia, M.D., Girelli, L., & Vallar, G. (2006). Numbers and space: a cognitive illusion? *Experimental Brain Research*, 168, 254-264
- de Hevia, M.D., & Spelke, E.S. (2009). Spontaneous mapping of number and space in adults and young children. *Cognition*, 110, 198-207
- De Rammelaere, S., Stuyven, E., & Vandierendonck, A. (1999). The contribution of working memory resources in the verification of simple mental arithmetic sums. *Psychological Research/Psychologische Forschung*, 62, 72-77
- De Rammelaere, S., Stuyven, E., & Vandierendonck, A. (2001). Verifying simple arithmetic sums and products: are the phonological loop and the central executive involved? *Memory & Cognition*, 29, 267-273
- De Smedt, B., Verschaffel, L., & Ghesquière, P. (2009). The predictive value of numerical magnitude comparison for individual differences in mathematics achievement. *Journal of Experimental Child Psychology*, 103(4), 469-479
- deCharms, C.R. (2008). Applications of real-time fMRI. *Nature Reviews Neuroscience*, 9, 720-729
- Dehaene, S. (1992). Varieties of numerical abilities. *Cognition*, 44, 1-42
- Dehaene, S. (1997). *The number sense: How the mind creates mathematics*, New York: Oxford University Press
- Dehaene, S. (2003). The neural basis of the Weber-Fechner law: a logarithmic mental number line. *Trends in Cognitive Sciences*, 7, 145-147

- Dehaene, S. (2009). Origins of mathematical intuitions: the case of arithmetic. *Annals of the New York Academy of Sciences*, 1156, 232-259. Review
- Dehaene, S., Bossini, S., & Giraux, P. (1993). The mental representation of parity and number magnitude. *Journal of Experimental Psychology*, 122(3), 371-396
- Dehaene, S., & Changeux, J.P. (1993). Development of elementary numerical abilities: A neuronal model. *Journal of Cognitive Neuroscience*, 5, 390-407
- Dehaene, S. & Coehen, L. (2007). Cultural recycling of cortical maps. *Neuron*, 56, 384-398
- Dehaene, S., & Cohen, L. (2011). The unique role of the visual word form area in reading. *Trends in Cognitive Sciences*, 15(6), 254-262
- Dehaene, S., Dupoux, E., & Mehler, J. (1990). Is numerical comparison digital? Analogical and symbolic effects in two-digital comparison. *Journal of Experimental Psychology: Human Perception and Performance*, 16, 626-641
- Dehaene, S., Izard, V., Spelke, E., & Pica, P. (2008). Log or linear? Distinct intuitions of the number scale in Western and Amazonian indigene cultures. *Science*, 320(5880), 1217-1220
- Dehaene, S., Molko, N., Cohen, L. & Wilson, A.J. (2004). Arithmetic and the brain. *Current Opinion in Neurobiology*, 14, 218-224. Review
- Dehaene, S., Piazza, M., Pinel, P., & Cohen, L. (2003). Three parietal circuits for number processing. *Cognitive Neuropsychology*, 20, 487-506
- Dehaene, S., Tzourio, N., Frak, V., Raynaud, L., Cohen, L., Mehler, J., & Mazoyer, B. (1996). Cerebral activations during number multiplication and comparison: a PET study. *Neuropsychologia*, 34(11), 1097-106
- Delazer, M., Domahs, F., Barthä, L., Brenneis, C., Lochy, A., Trieb, T., & Benke, T. (2003). Learning complex arithmetic – an fMRI study. *Cognitive Brain Research*, 18(1), 76-88
- Delazer, M., Ischebeck, A., Domahs, F., Zamarian, L., Koppelstaetter, F., Siedentopf, C.M., ... & Felber, S. (2005). Learning by strategies and learning by drill - evidence from an fMRI study. *NeuroImage*, 25(3), 838-849
- Department for Education and Skills -DfES - (2001). *Guidance to support pupils with dyslexia and dyscalculia* (DfES 0512/2011). London: Department for Education and Skills
- DfEE (1999). *The National Numeracy Strategy: Framework for teaching mathematics from Reception to Year 6*. Sudbury, Suffolk: DfEE Publications
- Di Martino, A., Ross, K., Uddin, L.Q., Sklar, A.B., Castellanos, F.X., & Milham, M.P. (2009). Functional brain correlates of social and non-social processes in autism spectrum disorders: an activation likelihood estimation meta-analysis. *Biological Psychiatry*, 65(1), 63-74

- Dickerson Mayes, S., & Calhoun, S. (2003). Ability profiles in children with autism: influence of age and IQ. *Autism*, 7, 65-80
- Diester, I. & Nieder, A. (2007). Semantic associations between signs and numerical categories in the prefrontal cortex. *PLoS Biology*, 5, e294
- Donlan, C., Bishop, D. V. M., & Hitch, G. J. (1998). Magnitude comparisons by children with specific language impairments: evidence of unimpaired symbolic processing. *International Journal of Language & Communication Disorders*, 33, 149-160
- Donlan, C., Cowan, R., Newton, E. J., & Lloyd, D. (2007). The role of language in mathematical development: Evidence from children with specific language impairments. *Cognition*, 103, 23-33
- Dosenbach, N.U.F., Nardos, B., Cohen, A.L., Fair, D.A., Power, J.D., Church, J.A., ... & Schlaggar, B.L. (2010). Prediction of individual brain maturity using fMRI. *Science*, 329, 1358-1361
- Dowker, A. D. (2003). Interventions in numeracy: individualized approaches. In: I. Thompson (Ed.), *Enhancing Primary Mathematics Teaching*. Maidenhead: Open University Press
- Dowker, A. D. (2005). *Individual differences in arithmetic: Implications for psychology, neuroscience and education*. Hove, East Sussex: Psychology
- Duchesne, S., Caroli, A., Geroldi, C., Collins, D. L., & Frisoni, G. B. (2009). Relating one-year cognitive change in mild cognitive impairment to baseline MRI features. *NeuroImage*, 47, 1363-1370
- Duncan, G.J., Dowsett, C.J., Claessens, A., Magnuson, K., Huston, A.C., Klebanov, P., ... & Japel, C. (2007). School readiness and later achievement. *Developmental Psychology*, 43(6), 1428-1446
- Duncan, J. (2001). An adaptive coding model of neural function in prefrontal cortex. *Nature Reviews Neuroscience*, 2(11), 820-829
- Ebersbach, M., Luwel, K., Frick, A., Onghena, P., & Verschaffel, L. (2008). The relationship between the shape of the mental number line and familiarity with numbers in 5-to 9-year old children: evidence for a segmented linear model. *Journal of Experimental Child Psychology*, 99(1), 1-17
- Ecker, C., Marwuand, A., Mourao-Miranda, J., Johnston, P., Daly, E.M., Brammer, M.J., ... & Murphy, D.G.M. (2010). Describing the brain in autism in five dimensions – magnetic resonance imaging-assisted diagnosis of Autism Spectrum Disorder using a Multiparameter Classification Approach. *The Journal of Neuroscience*, 30(32), 10612-10623
- Edward H.F., de Haan, E.H.F., & Cowey, A., (2011). On the usefulness of ‘what’ and ‘where’ pathways in vision. *Trends in Cognitive Sciences*, 10, 460-466
- Elliott, C.D., Smith, P., & McCulloch, K. (1996). *British Ability Scales Second Edition (BAS-II)*. UK: nferNelson Publishing Company Ltd.

- Emmerton, J., Lohmann, A., & Niemann, J. (1997). Pigeon's serial ordering of numerosity with visual arrays. *Animal Learning and Behaviour*, 25(2), 234-244
- Ericsson, K.A., & Charness, N. (1994). Expert performance: its structure and acquisition. *American Psychologist*, 49(8), 725-747
- Ericsson, K. A. & Kintsch, W. (1995). Long-term working memory. *Psychological Review*, 102, 211-245
- Every Child a Chance Trust, *Annual Report 2009/2010*. Published online February 2011 www.everychildachancetrust.org
- Fabbri, S., Tang, J., Zorzi, M., & Butterworth, B. (2009). Interaction between numerosity and proportion in the processing of non-symbolic numerical fractions. Poster presented at *Concepts, Actions and Objects, functional and neural perspectives Workshop*. University of Trento, Italy. 22nd-25th April 2009
- Fehr, T., Weber, J., Willmes, K., & Herrmann, M. (2010). Neural correlates in exceptional mental arithmetic – about the neural architecture of prodigious skills. *Neuropsychologia*, 48, 1407-1416
- Feigenson, L., Dehaene, S., & Spelke, E.S. (2004). Core systems of number. *Trends in Cognitive Sciences*, 8(7), 307-314
- Feigenson, L., Spelke, E.S., & Carey, S. (2002). Infants' discrimination of number vs. continuous extent. *Cognitive Psychology*, 44(1), 33-66
- Fernell, E., & Gillberg, C. (2010). Autism spectrum disorder diagnoses in Stockholm preschoolers. *Research in Developmental Disabilities*, 31, 680-685
- Fias, W. (2001). Two routes for the processing of verbal numbers: evidence from the SNARC effect. *Psychological Research*, 65, 250-259
- Fias, W., Brysbaert, M., Geypens, F., & d'Ydewalle, G. (1996). The importance of magnitude information in numerical processing: evidence from the SNARC effect. *Mathematical Cognition*, 2, 95-110
- Finger, S. (1994). *Origins of Neuroscience. A history of Exploration into Brain Functions*. Oxford University Press
- Fodor, J.A. (1983). *Modularity of Mind*. Cambridge, MA: MIT Press
- Forman, S.D., Cohen, J.D., Fitzgerald, M., Eddy, W.F., Mintun, M.A., & Noll, D.C. (1995). Improved assessment of significant activation in functional magnetic resonance imaging (fMRI): use of a cluster-size threshold. *Magnetic Resonance in Medicine*, 33(5), 636-647
- Frazier, T.W., & Hardan, A.Y. (2009). A meta-analysis of the corpus callosum in autism. *Biological Psychiatry*, 66, 935-941
- Friston, K.J., Holmes, A.P., Worsley, K.J., Poline, J-B., Frith, C.D. & Frackowiak, R.S.J. (1995). Statistical Parametric Maps in functional imaging: A general linear approach. *Human Brain Mapping*, 2, 189-210

- Friston, K.J., Zarahn, E., Josephs, O., Henson, R.N., & Dale, A.M. (1999). Stochastic designs in event-related fMRI. *NeuroImage*, 10(5), 607-619
- Frith, U. (1989). *Autism: explaining the enigma* (pp. 95-102). Oxford: Basil Blackwell
- Fujita, I., Tanaka, K., Ito, M. & Cheng, K. (1992). Columns for visual features of objects in monkey inferotemporal cortex. *Nature*, 360, 343-346
- Fuson, K.C. (1988). *Children's counting and the concepts of number*. New York: Springer-Verlag
- Gabrieli, J.D., Brewer, J.B., Desmond, J.E., & Glover, G.H. (1997). Separate neural bases of two fundamental memory processes in the human medial temporal lobe. *Science*, 276, 264-266
- Gallistel, C.R., & Gelman, R. (1992). Preverbal and verbal counting and computation. *Cognition*, 44, 43-74
- Gallistel, C.R., & Gelman, R. (2000). Non-verbal numerical cognition: From reals to integers. *Trends in Cognitive Sciences*, 4(2), 59-65
- Galton, F. (1880). Visualized numerals. *Nature*, 21, 252-256
- Galton, F. (1979). *Hereditary genius: an inquiry into its laws and consequences* (originally published in 1869). London: Julian Friedman Publishers
- Gardner, H. (1983). *Frames of Mind: The theory of Multiple Intelligences*. New York: Basic Books
- Gathercole, S. E. & Pickering, S. J. (2000). Working memory deficits in children with low achievements in the national curriculum at 7 years of age. *British Journal of Educational Psychology*, 70, 177-194
- Gauthier, I., Skudlarski, P., Gore, J. C. & Anderson, A. W. (2000). Expertise for cars and birds recruits brain areas involved in face recognition. *Nature Neuroscience*, 3, 191-197
- Gauthier, I., Tarr, M. J., Anderson, A. W., Skudlarski, P. & Gore, J. C. (1999). Activation of the middle fusiform face area increases with expertise in recognizing novel objects. *Nature Neuroscience*, 2, 568-573
- Geary, D. C. (1990). A componential analysis of an early learning deficit in mathematics. *Journal of Experimental Child Psychology*, 49, 363-383
- Geary, D. C. (1993). Mathematical disabilities: Cognition, neuropsychological and genetic components. *Psychological Bulletin*, 114, 345-362
- Geary, D. C. (1994). *Children's mathematical development: Research and practical applications*. Washington, DC: American Psychological Association
- Geary, D.C. (2011a). Cognitive predictors of achievement growth in mathematics: a 5-year longitudinal study. *Developmental Psychology*, 47(6), 1539-1552
- Geary, D.C. (2011b). Consequences, characteristics, and causes of mathematical learning disabilities and persistent low achievement in mathematics. *Journal of Developmental and Behavioral Pediatrics*, 32(3), 250-263

- Geary, D.C., Bailey, D.H., Littlefield, A., Wood, P., Hoard, M.K., & Nugent, L. (2009). First-grade predictors of mathematical learning disability: a latent class trajectory analysis. *Cognitive Development*, 24(4), pii:4
- Geary, D.C., & Brown, S. (1991). Cognitive addition: strategy choice and speed-of-processing differences in gifted, normal, and mathematically disabled children. *Developmental Psychology*, 27, 398-406
- Geary, D.C., Hamson, C.O., & Hoard, M.K. (2000). Numerical and arithmetical cognition: a longitudinal study of process and concept deficits in pupils with learning disability. *Journal of Experimental Pupil Psychology*, 77, 236-263
- Geary, D.C., & Hoard, M.K. (2001). Numerical and arithmetical deficits in learning-disabled pupils: relation to dyscalculia and dyslexia. *Aphasiology*, 15(7), 635-647
- Geary, D. C., & Hoard, M. K. (2005). Learning disabilities in arithmetic and mathematics: Theoretical and empirical perspectives. In: J. I. D. Campbell (Ed.), *Handbook of Mathematical Cognition* (pp. 253-268). New York: Psychology Press
- Geary, D.C., Hoard, M.K., Byrd-Craven, J., & DeSoto, C.M. (2004). Strategy choices in simple and complex addition: contributions of working memory and counting knowledge for children with mathematical disability. *Journal of Experimental Child Psychology*, 88, 121-151
- Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development*, 78, 1343-1359
- Geary, D.C., Hoard, M.K., & Hamson, C.O. (1999). Numerical and arithmetical cognition: patterns of functions and deficits in children at risk for a mathematical disability. *Journal of Experimental Child Psychology*, 74(3), 213-239
- Geary, D.C., Hoard, M.K., Nugent, L., & Byrd-Craven, J. (2008). Development of number line representations in children with mathematical learning disability. *Developmental Neuropsychology*, 33, 277-299
- Gelman, R., & Butterworth, B. (2005). Number and language: how are they related? *Trends in Cognitive Sciences*, 9(1), 6-10
- Gelman, R., & Gallistel, C.R. (1978). *The child's understanding of number*. Cambridge, Mass: Harvard University Press
- Giaquinto, M. (1995). Concepts and calculation. *Mathematical Cognition*, 1, 61-81
- Gibson, E.P., Gibson, J.J., Pick, A.D., & Osser, H. (1962). A developmental study of the discrimination of letter-like forms. *Journal of Comparative and Physiological Psychology*, 55, 897-906
- Gilmore, C. K., McCarthy, S. E., & Spelke, E. S. (2007). Symbolic arithmetic knowledge without instruction. *Nature*, 447, 589-592

- Gilmore, C., McCarthy, S.E., & Spelke, E.S. (2010). Non-symbolic arithmetic abilities and mathematics achievement in the first year of formal schooling. *Cognition*, 115(3), 394-406
- Ginsburg, H.P. (1997). Mathematics learning disabilities: a view from developmental psychology. *Journal of Learning Disabilities*, 30, 20-33
- Girelli, L., Lucangeli, D., & Butterworth, B. (2000). The development of automaticity in accessing number magnitude. *Journal of Experimental Child Psychology*, 76(2), 104-122
- Grabner, R.H., Ansari, D., Koschutnig, K., Reishofer, G., Ebner, F., & Neuper, C. (2009). To retrieve or to calculate? Left angular gyrus mediates the retrieval of arithmetic facts during problem solving. *Neuropsychologia*, 47(2), 604-608
- Grabner, R.H., Ansari, D., Reishofer, G., Stern, E., Ebner, F., & Neuper C. (2007). Individual differences in mathematical competence predict parietal brain activation during mental calculation. *NeuroImage*, 38(2), 346-356
- Grelotti, D., Gauthier, I. & Schultz, R. T. (2001). Social interest and the development of cortical face specialization: what autism teaches us about face processing. *Developmental Psychobiology*, 40, 213-225
- Gross-Tsur, V., Manor, O. & Shalev, R.S. (1996). Developmental Dyscalculia: Prevalence and demographic features. *Developmental Medicine and Child Neurology*, 38(1), 25-33
- Gruber, O., Indefrey, P., Steinmetz, H., & Kleinschmidt, A. (2001). Dissociating neural correlates of cognitive components in mental calculation. *Cerebral Cortex*, 11(4), 350-359
- Hadjikhani, N., Joseph, R.M., Snyder, J., & Tager-Flusberg, H. (2006). Anatomical differences in the mirror neuron system and social cognition network in autism. *Cerebral Cortex*, 16, 1276-1282
- Halberda, J., Mazocco, M. & Feigenson, L. (2008). Individual differences in nonverbal number acuity predict maths achievement. *Nature*, 455, 665-668
- Halgren, E., Baudena, P., Heit, G., Clarke, J.M., Marinkovic, K., & Clarke, M. (1994). Spatio-temporal stages in face and word processing. I. Depth-recorder potentials in the human occipital, temporal and parietal lobes. *Journal of Physiology - Paris*, 88(1), 1-50
- Hanich, L.B., Jordan, N.C., Kaplan, D., & Dick, J. (2001). Performance across different areas of mathematical cognition in children with learning difficulties. *Journal of Educational Psychology*, 93(3), 615-626
- Happè, F.G. (1994). Wechsler IQ profile and theory of mind in autism: a research note. *Journal of Child Psychology and Psychiatry*, 35, 1461-1471
- Happè, F.G., & Frith, U. (2006). The weak coherence account: detail-focused cognitive style in autism spectrum disorders. *Journal of Autism and Developmental Disorders*, 36, 5-25

- Happè, F. G. & Frith, U. (2010). *Autism and talent*. New York: Oxford University press
- Hardan, A.Y., Libove, R.A., Keshavan, M.S., Melhem, N.M., & Minshew, N.J. (2009). A preliminary longitudinal magnetic resonance imaging study of brain volume and cortical thickness in autism. *Biological Psychiatry*, 66, 320-326
- Hardan, A.Y., Minshew, N.J., Mallikarjunn, M., & Keshavan, M.S. (2001). Brain volume in autism. *Journal of Child Neurology*, 16, 421-424
- Hartnett, P. M., & Gelman, R. (1998). Early understandings of numbers: Paths or barriers to the construction of new understandings? *Learning and Instruction: The Journal of the European Association for Research in Learning and Instruction*, 8(4), 341-374
- Hasemann, K. (1981). On difficulties with fractions. *Educational Studies in Mathematics*, 12, 71-87
- Haxby, J.V., Hoffman, E.A., & Gobbini, M.I. (2000). The distributed human neural system for face perception. *Trends in Cognitive Sciences*, 4, 223-233
- Hayes, J.R. (1973). On the function of visual imagery in elementary mathematics. In: W. Chase (Ed.), *Visual Information Processing* (pp. 177-214). New York: Academic Press
- Haynes, J.D., Katsuyuki Sakai, K., Rees, G., Gilbert, S., Frith, C., & Passingham, R.E. (2007). Reading hidden intentions in the human brain. *Current Biology*, 17, 323-328
- Haynes, J.D., & Rees, G. (2006). Decoding mental states from brain activity in humans. *Nature Reviews Neuroscience*, 7, 523-534
- Hazlett, H.C., Poe, M.D., Gerig, G., Smith, R.G., & Piven, J. (2006). Cortical gray and white brain tissue volume in adolescents and adults with autism. *Biological Psychiatry*, 59, 1-6
- Haznedar, M.M., Buchsbaum, M.S., Metzger, M., Solimando, A., Spiegel-Cohen, J., & Hollander, E. (1997). Anterior cingulate gyrus volume and glucose metabolism in autistic disorder. *American Journal of Psychiatry*, 154, 1047-1050
- Heathcote, D. (1994). The role of visuo-spatial working memory in the mental addition of multi-digit addends. *Current Psychology of Cognition*, 13, 207-245
- Heavey, L., Pring, L., & Hermelin, B. (1999). A date to remember: the nature of memory in savant calendrical calculators. *Psychological Medicine*, 29, 145-160
- Hecht, S.A. (2002). Counting on working memory in simple arithmetic when counting is used for problem solving. *Memory & Cognition*, 30, 447-455
- Herbert, M.R., Harris, G.J., Adrien, K.T., Ziegler, D.A., Makris, N., Kennedy, D.N., ... & Caviness Jr., V.S. (2002). Abnormal asymmetry in language association cortex in autism. *Annals of Neurology*, 52, 588-596

- Herbert, M.R., Ziegler, D.A., Deutsch, C.K., O'Brien, L.M., Lange, N., Bakardjiev, A., ... & Caviness, V.S. Jr. (2003). Dissociations of cerebral cortex, subcortical and cerebral white matter volumes in autistic boys. *Brain*, 126, 1182-1192
- Herbert, M.R., Ziegler, D.A., Makris, N., Filipek, P.A., Kemper, T.L., Normandin, J.J., ... & Caviness, V.S. Jr. (2004). Localization of white matter volume increase in autism and developmental language disorder. *Annals of Neurology*, 55, 530-540
- Hermelin, B., & O'Connor, N. (1986). Idiot savant calendrical calculators: rules and regularities. *Psychological Medicine*, 16, 885-893
- Hetch, S.A., Torgensen, J.K., Wagner, R.K., & Rashotte, C.A. (2001). The relations between phonological processing abilities and emerging individual differences in mathematical computation skills: a longitudinal study from second to fifth grades. *Journal of Experimental Pupil Psychology*, 79, 192-227
- Hinrichs, J.V., Yurko, D.D., & Hu, J. (1981). Two-digit number comparison: use of place information. *Journal of Experimental Psychology: Human Perception and Performance*, 7, 890-901
- Hitch, G. J. (1978). The role of short-term working memory in mental arithmetic. *Cognitive Psychology*, 10, 302-323
- Hitch, G., & McCauley, E. (1991). Working memory in children with specific arithmetical learning difficulties. *British Journal of Psychology*, 82(3), 375-386
- Hobson, R. P., Ouston, J. & Lee, A. (1988). What's in a face? The case of autism. *British Journal of Psychology*, 79, 441-453
- Holland, R, Leff, A.P., Josephs, O., Galea, J.M., Desikan, M., Price, C.J., ... & Crinion, J. (2011). Speech facilitation by left inferior frontal cortex stimulation. *Current Biology*, 21(16), 1403-1407
- Hollander, E., Anagnostou, E., Chaplin, W., Esposito, K., Haznedar, M.M., Licalzi, E., ... & Buchsbaum, M. (2005). Striatal volume on magnetic resonance imaging and repetitive behaviors in autism. *Biological Psychiatry*, 58, 226-232
- Holloway, I.D., & Ansari D. (2009). Mapping numerical magnitudes onto symbols: the numerical distance effect and individual differences in children's mathematics achievement. *Journal of Experimental Child Psychology*, 103(1), 17-29
- Horwitz, W.A., Deming, W.E., & Winter, R.F. (1969). A further account of the idiot savants: Experts with the calendar. *American Journal of Psychiatry*, 126, 160-162
- Hoyles, C., Noss, R., & Pozzi, S. (2001). Proportional reasoning in nursing practice. *Journal for Research in Mathematics Education*, 32(1), 14-27
- Humphreys, K., Hasson, U., Avidan, G., Minshew, N., & Behrmann, M. (2008). Cortical patterns of category-selective activation for faces, places and objects in adults with autism. *Autism Research*, 1, 52-63

- Hyde, D.C. (2011). Two systems of non-symbolic numerical cognition. *Frontiers in Human Neuroscience*, doi: 10.3389/fnhum.2011.00150
- Hyde, D.C., Boas, D.A., Blair, C., & Carey S. (2010). Near-infrared spectroscopy shows right parietal specialization for number in pre-verbal infants. *NeuroImage*, 53(2), 647-652
- Hyde, K.L., Samson, F., Evans, A.C., & Mottron, L. (2010). Neuroanatomical differences in brain areas implicated in perceptual and other core features of autism revealed by cortical thickness analysis and voxel-based morphometry. *Human Brain Mapping*, 31, 556-566
- Isaacs, E. B., Edmonds, C. J., Lucas, A., & Gadian, D. G. (2001). Calculation difficulties in children of very low birth weight: A neural correlate. *Brain*, 124, 1701-1707
- Ischebeck, A., Schocke, M., & Delazer, M. (2007). The processing and representation of fractions within the brain: an fMRI investigation. *NeuroImage*, 47(1), 403-413
- Ischebeck, A., Zamarian, L., Eger, K., Schocke, M., & Delazer, M. (2007). Imaging early practice effects in arithmetic. *NeuroImage*, 36, 993-1003
- Ischebeck, A., Zamarian, L., Siedentopf, C., Koppelstätter, F., Benke, T., Felber, S., & Delazer, M. (2006). How specifically do we learn? Imaging the learning of multiplication and subtraction. *NeuroImage*, 30(4), 1365-1375
- Ito, Y., & Hatta, T. (2004). Spatial structure of quantitative representation of numbers: evidence from the SNARC effect. *Memory and Cognition*, 32, 662-673
- Izard, V., Dehaene-Lambertz, G., & Dehaene, S. (2008). Distinct cerebral pathways for object identity and number in human infants. *PLoS Biology*, 6(2):e11
- Jacob, S.N., & Nieder, A. (2009). Notation-independent representation of fractions in the human parietal cortex. *The Journal of Neuroscience*, 29(14), 4652-4657
- James, W. (1890). *Principles of Psychology* (pp. 403-404). New York: Holt
- Jarrold, C., Russell, J. (1997). Counting abilities in autism: possible implications for central coherence theory. *Journal of Autism and Developmental Disorders*, 27(1), 25-37
- Jensen, A.R. (1990). Speed of information-processing in a calculating prodigy. *Intelligence*, 14(3), 259-274
- Jolliffe, T. & Baron-Cohen, S. (1997). Are people with autism and Asperger Syndrome faster than normal on the Embedded Figures Test? *Journal of Child Psychology and Psychiatry*, 38, 527-553
- Jones, C.R., Happé, F., Golden, H., Marsden, A.J., Tregay, J., Simonoff, E., ... & Charman, T. (2009). Reading and arithmetic in adolescents with autism spectrum disorders: peaks and dips in attainment. *Neuropsychology*, 23(6), 718-728

- Jordan, N. C., Kaplan, D., & Hanich, L. B. (2002). Achievement growth in children with learning difficulties in mathematics: findings of a two-year longitudinal study. *Journal of Educational Psychology*, 94(3), 586-597
- Jordan, N. & Montani, T. (1997). Cognitive arithmetic and problem solving: a comparison of pupils with specific and general mathematics difficulties. *Journal of Learning Disabilities*, 30(6), 624-634
- Just, M.A., Cherkassky, V.L., Keller, T.A., & Minshew, N.J. (2004). Cortical activation and synchronization during sentence comprehension in high-functioning autism: evidence of underconnectivity. *Brain*, 127, 1811-1821
- Kahneman, D. (2003). A perspective on judgement and choice: mapping bounded rationality. *American Psychologist*, 58(9), 697-720
- Kahnt, T., Heinzle, J., Park, S. Q., & Haynes, J-D. (2011). Decoding different roles for vmPFC and dlPFC in multi-attribute decision making. *NeuroImage*, 56, 709-715
- Kallay, A.Y., Tzelgov, J. (2009). A generalized fraction: an entity smaller than one of the mental number line. *Journal of Experimental Psychology: Human Perception and Performance*, 35(6), 1845-1864
- Kalu, U.G., Sexton, C.E., Loo, C.K., & Ebmeier, K.P. (2012). Transcranial direct current stimulation in the treatment of major depression: a meta-analysis. *Psychological Medicine*, 12, 1-10, doi:10.1017/S0033291711003059
- Kanner, L. (1943). Autistic disturbances of affective contact. *Nerv. Child*, 2, 217-250
- Kanwisher, N. (2000). Domain specificity in face perception. *Nature Neuroscience*, 3, 759-763
- Karolis, V., Iuculano, T., & Butterworth, B. (2011). Mapping numerical magnitudes along the right lines: Differentiating between scale and bias. *Journal of Experimental Psychology: General*, 140(4), 693-706
- Kaufmann, L., Koppelstaetter, F., Siedentopf, C., Haala, I., Haberlandt, E., Zimmerhackl, L.B., ... & Ischebeck, A. (2006). Neural correlates of the number-size interference task in children. *Neuroreport*, 17(6), 587-591
- Kemner, C., Verbaten, M.N., Cuperus, J.M., Camferman, G., & van Engeland, H. (1995). Auditory event-related brain potentials in autistic children and three different control groups. *Biological Psychiatry*, 38, 150-165
- Kennedy, D.P., & Courchesne, E. (2008). Functional abnormalities of the default network during self- and other-reflection in autism. *Social Cognitive and Affective Neuroscience*, 3, 177-190
- Keselman, H.J., & Rogan, J.C. (1980). Repeated measures F tests and psychophysiological research: Controlling the number of false positives. *Psychophysiology*, 17, 499-503

- Kim, D.H., Adalsteinsson, E., Glover, G.H., & Spielman, D.M. (2002). Regularized higher-order in vivo shimming. *Magnetic Resonance in Medicine*, 48(4), 715-722
- Kirby, J.R., & Becker, L.D. (1988). Cognitive components of learning problems in arithmetic. *Remedial and Special Education*, 9, 7-16
- Kirwan, C.B., & Stark, C.E. (2004). Medial temporal lobe activation during encoding and retrieval of novel face-name pairs. *Hippocampus*, 14, 919-930
- Klin, A., Sparrow, S. S., de Bildt, A., Cicchetti, D. V., Cohen, D. J. & Volkmar, F. R. (1999). A normed study of face recognition in autism and related disorders. *Journal of Autism and Developmental Disorders*, 29, 499-508
- Knaus, T.A., Silver, A.M., Dominick, K.C., Schuring, M.D., Shaffer, N., Lindgren, K.A., ... & Tager-Flusberg, H. (2009). Age-related changes in the anatomy of language regions in autism spectrum disorder. *Brain Imaging and Behavior*, 3, 51-63
- Knops, A., Nuerk, H.C., Fimm, B., Vohn, R., & Willmes, K. (2006). A special role for numbers in working memory? An fMRI study. *NeuroImage*, 29(1), 1-14
- Koehler, O. (1951). The ability of birds to 'count'. *Bulletin of Animal Behavior*, 9, 41-45
- Koontz, K.L., & Berch, D.B. (1996). Identifying simple numerical stimuli: processing inefficiencies exhibited by arithmetic learning disabled pupils. *Mathematical Cognition*, 2(1), 1-23
- Kosaka, H., Omori, M., Munesue, T., Ishitobi, M., Matsumura, Y., Takahashi, T., ... & Wada, Y. (2010). Smaller insula and inferior frontal volumes in young adults with pervasive developmental disorders. *NeuroImage*, 50, 1357-1363
- Kosc, L. (1974). Developmental dyscalculia. *Journal of Learning Disabilities*, 7, 165-171
- Koshino, H., Carpenter, P.A., Minshew, N.J., Cherkassky, V.L., Keller, T.A., & Just, M.A. (2005). Functional connectivity in an fMRI working memory task in high-functioning autism. *NeuroImage*, 24, 810-821
- Kovas, Y., Harlaar, N., Petrill, S.A., & Plomin, R. (2006). 'Generalist genes' and mathematics in 7-year-old twins. *Intelligence*, 35(5), 473-489
- Kriegeskorte, N., Goebel, R., & Bandettini, P. (2006). Information-based functional brain mapping. *Proceedings of the National Academy of Sciences of the United States of America*, 103, 3863-3868
- Kucian, K., Grond, U., Rotzer, S., Henzi, B., Schönmann, C., Plangger, F., ... & von Aster, M. (2011). Mental number line training in children with developmental dyscalculia. *NeuroImage*, 57(3), 782-795

- Kucian, K., Loenneker, T., Dietrich, T., Dosch, M., Martin, E., & von Aster M. (2006). Impaired neural networks for approximate calculation in dyscalculic children: a functional MRI study. *Behavioral and Brain Functions*, 5, 2-31
- Kuriki, S., Takeuchi, F., & Hirata, Y. (1998). Neural processing of words in the human extrastriate visual cortex. *Brain Research Cognitive Brain Research*, 6(3), 193-203
- Kwong, K.K., Belliveau, J.W., Chesler, D.A., Goldberg, I.E., Weisskoff, R.M., Poncelet, B.P., ... & Turner, R. (1992). Dynamic magnetic resonance imaging of human brain activity during primary sensory stimulation. *Proceedings of the National Academy of Sciences of the United States of America*, 89, 5675-5679
- Landerl, K., Bevan, A., & Butterworth, B. (2004). Developmental Dyscalculia and basic numerical capacities: a study of 8-9 year old students. *Cognition*, 93, 99-125
- Landerl, K., Fussenegger, B., Moll, K., & Willburger, E. (2009). Dyslexia and dyscalculia: two learning disorders with different cognitive profiles. *Journal of Experimental Child Psychology*, 103(3), 309-324
- Landerl, K., & Kölle, C. (2009). Typical and atypical development of basic numerical skills in elementary school. *Journal of Experimental Child Psychology*, 103(4), 546-565
- Le Corre, M., & Carey, S. (2007). One, two, three, four, nothing more: An investigation of the conceptual sources of the verbal counting principles. *Cognition*, 105(2), 395-438
- LeFevre, J.A., Greenham, S.L., & Waheed, N. (1993). The development of procedural and conceptual knowledge in computational estimation. *Cognition and Instruction*, 11, 95-132
- LeFevre, J.A., Smith-Chant, B.L., Hiscock, K., Daley, K.E., & Morris, J. (2003). Young adults' strategies in simple arithmetic: implications for the development of mathematical representation. In: A. Baroody & A. Dowker (Eds.), *The development of arithmetical concepts and skills* (pp. 203-228). Mahwah, NJ: Lawrence Erlbaum Associates, Inc
- Leung, H.C., Gore, J.C., & Goldman-Rakic, P.S. (2002). Sustained mnemonic response in the human middle frontal gyrus during on-line storage of spatial memoranda. *Journal of Cognitive Neuroscience*, 14(4), 659-671
- Levitt, J.G., Blanton, R.E., Smalley, S., Thompson, P.M., Guthrie, D., McCracken, J.T., ... & Toga, A.W. (2003). Cortical sulcal maps in autism. *Cerebral Cortex*, 13, 728-735
- Lewis, C., Hitch, G., & Walker, P. (1994). The prevalence of specific arithmetic difficulties and specific reading difficulties in 9-and 10-year old boys and girls. *Journal of Pupil Psychology and Psychiatry*, 35, 283-292
- Lipton, J.S., & Spelke, E.S. (2003). Origins of number sense. Large-number discrimination in human infants. *Psychological Science*, 14(5), 396-401

- Logie, R. H., Gilhooly, K. J., & Wynn, V. (1994). Counting on working memory in arithmetic problem solving. *Memory and Cognition*, 22, 395-410
- Mack, N. (1995). Confounding whole-number and fraction concepts when building on informal knowledge. *Journal for Research Mathematics Education*, 26, 422-441
- Mahon, B.Z., & Caramazza, A. (2011). What drives the organization of object knowledge in the brain? *Trends in Cognitive Sciences*, 15(3), 97-103
- Martos-Pérez, J., & Ayuda-Pascual, R. (2003). Autism and hyperlexia. *Rev. Neurol. (Paris)*, 36(1), S57-S60
- Matsuzawa, T. (1985). Use of numbers by chimpanzee. *Nature*, 315(6014), 57-59
- Mayes, S.D., & Calhoun, S.L. (2006). Frequency of reading, math, and writing disabilities in children with clinical disorders. *Learning and Individual Differences*, 16(2), 145-157
- Mayes, S.D., & Calhoun, S.L. (2007). Learning, attention, writing, and processing speed in typical children and children with ADHD, autism, anxiety, depression, and oppositional-defiant disorder. *Child Neuropsychology*, 13(6), 469-493
- Mayes, S.D., & Calhoun, S.L. (2008). WISC-IV and WIAT-II profiles in children with high-functioning autism. *Journal of Autism and Developmental Disorders*, 38(3), 428-439
- Mazzocco, M.M.M. (2007). Defining and differentiating mathematical learning disabilities and difficulties. In: D.B. Berch & M.M.M. Mazzocco, (Eds.), *Why is math so hard for some children?* (pp. 29-47). Baltimore: Paul H. Brookes Publishing Co
- Mazzocco, M.M.M., & McCloskey, M. (2005). Math performance in girls with Turner or Fragile X syndrome. In: J. I. D. Campbell (Ed.), *Handbook of Mathematical Cognition* (pp. 269-297). New York: Psychology Press
- Mazzocco, M.M.M., & Devlin, K.T. (2008). Parts and 'holes': gaps in rational number sense among children with vs. without mathematical learning disabilities. *Developmental Science*, 11(5), 681-691
- Mazzocco, M.M.M., Feigenson, L., & Halberda, J. (2011). Impaired acuity of the approximate number system underlies mathematical learning disability (Dyscalculia). *Child Development*, 82(4), 1224-1237
- Mazzocco, M.M.M. & Myers, G.F. (2003). Complexities in identifying and defining mathematics learning disability in the primary school years. *Annals of Dyslexia*, 53, 218-253
- McAlonan, G.M., Cheung, V., Cheung, C., Suckling, J., Lam, G.Y., Tai, K.S., ... & Chua, S.E., (2005). Mapping the brain in autism. A voxel-based MRI study of volumetric differences and intercorrelations in autism. *Brain*, 128, 268-276

- McComb, K., Packer, C., & Pusey, A. (1994). Roaring and numerical assessment in contests between groups of female lions, *Panthera leo*. *Animal Behaviour*, 47, 379-387
- McLean, J.F., & Hitch, G.J. (1999). Working memory impairments in children with specific arithmetic learning difficulties. *Journal of Experimental Child Psychology*, 74(3), 240-260
- McMullen, P. (2000). The gifted side of Autism. *Focus on Autism and Other Developmental Disabilities*, 15, 239-242
- Mechner, F. (1958). Probability relations within response sequences under ratio reinforcement. *Journal of Experimental Animal Behavior Processes*, 1(2), 109-121
- Meck, W.H., & Church, R.M. (1983). A mode control model of counting and timing processes. *Journal of Experimental Psychology: Animal Behavior Processes*, 9(3), 320-334
- Meert, G., Grégoire, J., & Noël, M-P. (2009). Rational numbers: componential versus holistic representation of fractions in a magnitude comparison task. *The Quarterly Journal of Experimental Psychology*, 62(8), 1598-1616
- Mei, L., Xue, G., Chen, C., Xue, F., Zhang, M., & Dong, Q. (2010). The "visual word form area" is involved in successful memory encoding of both words and faces. *NeuroImage*, 52(1), 371-378
- Mejias, S., Mussolin, C., Rousselle, L., Grégoire, J., & Noël M-P. (2011). Numerical and nonnumerical estimation in children with and without mathematical learning disabilities. *Child Neuropsychology*, doi:10.1080/09297049.2011.625355
- Menon, V., Rivera, S.M., White, C.D., Glover, G.H., & Reiss, A.L. (2000). Dissociating prefrontal and parietal cortex activation during arithmetic processing. *NeuroImage*, 12(4), 357-365
- Milner, A.D. & Goodale, M.A. (1995). *The visual brain in action*. Oxford: Oxford University Press
- Miniussi, C., Cappa, S.F., Cohen, L.G., Floel, A., Fregni, F., Nitsche, M.A., ... & Walsh V. (2008). Efficacy of repetitive transcranial magnetic stimulation/transcranial direct current stimulation in cognitive neurorehabilitation. *Brain Stimulation*, 1(4), 326-336. Review
- Mitchell, F.D. (1907). Mathematical prodigies. *American Journal of Psychology*, 18(1), 61-143
- Miyake, A., Friedman, N. P., Emerson, M. J., Witzki, A. H., Howerter, A., & Wager, T. D. (2000). The unity and diversity of executive functions and their contributions to complex "frontal lobe" tasks: a Latent Variable Analysis. *Cognitive Psychology*, 41, 49-100

- Moeller, K., Pixner, S., Kaufmann, L., & Nuerk, H-C. (2009). Children's early mental number line: Logarithmic or decomposed linear? *Journal of Experimental Child Psychology*, 103, 503-515
- Molko, N., Cachia, A., Rivière, D., Mangin, J.F., Bruandet, M., Le Bihan, D., ... & Dehaene, S. (2003). Functional and structural alterations of the intraparietal sulcus in a developmental dyscalculia of genetic origin. *Neuron*, 40(4), 847-858
- Molko, N., Cachia, A., Riviere, D., Mangin, J.F., Bruandet, M., LeBihan, D., ... & Dehaene S. (2004). Brain anatomy in Turner syndrome: evidence for impaired social and spatial-numerical networks. *Cerebral Cortex*, 14(8), 840-850
- Monuteaux, M. C., Faraone, S. V., Herzig, K., Navsaria, N., & Biederman, J. (2005). ADHD and Dyscalculia: evidence for independent familial transmission. *Journal of Learning Disabilities*, 38(1), 89-93
- Moro, R. (2008). *Control and maintenance processes in Working Memory: neuropsychological investigations*. PhD thesis, University College London
- Moyer, R.S., & Landauer, T.K. (1967). The time required for judgments of numerical inequality. *Nature*, 215, 1519-1520
- Muller, K.R., Mika, S., Ratsch, G., Tsuda, K., & Scholkopf, B. (2001). An introduction to kernel-based learning algorithms. *IEEE Transactions on Neural Networks*, 12, 181-201
- Muller, R.A. (2008). From loci to networks and back again: anomalies in the study of autism. *Annals of the New York Academy of Sciences*, 1145, 300-315
- Mundy, E., & Gilmore, C.K. (2009). Children's mapping between symbolic and nonsymbolic representations of number. *Journal of Experimental Child Psychology*, 103(4), 490-502
- Nardone, R., Bergmann, J., Christova, M., Caleri, F., Tezzon, F., Ladurner, G., ... & Golaszewski, S. (2012). Effect of transcranial brain stimulation for the treatment of Alzheimer disease: a review. *International Journal of Alzheimer's Disease*, doi: 10.1155/2012/687909
- Nathan, S.S., Sinha, S.R., Gordon, B., Lesser, R.P., & Thakor, N.V. (1993). Determination of current density distributions generated by electrical stimulation of the human cerebral cortex. *Electroencephalography and Clinical Neurophysiology*, 86, 183-192
- National Mathematics Advisory Panel (NMAP) (2008). *The final report of the national mathematics advisory panel*. Washington DC: US. Department of Education
- Nestor, A., Plaut, D.C., & Behrmann, M. (2011). Unrevealing the distributed neural code of facial identity through spatiotemporal pattern analysis. *Proceedings of the National Academy of Sciences of the United States of America*, 108(24), 9998-10003

- Newell, A., & Rosenbloom, P. (1981). Mechanisms of skill acquisition and the law of practice. In: *Cognitive skills and their acquisition* (pp.1-55). J. R. Anderson (Ed.), Hillsdale, NJ: Erlbaum
- Ni, Y., & Zhou, Y. (2005). Teaching and learning fraction and rational numbers: the origins and implications of whole number bias. *Educational Psychologist*, 40, 27-52
- Nichelli, P., Rinaldi, M., & Cubelli, R. (1989). Selective spatial attention and length representation in normal subjects and in patients with unilateral spatial neglect. *Brain and Cognition*, 9, 57-70
- Nieder, A. (2005). Counting on neurons: the neurobiology of numerical competence. *Nature Reviews Neuroscience*, 6(3), 177-190
- Nieder, A., Freedman, D.J., & Miller, E.K. (2002). Representation of the quantity of visual items in the primate prefrontal cortex. *Science*, 297(5587), 1708-1711
- Nitsche, M.A., Cohen, L.G., Wassermann, E.M., Priori, A., Lang, N., Antal, A., ... & Pascual-Leone, A. (2008). Transcranial direct current stimulation: State of the art 2008. *Brain Stimulation*, 1(3), 206-223. Review
- Noël, M.-P., Seron, X., & Trovarelli, F. (2004). Working memory as a predictor of addition skills and addition strategies in children. *Current Psychology of Cognition*, 22, 3-25
- Nuerk, H.-C., Weger, U., & Willmes, K. (2001). Decade breaks in the mental number line? Putting the tens and units back in different bins. *Cognition*, 82(1), 25–33
- Núñez-Peña, M.I., Gracia-Bafalluy, M., & Tubau, E. (2011). Individual differences in arithmetic skill reflected in event-related brain potentials. *International Journal of Psychophysiology*, 80(2), 143-149
- O'Connor, N., Cowan, R., & Samella, K. (2000). Calendrical calculation and intelligence. *Intelligence*, 28(1), 31-48
- Ogawa, S., Lee, T.M., Kay, A.R., & Tank, D.W. (1990). Brain magnetic resonance imaging with contrast dependent on blood oxygenation. *Proceedings of the National Academy of Sciences of the United States of America*, 87, 9868-9872
- Ogawa, S., Tank, D.W., Menon, R., Ellermann, J.M., Kim, S.G., Merkle, H., & Ugurbil, K. (1992). Intrinsic signal changes accompanying sensory stimulation: functional brain mapping with magnetic resonance imaging. *Proceedings of the National Academy of Sciences of the United States of America*, 89, 5951-5955
- Opfer, J.E., & DeVries, J.M. (2008). Representational change and magnitude estimation: why young children can make more accurate salary comparisons than adults. *Cognition*, 108(3), 843-849
- Opfer, J. E., & Thompson, (2006). Even early representations of numerical magnitude are spatially organized: Evidence for a directional mapping bias in pre-reading preschoolers. In: R. Sun & N. Miyake (Eds.) *Proceedings of the 28th annual*

- conference of the Cognitive Science Society (pp. 639-644). Vancouver, BC: Cognitive Science Society
- Organization for Economic Co-operation and Development (OECD). (2010). *The High Cost of Low Educational Performance: The Long-Run economic impact of Improving Educational Outcomes*, Paris, 2010
- Osterling, J. & Dawson, G. (1994). Early recognition of children with autism: a study of first birthday home video tapes. *Journal of Autism and Developmental Disorders*, 24, 247-257
- Pagani, M., Manouilenko, I., Stone-Elander, S., Odh, R., Salmaso, D., Hatherly, R., ... & Bejerot, S. (2012). Brief report: alterations in cerebral blood flow as assessed by PET/CT in adults with Autism Spectrum Disorder with normal IQ. *Journal of Autism and Developmental Disorders*, 42(2), 313-318
- Paivio, A. (1991). Dual coding theory: Retrospect and current status. *Canadian Journal of Psychology*, 45, 255-287
- Paivio, A., Walsh, M., Bons, T. (1994). Concreteness effects on memory: When and why. *Journal of Experimental Psychology: Learning, Memory, and Cognition*, 20, 1196-1204
- Palladino, P., Cornoldi, C., De Beni, R., & Pazzaglia, F. (2001). Working memory and updating processes in reading comprehension. *Memory and Cognition*, 29, 344-354
- Palmen, S.J., Hulshoff Pol, H.E., Kemner, C., Schnack, H.G., Durston, S., Lahuus, B.E., ... & van Engeland, H. (2005). Increased gray-matter volume in medication-naïve high functioning children with autism spectrum disorder. *Psychological Medicine*, 35, 562-570
- Parsons, S., & Bynner, J. (2005). *Does numeracy matter more?* London: National Research and Development Centre for Adult Literacy and Numeracy, Institute of Education.
- Pascual-Leone, A., & Torres, F. (1993). Plasticity of the sensorimotor cortex representation of the reading finger in Braille readers. *Brain*, 116, 39-52
- Passolunghi, M.C., Siegel, L.S. (2001). Working memory and access to numerical information in children with disability in mathematics. *Journal of Experimental Child Psychology*, 88(4), 348-367
- Passolunghi, M.C., Vercelloni, B., & Schadee, H. (2007). The precursors of mathematics learning: Working memory, phonological ability and numerical competence. *Cognitive Development* 22(2), 165-184
- Perea, M., Duñabeitia, J.A., & Carreiras, M. (2008). R34D1NG W0RD5 W1TH NUMB3R5. *Journal of Experimental Psychology: Human Perception and Performance*, 34(1), 237-241
- Pereira, F., Mitchell, T., & Botvinick, M. (2009). Machine learning classifiers and fMRI: a tutorial overview. *NeuroImage*, 45, S199-S209

- Pesenti, M., Seron, X., Samson, D., & Duroux, B. (1999). Basic and exceptional calculation abilities in a calculating prodigy: a case study. *Mathematical Cognition*, 5, 97-148
- Pesenti, M., Zago, L., Crivello, F., Mellet, E., Samson, D., Duroux, B., ... & Tzourio-Mazoyer, N. (2001). Mental calculation in a prodigy is sustained by right prefrontal and medial temporal areas. *Nature Neuroscience*, 4(1), 103-107
- Piazza, M. (2010). Neurocognitive start-up tools for symbolic number representations. *Trends in Cognitive Sciences*, 14(12), 542-551. Review
- Piazza, M., Facoetti, A., Trussardi, A.N., Berteletti, I., Conte, S., Lucangeli, D., ... & Zorzi, M. (2010). Developmental trajectory of number acuity reveals a severe impairment in developmental dyscalculia. *Cognition*, 116(1), 33-41
- Piazza, M., & Izard, V. (2009). How humans count: numerosity and the parietal cortex. *The Neuroscientist*, 15(3), 261-273
- Piazza, M., Izard, V., Pinel, P., Le Bihan, D., & Dehaene, S. (2004). Tuning curves for approximate numerosity in the human intraparietal sulcus. *Neuron*, 44, 547-555
- Piazza, M., Mechelli, A., Butterworth, B., & Price, C. (2002). Are subitizing and counting: implemented as separate or functionally overlapping processes? *NeuroImage*, 15(2), 435-446
- Piazza, M., Mechelli, A., Price, C. J., & Butterworth, B. (2006). Exact and approximate judgements of visual and auditory numerosity: an fMRI study. *Brain Research*, 1106, 177-188
- Piazza M., Pinel, P., Le Bihane, D., & Dehaene, S. (2007). A magnitude code common to numerosities and number symbols in human intraparietal cortex. *Neuron*, 53(2), 293-305
- Pica, P., Lemer, C., Izard, V., & Dehaene, S. (2004). Exact and approximate arithmetic in an Amazonian indigene group. *Science*, 306(5695), 499-503
- Pickering, S., & Gathercole, S. (2001). *Working memory test battery for children*. London: The Psychological Corporation
- Pierce, K., & Redcay, E. (2008). Fusiform function in children with an autism spectrum disorder is a matter of "who". *Biological Psychiatry*, 64, 552-560
- Piffer L., Agrillo C., & Hyde D.C. (2011). Small and large number discrimination in guppies. *Animal Cognition*, doi: 10.1007/s10071-011-0447-9
- Piggot, J., Kwon, H., Mobbs, D., Blasey, C., Lotspeich, L., Menon, V., ... & Reiss, A.L. (2004). Emotional attribution in high-functioning individuals with autistic spectrum disorder: a functional imaging study. *Journal of the American Academy of Child and Adolescent Psychiatry*, 43, 473-480
- Polk, T.A., Reed, C.L., Keenan, J.M., Hogarth, P., & Anderson, C.A. (2002). A dissociation between symbolic number knowledge and analogues magnitude information. *Brain and Cognition*, 47(3), 545-563

- Price, C.J., & Friston, K.J. (2002). Degeneracy and cognitive anatomy. *Trends in Cognitive Sciences*, 6(10), 416-421
- Price, C.J., & Friston, K.J. (2003). Degeneracy and redundancy in cognitive anatomy. *Trends in Cognitive Sciences*, 7(4), 151-152
- Price, G., Holloway, I., Räsänen, P., Vesterinen, M., & Ansari, D. (2007). Impaired parietal magnitude processing in developmental dyscalculia. *Current Biology*, 17(24), R1042-R1043
- Räsänen, P., Salminen, J., Wilson, A.J., Aunio, P., & Dehaene, S. (2009). Computer-assisted intervention for children with low numeracy skills. *Cognitive Development*, 24, 450-472
- Reigosa-Crespo, V., Valdés-Sosa, M., Butterworth, B., Estévez, N., Rodríguez, M., Santos, E., ... & Lage, A. (2012). Basic numerical capacities and prevalence of developmental dyscalculia: The Havana Survey. *Developmental Psychology*, 48(1), 123-135
- Rice, C. (2009). Prevalence of autism spectrum disorders – Autism and Developmental Disabilities Monitoring Network, *MMWR Surveillance Summaries*, 58, 1-20
- Rickard, T.C., Romero, S.G., Basso, G., Wharton, C., Flitman, S., & Grafman, J. (2000). The calculating brain: an fMRI study. *Neuropsychologia*, 38(3), 325-335
- Ring, H., Baron-Cohen, S., Wheelwright, S., Williams, S., Brammer, M., Andrew, C., & Bullmore, E.T. (1999). Cerebral correlates of preserved cognitive skills in autism. A functional MRI study of Embedded Figures task performance. *Brain*, 122, 1305-1315
- Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: an iterative process. *Journal of Educational Psychology*, 91(2), 346-362
- Rivera, S.M., Menon, V., White, C.D., Glaser, B., & Reiss, A.L. (2002). Functional brain activation during arithmetic processing in females with fragile X Syndrome is related to FMR1 protein expression. *Human Brain Mapping*, 16(4), 206-218
- Rivera, S. M., Reiss, A. L., Eckert, M. A. & Menon, V. (2005). Developmental changes in mental arithmetic: evidence for increased functional specialization in the left inferior parietal cortex. *Cerebral Cortex* 15, 1779-1790
- Rizk-Jackson, A., Stoffers, D., Sheldon, S., Kuperman, J., Dale, A., Goldstein, J., ... & Aron, A. R. (2011). Evaluating imaging biomarkers for neurodegeneration in pre-symptomatic Huntington's disease using machine learning techniques. *NeuroImage*, 56, 788-796
- Rosenberg-Lee, M., Barth, M. & Menon, V. (2011). What difference does a year of schooling make? Maturation of brain response and connectivity between 2nd and 3rd grades during arithmetic problem solving. *NeuroImage*. 57(3), 796-808

- Rossetti, Y., Jacquin,-Courtois, S., Rode, G., Ota, H., Michel, C., & Boisson, D. (2004). Does action make the link between number and space representation? *Psychological Science*, 15, 426-430
- Rotzer S, Kucian K, Martin E, von Aster M, Klaver P, & Loenneker T. (2008). Optimized voxel-based morphometry in children with developmental dyscalculia. *NeuroImage*, 39(1), 417-422
- Rousselle, L., & Noël, M-P. (2007). Basic numerical skills in children with mathematics learning disabilities: a comparison of symbolic vs non-symbolic number magnitude processing. *Cognition*, 102, 361-395
- Rousselle, L., Palmers, E., & Noël, M-P. (2004). Magnitude comparison in preschoolers: what counts? Influence of perceptual variables. *Journal of Experimental Child Psychology*, 87, 57-84
- Rovet, J., Szekely, C., & Hockenberry, M.N. (1994). Specific arithmetic calculation deficits in children with Turner syndrome. *Journal of Clinical Experimental Neuropsychology*, 16(6), 820-839
- Rubinsten O. & Henik, A. (2006). Double dissociation of functions in developmental dyslexia and dyscalculia. *Journal of Educational Psychology*, 98(4), 854-867
- Rubinsten, O., & Henik, A. (2009). Developmental dyscalculia: heterogeneity might not mean different mechanisms. *Trends in Cognitive Sciences*, 13(2), 92-99
- Rubinsten, O., Henik, A., Berger, A., & Shahar-Shalev, S. (2002). The development of internal representations of magnitude and their association with Arabic numerals. *Journal of Experimental Child Psychology*, 81(1), 74-92
- Rueckert, L., Lange, N., Partiot, A., Appollonio, I., Litvan, I., Le Bihan, D., & Grafman J. (1996). Visualizing cortical activation during mental calculation with functional MRI. *NeuroImage*, 3(2), 97-103
- Russell, R.L., & Ginsburg, H.P. (1984). Cognitive analysis of pupils' mathematical difficulties. *Cognition and Instruction*, 1, 217-244
- Rykhlevskaia, E., Uddin, L.Q., Kondos, L., & Menon, V. (2009). Neuroanatomical correlates of developmental dyscalculia: combined evidence from morphometry and tractography. *Frontiers in Human Neuroscience*, 3(51), 1-13
- Sacks, O. (1986). *The Man Who Mistook His Wife for a Hat*. London: Picador, p. 185
- Schlaug, G., Jancke, L., Huang, Y.X., Staiger, J.F., & Steinmetz, H. (1995). Increased corpus callosum size in musicians. *Neuropsychologia*, 33, 1047-1055
- Schlaug, G., Jancke, L., Huang, Y.X., & Steinmetz, H. (1995). In-vivo evidence of structural brain asymmetry in musicians. *Science*, 267, 699-701
- Schneider, M., Grabner, R.H., & Paetsch, J. (2009). Mental number line, number line, number line estimation, and mathematical achievement: their interrelations in grades 5 and 6. *Journal of Educational Psychology*, 101(2), 359-372

- Schultz, R.T., Gauthier, I., Klin, A., Fulbright, R.K., Anderson, A.W., Volkmar, F., ... & Gore, J.C., (2000). Abnormal ventral temporal cortical activity during face discrimination among individuals with autism and Asperger syndrome. *Archives of General Psychiatry*, 57, 331-340
- Schultz R.T., Grelotti, D.J., Klin, A., Kleinman, J., Van der Gaag, C., Marois, R., & Skudlarski, P. (2003). The role of the fusiform face area in social cognition: implications for the pathobiology of autism. *Philosophical Transactions of the Royal Society B: Biological Sciences*, 358, 415-427
- Schumann, C.M., Hamstra, J., Goodlin-Jones, B.L., Lotspeich, L.J., Kwon, H., Buonocore, M.H., ... & Amaral, D.G., (2004). The amygdala is enlarged in children but not adolescents with autism; the hippocampus is enlarged at all ages. *The Journal of Neuroscience*, 24, 6392-6401
- Schwarz, W., & Ischebeck, A. (2003). On the relative speed account of number-size interference in comparative judgments of numerals. *Journal of Experimental Psychology: Human Perception and Performance*, 29(3), 507-522
- Seethaler, P.M., Fuchsm L,S, Starm J,R, & Bryantm, J. (2011). The cognitive predictors of computational skill with whole versus rational numbers: an exploratory study. *Learning and Individual Differences*, 21(5), 536-542
- Sekuler, R., & Mierkiewicz, D. (1977). Children's judgments of numerical inequality. *Child Development*, 48, 630-633
- Semenza, C., Miceli, L., & Girelli, L. (1997). A deficit for arithmetical procedures: Lack of knowledge or lack of monitoring? *Cortex*, 33, 483-498
- Semrud-Clikeman M., Fine, J.G., & Zhu D.C. (2011). The role of the right hemisphere for processing of social interactions in normal adults using functional magnetic resonance imaging. *Neuropsychobiology*, 64, 47-51
- Seyler, D.J., Kirk, E.P., & Ashcraft, M.H. (2003). Elementary subtraction. *Journal of Experimental Psychology: Learning, Memory and Cognition*, 29, 1339-1352
- Shah, A., & Frith, U. (1983). An islet of ability in autistic children: a research note. *Journal of Child Psychology and Psychiatry*, 24, 613-620
- Shalev, R.S. (2007). Prevalence of Developmental dyscalculia. In: D.B. Berch & M.M.M. Mazzocco (Eds.). *Why is math so hard for some children? The nature and origins of mathematical learning difficulties and disabilities* (pp. 49-60). Baltimore, MD: Paul H Brookes Publishing Co
- Shalev R.S., & Gross-Tsur, V. (1993). Developmental Dyscalculia and medical assessment. *Journal of Learning Disabilities*, 26, 134-137
- Shalev, R.S., & Gross-Tsur, V. (2001). Developmental dyscalculia. Review article. *Pediatric Neurology*, 24, 337-342
- Shalev, R.S., Manor, O., & Gross-Tsur, V. (2005). Developmental dyscalculia: a prospective six-year follow-up. *Developmental Medicine and Child Neurology*, 47(2), 121-125

- Shalev, R.S., Manor, O., Kerem, B., Ayali, M., Badichi, N., Friedlander, Y., & Gross-Tsur, V. (2001). Developmental dyscalculia is a familial learning disability. *Journal of Learning Disabilities*, 34(1), 59-65
- Shallice, T., & Evans, M.E. (1978). The involvement of the frontal lobes in cognitive estimation. *Cortex*, 14(2), 294-303
- Shumaker, R.W., Palkovich, A.M., Beck, B.B., Guagnano, G.A., & Morowitz, H. (2001). Spontaneous use of magnitude discrimination and ordination by the orangutan (*Pongopygmaeus*). *Journal of Comparative Psychology*, 115(4), 385-391
- Siegel, L. S. & Ryan, E. B. (1989). The development of working memory in normally achieving and subtypes of learning disabled children. *Child Development*, 60, 973-980
- Siegler, R.S. (1987). The perils of averaging data over strategies: an example from children's addition. *Journal of Experimental Psychology: General*, 116, 250-264
- Siegler, R.S. (1996). *Emerging minds: The process of change in children's thinking*. New York: Oxford University Press
- Siegler, R.S., & Booth, J.L. (2004). Development of numerical estimation in young children. *Child Development*, 75(2), 428-444
- Siegler, R. S., & Opfer, J. E. (2003). The development of numerical estimation: Evidence for multiple representations of numerical quantity. *Psychological Science*, 14, 237-243
- Smith, C. L., Solomon, G. E. A., & Carey, S. (2005). Never getting to zero: elementary school students' understanding of the infinite divisibility of number and matter. *Cognitive Psychology*, 51, 101-140
- Smith, S.B. (1983). *The great mental calculators: the psychology, methods, and lives of calculating prodigies*. New York: Columbia University Press
- Soltész, F., Szucs, D., Dékány, J., Márkus, A., & Csépe, V. (2007). A combined event-related potential and neuropsychological investigation of developmental dyscalculia. *Neuroscience Letters*, 417(2), 181-186
- Soulières, I., Zeffiro, T.A., Girard, M.L., & Mottron, L. (2011). Enhanced mental image mapping in autism. *Neuropsychologia*, 49(5), 848-857
- Spearman, C. (1904). "General intelligence" objectively determined and measured. *American Journal of Psychology*, 15, 201-293
- Spelke, E.S., & Kinzler K.D. (2007). Core knowledge. *Developmental Science*, 10, 89-96
- Stagg, C.J., Best, J.G., Stephenson, M.C., O'Shea, J., Wylezinska, M., Kincses, Z.T., ... & Johansen-Berg, H. (2009). Polarity-sensitive modulation of cortical neurotransmitters by transcranial stimulation. *The Journal of Neuroscience*, 29(16), 5202-5206

- Stagg, C.J., O'Shea, J., Kincses, Z.T., Woolrich, M., Matthews, P.M., & Johansen-Berg, H. (2009). Modulation of movement-associated cortical activation by transcranial direct current stimulation. *European Journal of Neuroscience*, 30(7), 1412-1423
- Stanfield, A.C., McIntosh, A.M., Spencer, M.D., Philip, R., Gaur, S., & Lawrie, S.M., (2008). Towards a neuroanatomy of autism: a systematic review and meta-analysis of structural magnetic resonance imaging studies. *European Psychiatry*, 23, 289-299
- Starkey, P., & Cooper, R. G., Jr. (1980). Perception of numbers by human infants. *Science*, 210, 1033-1035
- Starkey, P., Spelke, E.S., & Gelman, R. (1990). Numerical abstraction by human infants. *Cognition*, 36(2), 97-127
- Stigler, K.A., McDonald, B.C., Anand, A., Saykin, A.J., & McDougle, C.J. (2011). Structural and functional magnetic resonance imaging of autism spectrum disorders. *Brain Research*, 1380, 146-161
- Swanson, H.L., & Sachse-Lee, C., (2001). Mathematical problem solving and working memory in pupils with learning disabilities: Both executive and phonological processes are important. *Journal of Experimental Pupils Psychology*, 79, 294-321
- Takahata, K., & Kato, M. (2008). Neural mechanism underlying autistic savant and acquired savant syndrome. *Brain Nerve*, 60(7), 861-869
- Temple, C.M. (1994). The cognitive neuropsychology of the developmental dyscalculias. *Current Psychology of Cognition*, 13, 351-370
- Thompson, C.A., & Opfer, J.E. (2008). Costs and benefits of representational change: effects of context on age and sex differences in symbolic magnitude estimation. *Journal of Experimental Child Psychology*, 101(1), 20-51
- Tzelgov, J., Meyer, J., & Henik, A. (1992). Automatic and intentional processing of numerical information. *Journal of Experimental Psychology: Learning, Memory and Cognition*, 18, 166-179
- Tzelgov, J., Yehene, V., Kotler, L., & Alon, A. (2000). Automatic comparisons of artificial digits never compared: learning linear ordering relations. *Journal of Experimental Psychology: Learning, Memory and Cognition*, 26, 103-120
- Uddin, L.Q., Menon, V., Young, C.B., Ryali, S., Chen, T., Khouzam, A., ... & Hardan, A.Y. (2011). Multivariate searchlight classification of structural MRI in children and adolescents with autism. *Biological Psychiatry*, 70(9), 833-841
- Utz, K.S., Dimova, V., Oppenländer, K., & Kerkhoff, G. (2010). Electrified minds: transcranial direct current stimulation (tDCS) and galvanic vestibular stimulation (GVS) as methods of non-invasive brain stimulation in neuropsychology--a review of current data and future implications. *Neuropsychologia*, 48(10), 2789-2810

- Valente, G., Martino, F. D., Esposito, Goebel, R., & Formisano, E. (2011). Predicting subject-driven actions and sensory experience in a virtual world with relevance vector machine regression of fMRI data. *NeuroImage*, 56, 651-661
- van Eimeren, L., Grabner, R.H., Koschutnig, K., Reishofer, G., Ebner, F., & Ansari, D. (2010). Structure-function relationships underlying calculation: a combined diffusion tensor imaging and fMRI study. *NeuroImage*, 52(1), 358-363
- van Harskamp, N. J., Rudge, P., & Cipolotti, L. (2002). Are multiplication facts implemented by the left supramarginal and angular gyri? *Neuropsychologia*, 40, 1789-1793
- van Leeuwen, T.M., den Ouden, H.E.M., & Hagoort, P. (2011). Effective connectivity determines the nature of subjective experience in grapheme-color synesthesia. *The Journal of Neuroscience*, 31(27), 9879-9884
- van Loosbroek, E., & Smitsman, A. (1990). Visual perception of numerosity in infancy. *Developmental Psychology*, 26(6), 916-922
- van Rijn, S., Swaab, H., Baas, D., de Haan, E., Kahn, R.S., & Aleman, A. (2011). Neural systems for social cognition in Klinefelter syndrome (47, XXY): evidence from fMRI. *Social Cognitive and Affective Neuroscience*, doi:10.1093/scan/nsr04
- Verguts, T., & Fias, W. (2004). Representation of number in animals and humans: a neural model. *Journal of Cognitive Neuroscience*, 16(9), 1493-1504
- Volkmar, F.R., Lord, C., Bailey, A., Shultz, R.T., & Klin, A. (2004). Autism and pervasive developmental disorders. *Journal of Child Psychology and Psychiatry*, 45(1), 135-170
- von Aster, M.G., & Shalev, R.S. (2007). Number development and developmental dyscalculia. *Developmental Medicine and Child Neurology*, 49(11), 868-873
- Vuilleumier, P., Ortigue, S., & Brugger, P. (2004). The number space and neglect. *Cortex*, 40, 399-410
- Wagner, T., Valero-Cabre, A., & Pascual-Leone, A. (2007). Noninvasive human brain stimulation. *Annual Reviews of Biomedical Engineering*, 9, 527-565
- Waiter, G.D., Williams, J.H., Murray, A.D., Gilchrist, A., Perrett, D.I., & Whiten, A., (2004). A voxel-based investigation of brain structure in male adolescents with autistic spectrum disorder. *NeuroImage*, 22, 619-625
- Wallace, G.L., Happé, F., & Giedd, J.N. (2009). A case study of a multiply talented savant with an autism spectrum disorder: neuropsychological functioning and brain morphometry. *Philosophical Transactions of the Royal Society B: Biological Sciences*, 364(1522), 1425-1432
- Wang, A.T., Dapretto, M., Hariri, A.R., Sigman, M., & Bookheimer, S.Y., (2004). Neural correlates of facial affect processing in children and adolescents with autism spectrum disorder. *Journal of the American Academy of Child and Adolescence Psychiatry*, 43, 481-490

- Wang, Y., Fan, Y., Bhatt, P., & Davatzikos, C. (2010). High-dimensional pattern regression using machine learning: from medical images to continuous clinical variables. *NeuroImage*, 50, 1519-1535
- Ward, B.D. (2000). Simultaneous Inference for FMRI Data. In: *AFNI 3dDeconvolve Documentation*, Medical College of Wisconsin
- Ward, M., & Alar, N. (2000). 'Being Autistic is part of who I am'. *Focus on Autism and Other Developmental Disabilities*, 15, 232-235
- Wechsler, D. (1996). *Manual for the Wechsler Intelligence Scale for Children - 3rd Edition UK*. The Psychological Corporation. Harcourt Brace & Company, Publishers
- Wechsler, D. (2001). *The Wechsler Individual Achievement Test – Second Edition (WIAT-II)*: The Psychological Corporation
- Wendelken, C., Chung, D., & Bunge, S.A. (2011). Rostrolateral prefrontal cortex: domain-general or domain-sensitive? *Human Brain Mapping*, doi: 10.1002/hbm.21336
- Westermann, G., Thomas, M.S.C., & Karmiloff-Smith, A. (2010). Neuroconstructivism. *The Wiley-Blackwell Handbook of Childhood Cognitive Development* (pp. 723-748): Wiley-Blackwell
- Westman, E., Simmons, A., Zhang, Y., Muehlboeck, J.S., Tunnard, C., Liu, Y., ... & Wahlund, L.O. (2010). Multivariate analysis of MRI data for Alzheimer's disease, mild cognitive impairment and healthy controls. *NeuroImage*, 54(2), 1178-1187
- Whitehead, A.N. (1948). *An Introduction to Mathematics* (Originally published in 1911 Eds). London: Oxford University Press
- Wilson, A.J., & Dehaene, S. (2007). Number sense and developmental dyscalculia. In: *Human Behavior, Learning and the Developing Brain: Atypical Development* (Coch, D. et al., Eds). Pp. 212-238, Guilford Publications
- Wilson, A.J., Revkin, S.K., Cohen, D., Cohen, L., & Dehaene, S. (2006). An open trial assessment of 'The Number Race', an adaptive computer game for remediation of dyscalculia. *Behavioral and Brain Functions*, 2(20), 1-16
- Wilson, M.L., Hauser, M.D., & Wrangham, R.W. (2001). Does participation in intergroup conflict depend on numerical assessment, range location, or rank for wild chimpanzees? *Animal Behaviour*, 61(6), 1203-1216
- Wilson, S.M., Ogar, J.M., Laluz, V., Growdon, M., Jang, J., Glenn, S., ... & Gorno-Tempini, M.L. (2009). Automated MRI-based classification of primary progressive aphasia variants. *NeuroImage*, 47(4), 1558-1567
- World Health Organization. (1994). *International Classification of Diseases - 10* (Tenth ed.). Geneva, Switzerland: World Health Organization.

- Wu, S.S., Chang, T.T., Majid, A., Caspers, S., Eickhoff, S.B., & Menon, V. (2009). Functional heterogeneity of inferior parietal cortex during mathematical cognition assessed with cytoarchitectonic probability maps. *Cerebral Cortex*, 19(12), 2930-2945
- Wu, S.S., Meyer, M.L., Maeda, U., Salimpoor, V., Tomiyama, S., Geary, D.C., & Menon, V. (2008). Standardized assessment of strategy use and working memory in early mental arithmetic performance. *Developmental Neuropsychology*, 33(3), 365-393
- Wynn, K. (1992). Addition and subtraction by human infants. *Nature*, 358, 749-750
- Wynn, K. (1998). Psychological foundations of number: numerical competence in human infants. *Trends in Cognitive Sciences*, 2, 296-303
- Wynn, K., Bloom, P., & Chiang, W.C. (2002). Enumeration of collective entities by 5-month-old infants. *Cognition*, 83(3), B55-B62
- Xu, F. (2003). Numerosity discrimination in infants: evidence for two systems of representations. *Cognition*, 89(1), B15-B25
- Xu, F., & Spelke, E. S. (2000). Large number discrimination in 6-month-old infants. *Cognition*, 74(1), B1-B11
- Yeo, D. (2003). A brief overview of some contemporary methodologies in primary maths. In: M. Johnson & L. Peer (Eds.), *The Dyslexia Handbook*. Reading: British Dyslexia Association
- Yoncheva, Y.N., Blau, V.C., Maurer, U., & McCandliss, B.D. (2010). Attentional focus during learning impacts N170 ERP responses to an artificial script. *Developmental Neuropsychology*, 35(4), 423-445
- Zago, L., Pesenti, M., Mellet, E., Crivello, F., Mazoyer, B., & Tzourio-Mazoyer, N. (2001). Neural correlates of simple and complex mental calculation. *NeuroImage* 13, 314-327
- Zamarian, L., Ischebeck A., & Delazer, M. (2009). Neuroscience of learning arithmetic - evidence from brain imaging studies. *Neuroscience and Biobehavioral Reviews*, 33(6), 909-925
- Zorzi, M., & Butterworth, B. (1999). A computational model of number comparison. In: M. Hahn, & S.C. Stoness (Ed.). *Proceedings of the Twenty First Annual Meeting of the Cognitive Science Society*. Mahwah, NJ: LEA
- Zorzi, M., Priftis, K., & Umiltà, C. (2002). Brain damage: neglect disrupts the mental number line. *Nature*, 417, 138-139
- Zorzi, M., Stoianov, I., & Umiltà, C. (2005). Computational modelling of numerical cognition. In: J. I. D. Campbell (Ed.), *Handbook of Mathematical Cognition* (pp. 67-84). Hove: Psychology Press

Appendix 1A

Sandwell Early Numeracy Test Revised (SENT–R)

Report form – Format 2 (five strands)

Name:

Date of birth:

School:

Year group:

Date:

XXXX completed the *Sandwell Early Numeracy Test Revised* Form to identify areas for development.

The test looks at understanding and skills within five strands and enables levelling between P6 and Level 2A.

XXXX was able to:

XXXX was inconsistent with:

XXXX was unable to:

The five strands are:

1. Identification of numbers

- Identify numbers at random to 3 [item 1: Level P6]
- Identify numbers at random to 6 [item 8: Level P7]
- Identify numbers at random to 9 [item 15: Level P8]
- Write numbers at random to 10 [item 19: Level 1C]
- Read numbers at random to 10 [item 20: Level 1C]
- Sort a selection of numbers within 20 as odd or even [item 39: Level 2C]
- Write a selection of random numbers from 10 to 50 [item 41: Level 2B]
- Value of number in the tens column [item 57: Level 2A]
- Draw a block graph to represent given information [item 61: Level 2A]
- Write an amount of money given in pounds and pence [item 64: Level 2A]
- Identify the values of columns on a block graph and use the information to answer ‘how many more/less than’ problems [item 67: Level 2A]

2. Oral counting

- Count by rote to 5 with a ‘start’ prompt [item 2: Level P6]
- Count by rote to 10 with a ‘start’ prompt [item 7: Level P7]
- Count by rote beyond 10 [item 13: Level P8]
- Count by rote to 20 with a ‘start’ prompt [item 21: Level 1C]
- Count backwards from 10 to 0 [item 26: Level 1C]
- Count on by rote from a given number to 10 [item 29: Level 1B]
- Count on by rote from a given number to at least 17 [item 31: Level 1A]
- Count on in tens from 0 to 100 [item 40: Level 2B]
- Count backwards from 20 to a given number [item 42: Level 2B]
- Count on in twos [item 46: Level 2B]

3. Value/computation

- Number/object matching to 3 [item 6: Level P7]
- Number/object matching to 5 [item 11: Level P7]

- Count a set and identify the correct number to 10 [item 18: Level P8]
- Give the total of two sets of objects to 10 [item 22: Level 1C]
- Find the total when taking away from a set to 10 [item 23: Level 1C]
- Count and record the number of objects in sets to 10 [item 24: Level 1C]
- Adding and subtracting 2 or 1 more or less, orally within 10 [item 27: Level 1B]
- Order a selection of numbers from smallest to biggest to 10 [item 28: Level 1B]
- Oral addition problems: units within 10 [item 30: Level 1B]
- Identify which column on a block graph represents a number of items shown pictorially [item 35: Level 2C]
- Oral addition problems: 10 add a unit [item 36: Level 2C]
- Count sets of 2 to at least 12 [item 37: Level 2C]
- Read and answer addition sums to 10 [item 38: Level 2C]
- Inverse operation: addition and subtraction within 10 [item 43: Level 2B]
- Continue and write number patterns, counting on in fives, tens and back in twos and tens [item 44: Level 2B]
- Calculate the missing number in an addition sum to 10 [item 47: Level 2B]
- Recognise coins to 50p. Make an identified amount to 50p using the coins [item 49: Level 2B]
- Number problem: doubling [item 50: Level 2B]
- Order a selection of numbers from smallest to biggest to 100 [item 51: Level 2B]
- Number problem: multiplication [item 52: Level 2A]
- Calculate the missing number in an addition sum to 20 [item 54: Level 2A]
- Number problem: division [item 55: Level 2A]
- Number problem: halving [item 56: Level 2A]
- Identify the greatest amount of money from a selection given in pence and pounds and pence [item 58: Level 2A]
- Read a two-digit number and identify the value of the numbers in the tens and units columns [item 59: Level 2A]
- Addition of multiples of 10 to 100 [item 60: Level 2A]
- Number problem: estimation [item 62: Level 2A]
- Number problem: multiplication [item 63: Level 2A]
- Give the total of a set of coins up to £2 [item 65: Level 2A]
- Number problem: subtraction [item 66: Level 2A]
- Number problem: division [item 68: Level 2A]

4. Object counting

- Count out objects to 5 with 1–1 correspondence [item 3: Level P6]
- Count objects to 5 with 1–1 correspondence [item 4: Level P6]
- Count out objects to 10 with 1–1 correspondence [item 12: Level P7]
- Count a small set of objects and recall the number when the set is removed [item 16: Level P8]

5. Language

- Select the set with ‘more’ [item 5: Level P6]
- Select the set with ‘less’ [item 9: Level P7]
- Identify the set with ‘more’ and the set with ‘less’ [item 10: Level P7]
- Identify ‘first’ and ‘last’ in a group [item 14: Level P8]
- Identify ‘second’ and ‘third’ in a group [item 17: Level P8]
- Select the set with the ‘greater’ number [item 25: Level 1C]
- Select the set with the ‘least’ number [item 32: Level 1A]
- Identify less than half of an object [item 33: Level 1A]
- Relate ‘subtract’, ‘minus’, ‘take away’, ‘more’, ‘add’, ‘plus’ to ‘+’ and ‘–’ symbols [item 34: Level 1A]
- Order a selection of numbers from smallest to biggest to 20 [item 45: Level 2B]
- Calculate halves and doubles of numbers within 20 [item 48: Level 2B]
- Sort shapes into a grid according to shape and colour [item 53: Level 2A]

Appendix 1B

Sandwell Early Numeracy Test Revised (SENT-R)

Report form – Format 1 (list of items)

Name:

Date of birth:

School:

Year group:

Date:

XXXX completed the *Sandwell Early Numeracy Test Revised* Form to identify areas for development.

The test looks at understanding and skills within five strands and enables levelling between P6 and Level 2A.

XXXX was able to:

XXXX was inconsistent with:

XXXX was unable to:

1. Identify numbers at random to 3 [Level P6]
2. Count by rote to 5 with a 'start' prompt [Level P6]
3. Count out objects to 5 with 1–1 correspondence [Level P6]
4. Count objects to 5 with 1–1 correspondence [Level P6]
5. Select the set with 'more' [Level P6]
6. Number/object matching to 3 [Level P7]
7. Count by rote to 10 with a 'start' prompt [Level P7]
8. Identify numbers at random to 6 [Level P7]
9. Select the set with 'less' [Level P7]
10. Identify the set with 'more' and the set with 'less' [Level P7]
11. Number/object matching to 5 [Level P7]
12. Count out objects to 10 with 1–1 correspondence [Level P7]
13. Count by rote beyond 10 [Level P8]
14. Identify 'first' and 'last' in a group [Level P8]
15. Identify numbers at random to 9 [Level P8]
16. Count a small set of objects and recall the number when the set is removed [Level P8]
17. Identify 'second' and 'third' in a group [Level P8]
18. Count a set and identify the correct number to 10 [Level P8]
19. Write numbers at random to 10 [Level 1C]
20. Read numbers at random to 10 [Level 1C]
21. Count by rote to 20 with a 'start' prompt [Level 1C]
22. Give the total of two sets of objects to 10 [Level 1C]
23. Find the total when taking away from a set to 10 [Level 1C]
24. Count and record the number of objects in sets to 10 [Level 1C]
25. Select the set with the 'greater' number [Level 1C]
26. Count backwards from 10 to 0 [Level 1C]
27. Adding and subtracting 2/1 more or less, orally within 10 [Level 1B]
28. Order a selection of numbers from smallest to biggest to 10 [Level 1B]
29. Count on by rote from a given number to 10 [Level 1B]
30. Oral addition problems: units within 10 [Level 1B]

31. Count on by rote from a given number to at least 17 [Level 1A]
32. Select the set with the 'least' number [Level 1A]
33. Identify less than half of an object [Level 1A]
34. Relate 'subtract', 'minus', 'take away', 'more', 'add', 'plus' to '+' and '-' symbols [Level 1A]
35. Identify which column on a block graph represents a number of items shown pictorially [Level 2C]
36. Oral addition problems: 10 add a unit [Level 2C]
37. Count sets of 2 to at least 12 [Level 2C]
38. Read and answer addition sums to 10 [Level 2C]
39. Sort a selection of numbers within 20 as odd or even [Level 2C]
40. Count on in tens from 0 to 100 [Level 2B]
41. Write a selection of random numbers from 10 to 50 [Level 2B]
42. Count backwards from 20 to a given number [Level 2B]
43. Inverse operation: addition and subtraction within 10 [Level 2B]
44. Continue and write number patterns, counting on in fives, tens and back in twos and tens [Level 2B]
45. Order a selection of numbers from smallest to biggest to 20 [Level 2B]
46. Count on in twos [Level 2B]
47. Calculate the missing number in an addition sum to 10 [Level 2B]
48. Calculate halves and doubles of numbers within 20 [Level 2B]
49. Recognise coins to 50p. Make an identified amount to 50p using the coins [Level 2B]
50. Number problem: doubling [Level 2B]
51. Order a selection of numbers from smallest to biggest to 100 [Level 2B]
52. Number problem: multiplication [Level 2A]
53. Sort shapes into a grid according to shape and colour [Level 2A]
54. Calculate the missing number in an addition sum to 20 [Level 2A]
55. Number problem: division [Level 2A]
56. Number problem: halving [Level 2A]
57. Value of number in the tens column [Level 2A]
58. Identify the greatest amount of money from a selection given in pence and pounds and pence [Level 2A]
59. Read a two-digit number and identify the value of the numbers in the tens and units columns [Level 2A]
60. Addition of multiples of 10 to 100 [Level 2A]
61. Draw a block graph to represent given information [Level 2A]
62. Number problem: estimation [Level 2A]
63. Number problem: multiplication [Level 2A]
64. Write an amount of money given in pounds and pence [Level 2A]
65. Give the total of a set of coins up to £2 [Level 2A]
66. Number problem: subtraction [Level 2A]
67. Identify the values of columns on a block graph and use the information to answer 'how many more/less than' problems [Level 2A]
68. Number problem: division [Level 2A]